6j. Colorimetry

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6j-1. Luminosity. *Photopic Luminosity.* RELATIVE PHOTOPIC LUMINOSITY ($\bar{y}$): Adopted in 1931 by International Commission on Illumination (C.I.E.)(intended to represent normal eyes, for fields subtending about 2 deg, having about 1 millilambert luminance).

ABSOLUTE PHOTOPIC LUMINOSITY (K_λ lumens per watt): 680 times photopic luminosities given in Table 6j-1.

LUMINOUS FLUX (lumens):

$$F = \sum_{\lambda=380}^{770} P_\lambda K_\lambda$$

for spectral distribution of radiant energy, P_λ (watts per 5-nm-wavelength band).

LUMINOUS TRANSMITTANCE:

$$t = \frac{\sum_{\lambda=380}^{\lambda=770} \tau_\lambda P_\lambda K_\lambda}{\sum_{\lambda=380}^{\lambda=770} P_\lambda K_\lambda}$$

or

$$t = \frac{\sum_{\lambda=380}^{\lambda=770} \tau_\lambda P_\lambda \bar{y}}{\sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{y}}.$$

for material with spectral transmittance τ_λ irradiated with spectral distribution P_λ.

LUMINOUS REFLECTANCE r: Substitute spectral reflectance ρ_λ for τ_λ in either of above.

Revisions of photopic relative luminosity data, recommended in 1951 by the United States Technical Committee on Colorimetry of C.I.E.:

370 nm 0.0001	380 0.0004	390 0.0015	400 0.0045	410 0.0093	420 0.0175	430 0.0273	440 0.0379	450 0.0468

These revisions have not been adopted by C.I.E.

Scotopic Luminosity. RELATIVE VALUES V' (Table 6j-1): Adopted in 1951 by C.I.E. (intended to represent normal eyes of young subjects, age ≤ 30, when observing at angles of not less than 5 deg from foveal center, under conditions of complete dark adaptation).

INTERNATIONAL PHOTOMETRIC STANDARD: Blackbody at temperature (2045 K) of solidification of platinum has luminance of 60 candela/cm^2 for both scotopic and photopic conditions.

ABSOLUTE SCOTOPIC LUMINOSITY K_λ': 1,725 times scotopic luminosities given in Table 6j-1.

SCOTOPIC LUMENS, SCOTOPIC, LUMINOUS TRANSMITTANCE, AND SCOTOPIC LUMINOUS REFLECTANCE: Substitute K_λ', V', for scotopic luminosity in formulas for corresponding photopic quantities.

6j-2. Standard Illuminants for Colorimetry. The C.I.E. makes a distinction between *illuminant* and *source*. *Source* refers to a physical emitter of light, such as a lamp or the sun and sky. *Illuminant* refers to a specific spectral power distribution, not necessarily provided directly by a source. The definitions of the standard sources are considered secondary, as it is conceivable that new developments in lamps and filters will bring about improved standard sources that represent the standard illuminants more accurately. STANDARD ILLUMINANT A is the spectral power distribution from a full (planckian, blackbody) radiator at 2858.7 K (IPTS, 1968, $c_2 = 0.014388$ m · K). STANDARD SOURCE A is a gas-filled, coiled-coil tungsten-filament lamp operating at a correlated color temperature of 2855.5 K. A lamp with a fused-quartz envelope or window is recommended if the spectral power distribution of illuminant A must be closely approximated in the ultraviolet. No recommendation has been made for a standard source representing standard illuminant D_{6500}.

STANDARD SOURCES B AND C and their spectral power distributions are omitted here because the sources are obsolete and their distributions are obsolescent. These sources, distributions, and colorimetric computation data based on them were published in the first and second editions of this Handbook (Tables 6j-4, 6j-5, and 6j-6). For general use in colorimetry, illuminants A and D_{6500} given in the new table 6j-2 should suffice.

STANDARD ILLUMINANT D_{6500} represents a phase of daylight with a correlated color temperature of approximately 6504 K. (The odd 4 K resulted from the change of the International Practical Temperature Scale, made by the Comité International des Poids et Mesures (CIPM) in 1968, after all details of illuminant D_{6500} had been adopted by the C.I.E., which had used the value of $c_2 = 0.014380$ m · K promulgated by the CIPM in 1960.)

OTHER ILLUMINANTS D: Whenever a phase of daylight other than D_{6500} is desired, the following rules may be used to define it:

1. *Chromaticity.* The C.I.E. 1931 x, y chromaticity coordinates of the daylight to be defined must satisfy $y_D = -3x_D^2 + 2.87x_D - 0.275$ and $0.25 \leq x_D \leq 0.38$.

TABLE 6j-1. PHOTOPIC AND SCOTOPIC LUMINOSITY DATA*

Wavelength, nm	Photopic $\bar{y}$	Scotopic V'	Wavelength, nm	Photopic $\bar{y}$	Scotopic V'
380	0.0000	0.00059	580	0.8700	0.1212
385	0.0001	0.00111	585	0.8163	0.0899
390	0.0001	0.00221	590	0.7570	0.0655
395	0.0002	0.00453	595	0.6949	0.0469
400	0.0004	0.00929	600	0.6310	0.03325
405	0.0006	0.01850	605	0.5668	0.02312
410	0.0012	0.03484	610	0.5030	0.01593
415	0.0022	0.0604	615	0.4412	0.01088
420	0.0040	0.0966	620	0.3810	0.00737
425	0.0073	0.1436	625	0.3210	0.00497
430	0.0116	0.1998	630	0.2650	0.003335
435	0.0168	0.2625	635	0.2170	0.002235
440	0.0230	0.3281	640	0.1750	0.001497
445	0.0298	0.3931	645	0.1382	0.001005
450	0.0380	0.4550	650	0.1070	0.000677
455	0.0480	0.5129	655	0.0816	0.000459
460	0.0600	0.5672	660	0.0610	0.0003129
465	0.0739	0.6205	665	0.0446	0.0002146
470	0.0910	0.6756	670	0.0320	0.0001480
475	0.1126	0.7337	675	0.0232	0.0001026
480	0.1390	0.7930	680	0.0170	0.0000716
485	0.1693	0.8509	685	0.0119	0.0000502
490	0.2080	0.9043	690	0.0082	0.00003533
495	0.2586	0.9491	695	0.0057	0.00002502
500	0.3230	0.9817	700	0.0041	0.00001780
505	0.4073	0.9984	705	0.0029	0.00001273
510	0.5030	0.9966	710	0.0021	0.00000914
515	0.6082	0.9750	715	0.0015	0.00000660
520	0.7100	0.9352	720	0.0010	0.00000478
525	0.7932	0.8796	725	0.0007	0.000003482
530	0.8620	0.8110	730	0.0005	0.000002546
535	0.9149	0.7332	735	0.0004	0.000001870
540	0.9540	0.6497	740	0.0003	0.000001379
545	0.9803	0.5644	745	0.0002	0.000001022
550	0.9950	0.4808	750	0.0001	0.000000760
555	1.0002	0.4015	755	0.0001	0.000000567
560	0.9950	0.3288	760	0.0001	0.000000425
565	0.9786	0.2639	765	0.0000	0.000000320
570	0.9520	0.2076	770	0.0000	0.000000241
575	0.9154	0.1602	775	0.0000	0.000000183
			780	......	0.000000139

* "The Science of Color." Available only from Optical Society of America, 2100 Pennsylvania Avenue NW, Washington, D.C. 20037.

Table 6j-2a. Spectral Power Distributions of C.I.E. Illuminants A and D_{6500}

λ, nm	A, $S(\lambda)$	D_{6500}, $S(\lambda)$	λ, nm	A, $S(\lambda)$	D_{6500}, $S(\lambda)$
300	0.93	0.03	575	110.80	96.1
05	1.13	1.7	80	114.44	95.8
10	1.36	3.3	85	118.08	92.2
15	1.62	11.8	90	121.73	88.7
20	1.93	20.2	95	125.39	89.3
325	2.27	28.6	600	129.04	90.0
30	2.66	37.1	05	132.70	89.8
35	3.10	38.5	10	136.34	89.6
40	3.59	39.9	15	139.99	88.6
45	4.14	42.4	20	143.62	87.7
350	4.74	44.9	625	147.23	85.5
55	5.41	45.8	30	150.83	83.3
60	6.15	46.6	35	154.42	83.5
65	6.95	49.4	40	157.98	83.7
70	7.82	52.1	45	161.51	81.9
375	8.77	51.0	650	165.02	80.0
80	9.80	50.0	55	168.51	80.1
85	10.90	52.3	60	171.96	80.2
90	12.09	54.6	65	175.38	81.2
95	13.36	68.7	70	178.76	82.3
400	14.71	82.8	675	182.11	80.3
05	16.15	87.1	80	185.42	78.3
10	17.68	91.5	85	188.70	74.0
15	19.29	92.5	90	191.93	69.7
20	21.00	93.4	95	195.11	70.7
425	22.79	90.1	700	198.26	71.6
30	24.67	86.7	05	201.35	73.0
35	26.64	95.8	10	204.40	74.3
40	28.70	104.9	15	207.40	68.0
45	30.85	110.9	20	210.36	61.6
450	33.09	117.0	725	213.26	65.7
55	35.41	117.4	30	216.11	69.9
60	37.81	117.8	35	218.91	72.5
65	40.30	116.3	40	221.66	75.1
70	42.87	114.9	45	224.35	69.3
475	45.52	115.4	750	226.99	63.6
80	48.24	115.9	55	229.58	55.0
85	51.04	112.4	60	232.11	46.4
90	53.91	108.8	65	234.58	56.6
95	56.85	109.1	70	237.00	66.8
500	59.86	109.4	775	239.36	65.1
05	62.93	108.6	80	241.67	63.4
10	66.06	107.8	85	243.91	63.8
15	69.25	106.3	90	246.11	64.3
20	72.50	104.8	95	248.24	61.9
525	75.79	106.2	800	250.32	59.5
30	79.13	107.7	05	252.34	55.7
35	82.52	106.0	10	254.30	52.0
40	85.95	104.4	15	256.21	54.7
45	89.41	104.2	20	258.06	57.4
550	92.91	104.0	825	259.85	58.9
55	96.44	102.0	30	261.59	60.3
60	100.00	100.0			
65	103.58	98.2			
70	107.18	96.3			

TABLE 6j-2b. SPECTRAL POWER DISTRIBUTIONS OF ILLUMINANTS D_{5500} AND D_{7500}

λ nm	D_{5500}, $S(\lambda)$	D_{7500}, $S(\lambda)$	λ nm	D_{5500}, $S(\lambda)$	D_{7500}, $S(\lambda)$
300	0.02	0.04	600	94.4	87.2
310	2.1	5.1	610	95.1	86.1
320	11.2	29.8	620	94.2	83.6
330	20.6	54.9	630	90.4	78.7
340	23.9	57.3	640	92.3	78.4
350	27.8	62.7	650	88.9	74.8
360	30.6	63.0	660	90.3	74.3
370	34.3	70.3	670	93.9	75.4
380	32.6	66.7	680	90.0	71.6
390	38.1	70.0	690	79.7	63.9
400	61.0	101.9	700	82.8	65.1
410	68.6	111.9	710	84.8	68.1
420	71.6	112.8	720	70.2	56.4
430	67.9	103.1	730	79.3	64.2
440	85.6	121.2	740	85.0	69.2
450	98.0	133.0	750	71.9	58.6
460	100.5	132.4	760	52.8	42.6
470	99.9	127.3	770	75.9	61.4
480	102.7	126.8	780	71.8	58.3
490	98.1	117.8	790	72.9	59.1
500	100.7	116.6	800	67.3	54.7
510	100.7	113.7	810	58.7	47.9
520	100.0	108.7	820	65.0	52.9
530	104.2	110.4	830	68.3	55.5
540	102.1	106.3			
550	103.0	104.9			
560	100.0	100.0			
570	97.2	95.6			
580	97.7	94.2			
590	91.4	87.0			

TABLE 6j-3. SPECTRAL POWER FUNCTIONS FOR COMPOSING ILLUMINANTS D

λ, nm	$P_0(\lambda)$	$P_1(\lambda)$	$P_2(\lambda)$	λ, nm	$P_0(\lambda)$	$P_1(\lambda)$	$P_2(\lambda)$
300	0.04	0.02	0.0	600	90.5	− 5.8	3.2
310	6.0	4.5	2.0	610	90.3	− 7.2	4.1
320	29.6	22.4	4.0	620	88.4	− 8.6	4.7
330	55.3	42.0	8.5	630	84.0	− 9.5	5.1
340	57.3	40.6	7.8	640	85.1	−10.9	6.7
350	61.8	41.6	6.7	650	81.9	−10.7	7.3
360	61.5	38.0	5.3	660	82.6	−12.0	8.6
370	68.8	42.4	6.1	670	84.9	−14.0	9.8
380	63.4	38.5	3.0	680	81.3	−13.6	10.2
390	65.8	35.0	1.2	690	71.9	−12.0	8.3
400	94.8	43.4	−1.1	700	74.3	−13.3	9.6
410	104.8	46.3	−0.5	710	76.4	−12.9	8.5
420	105.9	43.9	−0.7	720	63.3	−10.6	7.0
430	96.8	37.1	−1.2	730	71.7	−11.6	7.6
440	113.9	36.7	−2.6	740	77.0	−12.2	8.0
450	125.6	35.9	−2.9	750	65.2	−10.2	6.7
460	125.5	32.6	−2.8	760	47.7	− 7.8	5.2
470	121.3	27.9	−2.6	770	68.6	−11.2	7.4
480	121.3	24.3	−2.6	780	65.0	−10.4	6.8
490	113.5	20.1	−1.8	790	66.0	−10.6	7.0
500	113.1	16.2	−1.5	800	61.0	− 9.7	6.4
510	110.8	13.2	−1.3	810	53.3	− 8.3	5.5
520	106.5	8.6	−1.2	820	58.9	− 9.3	6.1
530	108.8	6.1	−1.0	830	61.9	− 9.8	6.5
540	105.3	4.2	−0.5				
550	104.4	1.9	−0.3				
560	100.0	0.0	0.0				
570	96.0	− 1.6	0.2				
580	95.1	− 3.5	0.5				
590	89.1	− 3.5	2.1				

TABLE 6j-4. CHROMATICITIES AND MULTIPLIERS FOR COMPOSING ILLUMINANTS D

T_c*	x_D	y_D	M_1	M_2
4000	0.3823	0.3838	−1.505	2.827
4100	0.3779	0.3812	−1.464	2.460
4200	0.3737	0.3786	−1.422	2.127
4300	0.3697	0.3760	−1.378	1.825
4400	0.3658	0.3734	−1.333	1.550
4500	0.3621	0.3709	−1.286	1.302
4600	0.3585	0.3684	−1.238	1.076
4700	0.3551	0.3659	−1.190	0.871
4800	0.3519	0.3634	−1.140	0.686
4900	0.3487	0.3610	−1.090	0.518
5000	0.3457	0.3587	−1.040	0.367
5100	0.3429	0.3564	−0.989	0.230
5200	0.3401	0.3541	−0.939	0.106
5300	0.3375	0.3519	−0.888	−0.005
5400	0.3349	0.3497	−0.837	−0.105
5500	0.3325	0.3476	−0.786	−0.195
5600	0.3302	0.3455	−0.736	−0.276
5700	0.3279	0.3435	−0.685	−0.348
5800	0.3258	0.3416	−0.635	−0.412
5900	0.3237	0.3397	−0.586	−0.469
6000	0.3217	0.3378	−0.536	−0.519
6100	0.3198	0.3360	−0.487	−0.563
6200	0.3179	0.3342	−0.439	−0.602
6300	0.3161	0.3325	−0.391	−0.635
6400	0.3144	0.3308	−0.343	−0.664
6500	0.3128	0.3292	−0.296	−0.688
6600	0.3112	0.3276	−0.250	−0.709
6700	0.3097	0.3260	−0.204	−0.726
6800	0.3082	0.3245	−0.159	−0.739
6900	0.3067	0.3231	−0.114	−0.749
7000	0.3054	0.3216	−0.070	−0.757
7100	0.3040	0.3202	−0.026	−0.762
7200	0.3027	0.3189	0.017	−0.765
7300	0.3015	0.3176	0.060	−0.765
7400	0.3003	0.3163	0.102	−0.763
7500	0.2991	0.3150	0.144	−0.760
7600	0.2980	0.3138	0.184	−0.755
7700	0.2969	0.3126	0.225	−0.748
7800	0.2958	0.3115	0.264	−0.740
7900	0.2948	0.3103	0.303	−0.730
8000	0.2938	0.3092	0.342	−0.720
8100	0.2928	0.3081	0.380	−0.708
8200	0.2919	0.3071	0.417	−0.695
8300	0.2910	0.3061	0.454	−0.682
8400	0.2901	0.3051	0.490	−0.667
8500	0.2892	0.3041	0.526	−0.652
9000	0.2853	0.2996	0.697	−0.566
9500	0.2818	0.2956	0.856	−0.471
10000	0.2788	0.2920	1.003	−0.369
10500	0.2761	0.2887	1.139	−0.265
11000	0.2737	0.2858	1.266	−0.160
12000	0.2697	0.2808	1.495	0.045
13000	0.2664	0.2767	1.693	0.239
14000	0.2637	0.2732	1.868	0.419
15000	0.2614	0.2702	2.021	0.586
17000	0.2578	0.2655	2.278	0.878
20000	0.2539	0.2603	2.571	1.231
25000	0.2499	0.2548	2.907	1.655
5503†	0.3324	0.3475	−0.785	−0.195
6504‡	0.3127	0.3291	−0.295	−0.689
7504§	0.2990	0.3150	0.145	−0.760

* All correlated color temperatures T_c are based on c_2 = 0.014388 m·K.
† Standard illuminant D_{5500}; T_c = 5500 × 1.4388/1.4380 = 5503 K (approximately).
‡ Standard illuminant D_{6500}; T_c = 6500 × 1.4388/1.4380 = 6504 K (approximately).
§ Standard illuminant D_{7500}; T_c = 7500 × 1.4388/1.4380 = 7504 K (approximately).

TABLE 6j-5. COLOR-MIXTURE FUNCTIONS FOR SMALL AND LARGE FIELDS

λ, nm	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$
380	0.0014	0.0000	0.0065	0.0002	0.0000	0.0007
385	0.0022	0.0001	0.0105	0.0007	0.0001	0.0029
390	0.0042	0.0001	0.0201	0.0024	0.0003	0.0105
395	0.0076	0.0002	0.0362	0.0072	0.0008	0.0323
400	0.0143	0.0004	0.0679	0.0191	0.0020	0.0860
405	0.0232	0.0006	0.1102	0.0434	0.0045	0.1971
410	0.0435	0.0012	0.2074	0.0847	0.0088	0.3894
415	0.0776	0.0022	0.3713	0.1406	0.0145	0.6568
420	0.1344	0.0040	0.6456	0.2045	0.0214	0.9725
425	0.2148	0.0073	1.0391	0.2647	0.0295	1.2825
430	0.2839	0.0116	1.3856	0.3147	0.0387	1.5535
435	0.3285	0.0168	1.6230	0.3577	0.0496	1.7985
440	0.3483	0.0230	1.7471	0.3837	0.0621	1.9673
445	0.3481	0.0298	1.7826	0.3867	0.0747	2.0273
450	0.3362	0.0380	1.7721	0.3707	0.0895	1.9948
455	0.3187	0.0480	1.7441	0.3430	0.1063	1.9007
460	0.2908	0.0600	1.6692	0.3023	0.1282	1.7454
465	0.2511	0.0739	1.5281	0.2541	0.1528	1.5549
470	0.1954	0.0910	1.2876	0.1956	0.1852	1.3176
475	0.1421	0.1126	1.0419	0.1323	0.2199	1.0302
480	0.0956	0.1390	0.8130	0.0805	0.2536	0.7721
485	0.0580	0.1693	0.6162	0.0411	0.2977	0.5701
490	0.0320	0.2080	0.4652	0.0162	0.3391	0.4153
495	0.0147	0.2586	0.3533	0.0051	0.3954	0.3024
500	0.0049	0.3230	0.2720	0.0038	0.4608	0.2185
505	0.0024	0.4073	0.2123	0.0154	0.5314	0.1592
510	0.0093	0.5030	0.1582	0.0375	0.6067	0.1120
515	0.0291	0.6082	0.1117	0.0714	0.6857	0.0822
520	0.0633	0.7100	0.0782	0.1177	0.7618	0.0607
525	0.1096	0.7932	0.0573	0.1730	0.8233	0.0431
530	0.1655	0.8620	0.0422	0.2365	0.8752	0.0305
535	0.2257	0.9149	0.0298	0.3042	0.9238	0.0206
540	0.2904	0.9540	0.0203	0.3768	0.9620	0.0137
545	0.3597	0.9803	0.0134	0.4516	0.9822	0.0079
550	0.4334	0.9950	0.0087	0.5298	0.9918	0.0040
555	0.5121	1.0000	0.0057	0.6161	0.9991	0.0011
560	0.5945	0.9950	0.0039	0.7052	0.9973	0.0000
565	0.6784	0.9786	0.0027	0.7938	0.9824	0.0000
570	0.7621	0.9520	0.0021	0.8787	0.9556	0.0000
575	0.8425	0.9154	0.0018	0.9512	0.9152	0.0000
580	0.9163	0.8700	0.0017	1.0142	0.8689	0.0000
585	0.9786	0.8163	0.0014	1.0743	0.8256	0.0000
590	1.0263	0.7570	0.0011	1.1185	0.7774	0.0000
595	1.0567	0.6949	0.0010	1.1343	0.7204	0.0000
600	1.0622	0.6310	0.0008	1.1240	0.6583	0.0000
605	1.0456	0.5668	0.0006	1.0891	0.5939	0.0000
610	1.0026	0.5030	0.0003	1.0305	0.5280	0.0000
615	0.9384	0.4412	0.0002	0.9507	0.4618	0.0000
620	0.8544	0.3810	0.0002	0.8563	0.3981	0.0000
625	0.7514	0.3210	0.0001	0.7549	0.3396	0.0000

TABLE 6j-5. COLOR-MIXTURE FUNCTIONS FOR SMALL AND LARGE FIELDS (*Continued*)

λ, nm	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$
630	0.6424	0.2650	0.0000	0.6475	0.2835	0.0000
635	0.5419	0.2170	0.0000	0.5351	0.2283	0.0000
640	0.4479	0.1750	0.0000	0.4316	0.1798	0.0000
645	0.3608	0.1382	0.0000	0.3437	0.1402	0.0000
650	0.2835	0.1070	0.0000	0.2683	0.1076	0.0000
655	0.2187	0.0816	0.0000	0.2043	0.0812	0.0000
660	0.1649	0.0610	0.0000	0.1526	0.0603	0.0000
665	0.1212	0.0446	0.0000	0.1122	0.0441	0.0000
670	0.0874	0.0320	0.0000	0.0813	0.0318	0.0000
675	0.0636	0.0232	0.0000	0.0579	0.0226	0.0000
680	0.0468	0.0170	0.0000	0.0409	0.0159	0.0000
685	0.0329	0.0119	0.0000	0.0286	0.0111	0.0000
690	0.0227	0.0082	0.0000	0.0199	0.0077	0.0000
695	0.0158	0.0057	0.0000	0.0138	0.0054	0.0000
700	0.0114	0.0041	0.0000	0.0096	0.0037	0.0000
705	0.0081	0.0029	0.0000	0.0066	0.0026	0.0000
710	0.0058	0.0021	0.0000	0.0046	0.0018	0.0000
715	0.0041	0.0015	0.0000	0.0031	0.0012	0.0000
720	0.0029	0.0010	0.0000	0.0022	0.0008	0.0000
725	0.0020	0.0007	0.0000	0.0015	0.0006	0.0000
730	0.0014	0.0005	0.0000	0.0010	0.0004	0.0000
735	0.0010	0.0004	0.0000	0.0007	0.0003	0.0000
740	0.0007	0.0002	0.0000	0.0005	0.0002	0.0000
745	0.0005	0.0002	0.0000	0.0004	0.0001	0.0000
750	0.0003	0.0001	0.0000	0.0003	0.0001	0.0000
755	0.0002	0.0001	0.0000	0.0002	0.0001	0.0000
760	0.0002	0.0001	0.0000	0.0001	0.0000	0.0000
765	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
770	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000
775	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
780	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 6j-6. MAXIMUM POSSIBLE LUMINOUS EFFICIENCY K_m
(In lumens per watt of sources having indicated chromaticities)

y	x = 0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.7	475	590	677*	(680 at x = 0.337, y = 0.659)			
0.6	425	548	620	670*			
0.5	375	500	553	590	610*		
0.4	310	430	480	505	500	480*	
0.3	245	350	380	385	370	320	226*
0.2	155	250	270	255	185		
0.1	80*	138	130				

* Entries marked with asterisks are for chromaticities beyond the domain of real colors. They are useful for interpolations only. Approximate values for intermediate chromaticities may be determined by tabular interpolation. A contour diagram permitting more accurate interpolation than Table 6j-8 was published in "The Science of Color," p. 308 (citation at bottom of Table 6j-1).

TABLE 6j-7. MAXIMUM POSSIBLE LUMINOUS REFLECTANCE
[For samples having indicated chromaticities when illuminated by standard illuminant D_{6500} (top) and A (bottom)]

y	x = 0.1	0.2	0.3	0.4	0.5	0.6	0.7
			r_D (or t_D), %				
0.7	32	53					
0.6	34	62	77	0*			
0.5	33	66	85	97	76*		
0.4	31	69	93	85	59	43*	
0.3	26	65	81	64	39	23	12*
0.2	14	36	43	35	14		
0.1	. . .	15	13				
			r_A (or t_A), %				
0.7	21	38					
0.6	22	42	61	0*			
0.5	20	45	67	83	77*		
0.4	18	36	48	71	86	64*	
0.3	14	22	27	36	52	38	21*
0.2	7	12	15	17	19		
0.1	0*	5	4				

* See footnote following Table 6j-6. Contour diagrams permitting more accurate interpolation for illuminants A and C than Table 6j-7 were published in "The Science of Color," pp. 310, 311 (citation at bottom of Table 6j-1). Similar diagrams for illuminants A, B, C, D_{6500}; blackbodies at 1600, 1800, 2000, 2200, 2360, 3000, 3200 K; "equal energy," two different neon lamps, two different mercury lamps, and a Lucalox 400-watt alkali-metal-vapor lamp are available in *ELECLAB Tech. Note* 200/68, which may be obtained from the Naval Ship Research and Development Center, Annapolis Division, Annapolis, Md. 21402.

The correlated color temperature T_c (c_2 = 0.014388 m · K) of daylight D_{T_c} determines

$$x_D = -\frac{4.607 \times 10^9}{T_c^3} + \frac{2.9678 \times 10^6}{T_c^2} + \frac{99.11}{T_c} + 0.244063$$

when 4000 K $\leq T_c \leq$ 7000 K, or

$$x_D = -\frac{2.00064 \times 10^9}{T_c^3} + \frac{1.9018 \times 10^6}{T_c^2} + \frac{247.48}{T_c} + 0.23704$$

when 7000 K $\leq T_c \leq$ 25,000 K.

2. *Relative Spectral Power Distribution*

$$P(\lambda) = P_0(\lambda) + M_1P_1(\lambda) + M_2P_2(\lambda)$$

where $P_0(\lambda)$, $P_1(\lambda)$, and $P_2(\lambda)$ are functions of wavelength λ, given in Table 6j-3. And

$$M_1 = \frac{-1.3515 - 1.7703x_D + 5.9114y_D}{0.0241 + 0.2562x_D - 0.7341y_D}$$

$$M_2 = \frac{0.03 - 31.4424x_D + 30.0717y_D}{0.0241 + 0.2562x_D - 0.7341y_D}$$

To facilitate the use of this recommendation, values of x_D, y_D, M_1, and M_2 for correlated color temperatures in the range from 4000 to 25,000 K are given in Table 6j-4. Although these formulas may be used to calculate the relative spectral power distributions for daylight of any desired correlated color temperature, the use of D_{6500} is recommended whenever that is possible. When D_{6500} cannot be used, one of the two other distributions D_{5500} or D_{7500} given in Table 6j-2 should be used, if possible.

TABLE 6j-8. CHROMATICITY COORDINATES OF BLACKBODIES
(c_2 = 0.014388 m · K)

T, K	x	y	u	v
1000	0.6525	0.3447	0.4476	0.3547
1500	0.5855	0.3932	0.3577	0.3603
1600	0.5730	0.3993	0.3449	0.3605
1700	0.5608	0.4043	0.3333	0.3604
1800	0.5490	0.4083	0.3229	0.3602
1900	0.5376	0.4112	0.3135	0.3597
2000	0.5265	0.4133	0.3049	0.3590
2100	0.5157	0.4146	0.2971	0.3582
2200	0.5054	0.4152	0.2900	0.3573
2300	0.4955	0.4152	0.2835	0.3563
2400	0.4860	0.4147	0.2775	0.3552
2500	0.4768	0.4137	0.2720	0.3540
2600	0.4680	0.4123	0.2670	0.3528
2700	0.4597	0.4106	0.2624	0.3516
2800	0.4517	0.4086	0.2581	0.3502
2900	0.4441	0.4064	0.2542	0.3489
3000	0.4368	0.4041	0.2505	0.3476
3100	0.4298	0.4015	0.2471	0.3462
3200	0.4232	0.3989	0.2439	0.3448
3300	0.4170	0.3962	0.2410	0.3435
3400	0.4109	0.3935	0.2382	0.3422
3500	0.4053	0.3906	0.2358	0.3408
3600	0.3997	0.3879	0.2332	0.3395
3700	0.3945	0.3849	0.2310	0.3381
3800	0.3896	0.3823	0.2289	0.3369
3900	0.3847	0.3794	0.2268	0.3356
4000	0.3804	0.3767	0.2251	0.3344
4500	0.3607	0.3635	0.2173	0.3284
5000	0.3450	0.3516	0.2114	0.3231
5500	0.3324	0.3410	0.2069	0.3183
6000	0.3220	0.3317	0.2033	0.3141
6500	0.3135	0.3236	0.2004	0.3103
7000	0.3063	0.3165	0.1981	0.3070
7500	0.3003	0.3103	0.1962	0.3041
8000	0.2952	0.3048	0.1946	0.3014
8500	0.2907	0.2999	0.1932	0.2990
9000	0.2869	0.2956	0.1921	0.2969
9500	0.2836	0.2918	0.1912	0.2950
10000	0.2806	0.2883	0.1903	0.2933
20000	0.2565	0.2577	0.1839	0.2771
∞	0.2399	0.2342	0.1800	0.2636

The spectral power distributions of daylight vary seasonally, particularly in the ultraviolet (λ <400 nm), but these formulas and tabulated data should be used pending recommendations of the C.I.E. concerning such variations. The formulas and data are believed to be sufficiently accurate for colorimetric uses, but they should not be used for other purposes if high accuracy is needed. Direct spectroradiometric measurements should be made in such cases.

6j-3. Colorimetry *Standard Color-mixture Data.* C.I.E. standard observer for color measurement is determined by the specifications for the equal-energy spectrum, as given in Table 6j-5. The *tristimulus values* are the amounts of three colors necessary to match equal energies of the indicated wavelengths. The value of $\bar{y}$ given in Table 6j-5 is the standard luminosity function or relative luminosity.

TRISTIMULUS VALUES:

$$X = 680 \sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{x}$$

$$Y = 680 \sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{y} = F \text{ (lumens)}$$

$$Z = 680 \sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{z}$$

for spectral distribution of radiant energy P_λ (watts per 5-nm-wavelength band). For material with spectral transmittance τ_λ:

$$X = \frac{\sum_{\lambda=380}^{\lambda=770} \tau_\lambda P_\lambda \bar{x}}{\sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{y}}$$

$$Y = \frac{\sum_{\lambda=380}^{\lambda=770} \tau_\lambda P_\lambda \bar{x}}{\sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{y}}$$

$$Z = \frac{\sum_{\lambda=380}^{\lambda=770} \tau_\lambda P_\lambda \bar{z}}{\sum_{\lambda=380}^{\lambda=770} P_\lambda \bar{y}}$$

Relative values of P_λ are sufficient for determining tristimulus values X, Y, Z of material. For reflecting materials, substitute ρ_λ for τ_λ in above formulas.

For samples subtending more than 5-deg visual angle, the values of $\bar{x}_{10}$, $\bar{y}_{10}$, $\bar{z}_{10}$ in Table 6j-5 are probably more appropriate than $\bar{x}$, $\bar{y}$, $\bar{z}$, for colorimetry, but Y based on $\bar{y}_{10}$ has no photometric significance.

Data designed to facilitate manual computation of tristimulus values based on $\bar{x}$, $\bar{y}$, $\bar{z}$ for illuminants A, B, and C; blackbody sources of 1000, 1500, 1900, 2360, 3000, 3500, 4800, 6000, 6500, 7000, 8000, 10,000, 24,000 K; and infinite temperature; for five phases of natural daylight and for three commercial sources of artificial daylight, are tabulated in "The Science of Color."[1]

[1] Citation at bottom of Table 6j-1 (p. 6–184).

Chromaticity Coordinates. Horizontal coordinate $x = X/(X + Y + Z)$. Vertical coordinate $y = Y/(X + Y + Z)$.

Illuminant	x	y
C.I.E. standard A	0.4476	0.4074
C.I.E. standard B	0.3484	0.3516
C.I.E. standard C	0.3101	0.3162
C.I.E. standard D_{6500}	0.3127	0.3290
D_{5500}	0.3324	0.3475
D_{7500}	0.2990	0.3150

C.I.E. 1931 (x,y) DIAGRAM: Produced by plotting the chromaticity coordinates, x horizontally, y vertically, to equal scales.

C.I.E. 1960 (u,v) DIAGRAM: Provisionally recommended for use whenever a projective transformation of the (x,y) diagram yielding more nearly uniform chromaticity spacing is desired; it is formed by plotting u horizontally and v vertically, to equal scales, where

$$u = \frac{4x}{3 - 2x + 12y}, \qquad v = \frac{6y}{3 - 2x + 12y}$$

Alternatively,

$$u = \frac{4X}{X + 15Y + 3Z}, \qquad v = \frac{6Y}{X + 15Y + 3Z}$$

C.I.E. 1964 U^*, V^*, W^*, COORDINATE SYSTEM: Provisionally recommended for use whenever a three-dimensional color-coordinate system perceptually more nearly uniformly spaced than the (X,Y,Z) system is desired. It is formed by plotting U^*, V^*, and W^* to equal scales along mutually orthogonal axes, where, with $1 \leq Y \leq 100$ and u_0, v_0 representing light that appears achromatic under the conditions prevailing in the application of interest (usually that is the illuminant),

$$W^* = 25Y^{\frac{1}{3}} - 17 \qquad U^* = 13W^*(u - u_0) \qquad V^* = 13W^*(v - v_0)$$

SPECTRUM LOCUS: Curve obtained by plotting chromaticity coordinates x, y or u, v for all wavelengths listed in Table 6j-5.

DOMINANT WAVELENGTH: Wavelength corresponding to the intersection of the spectrum locus with the straight line drawn from the point representing the light source or illuminant, through the point representing the light reflected from (or transmitted by) the sample.

COMPLEMENTARY WAVELENGTH: Wavelength corresponding to the intersection of the spectrum locus with the straight line drawn from the point representing the light from the sample through the point representing the light source or illuminant (used when dominant wavelength is not determinate).

PURITY: Ratio of distance from source point to sample point, compared with distance from source point to point on the spectrum locus representing the dominant wavelength (or, in case in which dominant wavelength is not determinate, ratio of distance from source point to sample point compared with distance from source point to collinear point on line joining extremities of the spectrum locus).

PLANCKIAN LOCUS: Curve produced when x, y in Table 6j-8 or the corresponding values of u, v, are plotted.

Correlated color temperature of an illuminant is the temperature corresponding to the point on the planckian locus which is at the foot of the perpendicular to that locus, from the point representing the illuminant in the C.I.E. 1960 (u,v) diagram.

C.I.E. 1964 COLOR-DIFFERENCE FORMULA: For evaluating difference between two closely similar colors specified by U_1^*, V_1^*, W_1^*, and U_2^*, V_2^*, W_2^*,

$$\Delta E = [(U_1^* - U_2^*)^2 + (V_1^* - V_2^*)^2 + (W_1^* - W_2^*)^2]^{\frac{1}{2}}$$

This and three other formulas proposed for test for the same purpose were published in *J. Opt. Soc. Am.* **58**, 291 (1968), which should be consulted for details.

The following formulas, to the end of the section on colorimetry, are not recommended by the C.I.E. or by any other organization. They are presented for trial and use by anyone who finds them to be applicable to his problems.

GEODESIC TRANSFORMATION OF (x,y) CHROMATICITY DIAGRAM: This nonlinear transformation of the (x,y) diagram provides the most nearly uniform plane representation of small-color-difference data for 14 observers.[1]

$$\xi = 3{,}751a^2 - 10a^4 - 520b^2 + 13{,}295b^3 + 32{,}327ab - 25{,}491a^2b - 41{,}672ab^2 + 10a^3b - 5{,}227\sqrt{a} + 2{,}952\sqrt[4]{a}$$

in which $a = 10x/(2.4x + 34y + 1)$ and $b = 10y/(2.4x + 34y + 1)$.

$$\eta = 404b - 185b^2 + 52b^3 + 69a(1 - b^2) - 3a^2b + 30ab^3$$

in which $a = 10x/(4.2y - x + 1)$ and $b = 10y/(4.2y - x + 1)$. ξ and η are given in units of root-mean-square errors of color matching by the 14 normal observers. All straight lines in the (ξ,η) diagram represent paths (geodesics) of minimum accumulated color differences, as evaluated according to the observer data.

According to the Schrödinger hypothesis,[2] such geodesics [straight lines in the (ξ,η) diagram] drawn outward from the achromatic point should represent series of colors of constant hue. The point representing C.I.E. standard illuminant D_{65} is at $\xi = 861.2$, $\eta = 395.7$.

CHROMATICITY DIFFERENCE between any two colors specified by (ξ_1,η_1) and (ξ_2,η_2) is

$$\Delta c = [(\xi_1 - \xi_2)^2 + (\eta_1 - \eta_2)^2]^{1/2}$$

SATURATION of color specified by ξ, η, Y: $s = w_1\,[(\xi - \xi_a)^2 + (\eta - \eta_a)^2]^{1/2}$, where $w_1 = 0.054 + 0.46Y^{1/3}$ $(1 < Y < 80)$, and ξ_a and η_a are the specifications of the achromatic color, usually the illuminant.

HUE (h) expressed as an angle clockwise from the vertical drawn downward from the point representing the achromatic color,

$$h = \tan^{-1}\frac{\xi_a - \xi}{\eta_a - \eta}$$

where
$0 < h < 90°$ when $\xi_a - \xi$ and $\eta_a - \eta$ are both positive
$h = 90°$ when $\xi_a - \xi$ is positive and $\eta = \eta_a$
$90° < h < 180°$ when $\xi_a - \xi$ is positive and $\eta_a - \eta$ is negative
$180° < h < 270°$ when $\xi_a - \xi$ and $\eta_a - \eta$ are both negative
$h = 270°$ when $\xi_a - \xi$ is negative and $\eta = \eta_a$
$270° < h < 360°$ when $\xi_a - \xi$ is negative and $\eta_a - \eta$ is positive.

COLOR DIFFERENCE between any two colors specified by (ξ_1,η_1), (ξ_2,η_2) and luminous reflectances Y_1 and Y_2 (in percent):

$$\Delta c = w_1\left[(\xi_1 - \xi_2)^2 + (\eta_1 - \eta_2)^2 + \frac{w_2^2(Y_1 - Y_2)^2}{Y^2}\right]^{1/2}$$

where $w_2 = 1$ for sharply juxtaposed samples subtending about 2 deg, and Y is the average of Y_1 and Y_2. For a less well-defined dividing line, w_2 may be considerably less, e.g., for 5-deg separation between large samples $w_2 \cong 0.07$. For extremely small samples, to be distinguished from a uniform-background color, $w_2 \cong 7$, and w_1 should be multiplied by about 0.1.

LIGHTNESS:[3] $L = 2.468Y^{1/3} - 1.636$, where $1.2 < Y < 79$.

[1] D. L. MacAdam, *J. Opt. Soc. Am.* **32**, 247 (1942); L. Silberstein and D. L. MacAdam, *ibid.* **35**, 32 (1945); W. R. J. Brown and D. L. MacAdam, *ibid.* **39**, 808 (1949); W. R. J. Brown, *ibid.* **47**, 137 (1957); D. L. MacAdam, Appl. Opt. **11** (January, 1971).

[2] E. Schrödinger, *Ann. Physik* **63**, 515 (1920).

[3] J. H. Ladd and J. E. Pinney, *Proc. Inst. Radio Engrs.* **43**, 1137 (1955).

CHROMATIC ADAPTATION: Colors that are viewed with eyes adapted to a chromaticity S but appear the same as other colors viewed with eyes adapted to some other chromaticity T may be calculated in terms of tristimulus values, R, G, B based on the set of primaries

$$X_R = 0.747 \qquad Y_R = 0.253$$
$$X_G = 1.75 \qquad Y_G = -0.75$$
$$X_B = 0.1785 \qquad Y_B = 0.0\ ,$$

by multiplication of constant *adaptation factors*, K_ρ, K_γ, K_β times

$$\rho = a_1 + b_1 R^{p(R,T)}$$
$$\gamma = a_2 + b_2 G^{p(G,T)}$$

and

$$\beta = a_3 + b_3 B^{p(B,T)}$$

and solution of similar equations for R', G', B' for the corresponding colors

$$R' = \frac{(K_\rho \rho - a_4)^{1/p(R,S)}}{b_4}$$
$$G' = \frac{(K_\gamma \gamma - a_5)^{1/p(G,S)}}{b_5}$$
$$B' = \frac{(K_\beta \beta - a_6)^{1/p(B,S)}}{b_6}$$

For complete adaptation, such that S and T look alike, the adaptation factors are $K_\rho = \sigma_T/\sigma_S$, $K_\gamma = \gamma_T/\gamma_S$, $K_\beta = \beta_T/\beta_S$. Incomplete adaptation can be provided for by using, instead, the ratios determined from any observed pair of corresponding colors, preferably not very different from S or T.

The formulas for R, G, B should be normalized so that the maximum values used in these formulas do not exceed 80. For values of X, Y, Z not exceeding 80, i.e., for ordinary colored materials in daylight (in illuminants C and D), whose spectral reflectance p_λ (and Y) are expressed in percent, the formulas for R, G, B based on the primaries specified above are

$$R = 0.32X + 0.74Y - 0.069Z$$
$$G = -0.46X + 1.36Y + 0.10Z$$
$$B = Z$$

For other illuminants, only the normalizations need be changed. Values of R, G, B, R', G', B' less than 1.0 should be avoided, so far as possible, by adjusting the normalizations. Any that are nevertheless less than 1 should immediately be set arbitrarily at 1. For data based on the normalization given above, the tristimulus values X', Y', Z' of the corresponding colors for eyes adapted to T may be computed from the values of R', G', B' by use of

$$X' = 1.75R' - 0.15G' + 0.216B'$$
$$Y' = 0.59R' + 0.41G'$$
$$Z' = B'$$

For other normalizations, the coefficients of R', G', B' in these formulas should be divided by the ratios of the corresponding adjusted formulas to the formulas given above for R, G, B.

Exponents: The exponents $p(R,S)$ and $p(R,T)$ of R and R' may be computed from the formula

$$p(R,i) = 12.82x^2 + 0.53y^2 - 7.20x + 3.75y - 11.05xy + 1.21$$

where x and y are the chromaticity coordinates of the color, S or T, to which the eyes are adapted in each instance. Whenever a value of p greater than 1.0 results from the use of this or the two following equations, the value 1 should be used instead. The exponents $p(G,S)$ and $p(G,T)$ of G and G' are computed from the formula $p(G,i) = 1 - 23(x - y)^2 + 2.3(x - y)$. The exponents $p(B,S)$ and $p(B,T)$ of B and B' are computed from the formula $p(B,i) = 1.8 - x - 2y$.

Constants: For each exponent p, the corresponding coefficients a and b, for use in the formulas for ρ, γ, β, R', G', B' are

$$b = \frac{6.1}{55.63^p - 2.422^p}$$
$$a = 7.8 - b(55.63)^p$$

e.g., for $p = 1$, $a = 1.42$, $b = 0.115$; for $p = \frac{1}{3}$, $a = 0.065$, $b = 1.046$.

TRANSFORMATIONS OF COLOR-MIXTURE DATA. In general, the tristimulus values R, G, B are the amounts (in a mixture matching a color specified by X, Y, Z) of a set of primaries located at (x_r,y_r), (x_g,y_g), and (x_b,y_b) in the (x,y) chromaticity diagram,

$$R = a_{11}X + a_{12}Y + a_{13}Z$$
$$G = a_{21}X + a_{22}Y + a_{23}Z$$
$$B = a_{31}X + a_{32}Y + a_{33}Z$$

Note that the color-mixture functions for the spectrum $\bar{x}$, $\bar{y}$, $\bar{z}$, $\bar{r}$, $\bar{g}$, $\bar{b}$ are simply special cases; their symbols may be substituted for X, Y, Z, R, G, B in these formulas. Let m_1 represent the slope, and let b_1 represent the y axis intercept of the line $y = m_1x + b_1$ drawn between the points (x_g,y_g), (x_b,y_b) representing the G and B primaries. Then

$$a_{12} = a_{11}\frac{b_1 - 1}{b_1 + m_1} \quad \text{and} \quad a_{13} = \frac{a_{11}b_1}{b_1 + m_1}$$

where a_{11} is arbitrary and may be used to normalize the R function. If, as is often the case, the line joining the G and B primaries is more than 45 deg from horizontal, accuracy is best preserved if it is specified in terms of its reciprocal slope M_1 and x-axis intercept B_1, $x = M_1y + B_1$. Then

$$a_{12} = a_{11}\frac{B_1 + M_1}{B_1 - 1} \quad \text{and} \quad a_{13} = \frac{a_{11}B_1}{B_1 - 1}$$

Similarly, in terms of the slope m_2 and y-axis intercept b_2 of the line through the R and B primaries, the coefficients in the formula for G are

$$a_{21} = a_{22}\frac{b_2 + m_2}{b_2 - 1} \qquad a_{23} = \frac{a_{22}b_2}{b_2 - 1}$$

where a_{22} is the normalization constant. Finally, in terms of the slope m_3 and y-axis intercept b_3 of the line through the R and G primaries the coefficients in the formula for B are

$$a_{31} = a_{33}\frac{b_3 + m_3}{b_3} \qquad a_{32} = a_{33}\frac{b_3 - 1}{b_3}$$

where a_{33} is the normalization constant. In terms of the two axis intercepts,

$$a_{31} = a_{33}(1 - B_3^{-1}) \qquad a_{32} = a_{33}(1 - b_3^{-1})$$

The reverse transformations are

$$X = A_1[(b_2 - b_3)R + (b_3 - b_1)G + (b_1 - b_2)B]$$
$$Y = A_2[(m_3b_2 - m_2b_3)R + (m_1b_3 - m_3b_1)G + (m_1b_2 - m_2b_1)B]$$
$$Z = A_3\left[(m_3 - m_2)R + (m_1 - m_3)G + (m_2 - m_1)W - \frac{X}{A_1} - \frac{Y}{A_2}\right]$$

where A_1, A_2, A_3 are constants that depend on the normalizations of R, G, B. They can be determined by calculating R, G, B and the corresponding values of the expressions in the brackets for some one color, e.g., the illuminant, and dividing those results into the original values of X, Y, Z. Note that the last two terms in the brackets in Z are simply the quantities that appear in brackets in the formulas for X and Y.