

## 6m. Magneto-, Electro-, and Elasto-optic Constants

WILLIAM R. COOK, JR. AND HANS JAFFE

*Gould, Inc.*

**6m-1. Magnetic Rotation (Faraday Effect).** The most important interaction between magnetic field and light wave propagation is a rotation of the plane of polarization of a light wave traveling parallel to a magnetic field component

$$\alpha = VHI \quad (6m-1)$$

where  $H$  is magnetic field strength, and  $l$  the path length. This is the Faraday effect. The coefficient  $V$  is known as the Verdet constant. The Faraday effect results from a difference in propagation velocity for left and right circular polarized light. For a constant value of this difference, the Verdet constant is inversely proportional to wavelength. The tables give  $V$  in angular minutes/oersted-cm. Positive sign indicates rotation of the polarization plane in the same sense as a positive current in a coil producing the field.

TABLE 6m-1a. VERDET CONSTANTS OF GASES AND LIQUIDS<sup>a</sup>

Gas <sup>b</sup>	$(n_D^{20} - 1) \times 10^3$	$10^6 V_0$	Liquid	$\lambda, \mu\text{m}$	$t, ^\circ\text{C}$	$n_D^{20, \lambda}$	$10^6 V$
He	0.036	+ 0.40	F	0.589	33		+13.3
Ar	2.81	+ 9.36	S	0.589	114	1.929 <sup>1100</sup>	+ 8.1
H <sub>2</sub>		+ 6.2 <sub>9</sub>	H <sub>2</sub> O <sup>c</sup>	0.546	25		+ 1.547
N <sub>2</sub>	0.297	+ 6.4 <sub>6</sub>	H <sub>2</sub> O	0.589	20	1.3330	+ 1.309
O <sub>2</sub>	0.272	+ 5.69	D <sub>2</sub> O	0.589	19.7	1.3384 <sup>1200</sup>	+ 1.257
Air	0.293	+ 6.27	H <sub>3</sub> PO <sub>4</sub>	0.578	97.4		+ 1.35
Cl <sub>2</sub>	0.773	+31.9	CS <sub>2</sub>	0.589	20	1.6255	+ 4.255
HCl	0.447	+21.5	CCl <sub>4</sub>	0.578-0.589	25.1	1.463 <sup>150</sup>	+ 1.60
H <sub>2</sub> S	0.63	+41.5	SbCl <sub>3</sub>	0.578	18	1.601 <sup>140</sup>	+ 7.45
NH <sub>3</sub>	0.37 <sub>6</sub>	+19.0	TiCl <sub>4</sub>	0.578	17	1.61	- 1.65
CO	0.34	+11.0	TiBr <sub>4</sub> <sup>f</sup>	0.578	46		- 5.3
CO <sub>2</sub>	0.45	+ 9.39	Methanol	0.589	18.7	1.3289	+ 0.958
NO	0.297	-58	Acetone	0.578-0.589	20.0	1.3585	+ 1.116
CH <sub>4</sub>	0.444	+17.4	Toluene	0.578-0.589	15.0	1.4950	+ 2.71
<i>n</i> -C <sub>4</sub> H <sub>10</sub>		+44.0	Benzene	0.578-0.589	15.0	1.5005	+ 3.00
			Chlorobenzene	0.589	15	1.5246	+ 2.92
			Nitrobenzene	0.589	15	1.5523	+ 2.17
			Bromoform	0.589	17.9	1.5960	+ 3.13

<sup>a</sup> Selected except as noted from R. de Malleman, "Tables des constantes selectionees, pouvoir rotatoire magnetique (effet Faraday)," Hermann & Cie, Paris, 1951.

<sup>b</sup>  $V_0$  for  $\lambda = 0.578 \mu\text{m}$  as reduced to  $0^\circ\text{C}$  and 760 mm Hg.

<sup>c</sup> "Handbook of Chemistry and Physics," Chemical Rubber Publishing Co., Cleveland, Ohio.

<sup>d</sup> Indices of refraction for organic chemicals from Eastman Kodak Co. Organic Chemicals List No. 39, 1954.

<sup>e</sup> V. Sivaramakrishnan, *Proc. Indian Acad. Sci.* **39**, 31 (1954); *J. Indian Inst. Sci.* **36**, 193 (1954).

<sup>f</sup> P. Fritsch, *Compt. Rend.* **217**, 447 (1943).

TABLE 6m-1b. VERDET CONSTANTS OF SOLIDS  
 (At room temperature except as noted)

Solid	V 0.633 $\mu\text{m}$	V 0.700 $\mu\text{m}$	Ref.
<i>Oxide glasses</i>			
39Tl <sub>2</sub> O·61SiO <sub>2</sub> (moles).....	0.13	0.10	8
20Tl <sub>2</sub> O·80PbO.....	0.14	0.127	8
24Pr <sub>2</sub> O <sub>3</sub> ·76B <sub>2</sub> O <sub>3</sub> .....	-0.26	-0.22	8
24Nd <sub>2</sub> O <sub>3</sub> ·76B <sub>2</sub> O <sub>3</sub> .....	-0.14	-0.105	8
85Bi <sub>2</sub> O <sub>3</sub> ·15B <sub>2</sub> O <sub>3</sub> .....	0.10	0.085	8
85PbO·15B <sub>2</sub> O <sub>3</sub> .....	0.115	0.093	8
85Tl <sub>2</sub> O·15B <sub>2</sub> O <sub>3</sub> .....	0.122	0.092	8
2.67Ce <sub>2</sub> O <sub>3</sub> ·P <sub>2</sub> O <sub>5</sub> .....	-0.174	-0.132	10
As <sub>2</sub> S <sub>3</sub> .....	0.26	0.194	9

  

Solid	n 0.5461 $\mu\text{m}$	V 0.5461 $\mu\text{m}$	n 0.5893 $\mu\text{m}$	V 0.5893 $\mu\text{m}$	Ref.
<i>Oxide glasses</i>					
SiO <sub>2</sub> .....	1.4601	0.01664	1.4585	0.01421	3b
Dense flint 18.....	1.8999	0.1180	1.8900	0.0969	3b
Lead glass (Corning 8363).....	.....	0.133	.....	0.107	9
<i>Oxide crystals</i>					
NH <sub>4</sub> Al(SO <sub>4</sub> ) <sub>2</sub> ·12H <sub>2</sub> O.....	.....	0.0151	1.4594	0.0128	3c
KAl(SO <sub>4</sub> ) <sub>2</sub> ·12H <sub>2</sub> O.....	.....	0.0144	1.4564	0.0124	3c
NH <sub>4</sub> Fe(SO <sub>4</sub> ) <sub>2</sub> ·12H <sub>2</sub> O at 26°C.....	.....	-0.00145	1.4848	-0.00058	1
Same at -111°C.....	.....	-0.0145	.....	-0.0111	1
NiSO <sub>4</sub> ·6H <sub>2</sub> O at 24°C.....	.....	0.0256	$\omega = 1.5109$	0.0221	4
Same at 1.36°K.....	.....	0.419	.....	.....	2
MgAl <sub>2</sub> O <sub>4</sub> (spinel).....	.....	.....	1.7181	0.021	6
CaCO <sub>3</sub> (calcite).....	.....	.....	$\omega = 1.6585$	0.019	7
NaClO <sub>3</sub> .....	.....	0.0105	1.5151	0.0081	3c
SiO <sub>2</sub> (quartz).....	$\omega = 1.5462$	0.01952	1.5443	0.01664	3b
Al <sub>2</sub> O <sub>3</sub> (corundum).....	$\omega = 1.7712$	0.0240	1.7685	0.0210	3d
<i>Cubic halide crystals</i>					
NaCl.....	.....	0.0410	1.5443	0.0345	3c
NaBr.....	.....	0.0621	1.6412	.....	3c
KCl.....	.....	0.0328	1.4904	0.0275	3c
KBr.....	1.5641	0.0500	1.5600	0.0425	3c
KI.....	1.6731	0.083	1.6664	0.070	3c
NH <sub>4</sub> Cl.....	.....	0.0430	1.6426	0.0362	3c
NH <sub>4</sub> Br.....	.....	0.0601	1.7108	0.0504	3c
CaF <sub>2</sub> .....	.....	.....	1.4338	0.00883	3a
<i>Tetrahedral cubic crystals</i>					
C, diamond.....	.....	0.0278	2.4172	0.0233	3a
CuCl.....	.....	0.20 $\pm$ 0.03	1.793	.....	5
ZnS.....	.....	0.287	2.3683	0.226	3a

## References to Table 6m-1b

1. Kaufmann, H.: *Ann. Physik* **18**, 251 (1933). (Paramagnetic rotation.)
2. Levy and van den Handel: *Physica* **15**, 717 (1951). (Paramagnetic rotation.)
3. Ramaseshan, S.: *Proc. Indian Acad. Sci.*: (a) **24**, 104 (1946); (b) **24**, 426 (1946); (c) **28**, 360 (1948); (d) **34**, 97 (1951); (e) *Current Sci. (India)* **20**, 150 (1951).
4. O'Connor, Beck, and Underwood: *Phys. Rev.* **60**, 443 (1941).
5. Gassmann, G.: *Ann. Physik* **35**, 638 (1939). A volume of 23.9 cm<sup>3</sup>/mole is assumed to derive this value of V.
6. DuBois: *Ann. Physik* **51**, 537 (1894).

7. Chauvin: *J. Phys.* 9, 5 (1890).  
 8. Borrelli, N. F.: Personal communication; also, *J. Chem. Phys.* 41, 3289 (1964).  
 9. Robinson, C. C.: *Appl. Opt.* 3, 1163 (1964).  
 10. Berger, S. B., C. B. Rubinstein, C. R. Kurkjian, and A. W. Treptow: *Phys. Rev.* 133A, 723 (1964).

6m-2. The Kerr Effect. The lowest-order effect of an electric field on the refractive index of an isotropic material permitted by symmetry is quadratic in the electric field. The observed effect is an induced birefringence, the Kerr effect. It has substantial magnitude in polar liquids. (See also Sec. 6m-6 for ferroelectric crystals.) The Kerr constant  $K$  is defined by the relation

$$\Gamma = \frac{(n_p - n_s)l}{\lambda} = lKE^2 \quad (6m-2)$$

where  $\Gamma$  is the retardation (path difference in fractions of the wavelength  $\lambda$ ),  $n_p$  and  $n_s$  are the refractive indices parallel and normal to the applied field  $E$ , and  $l$  is the path length. As customary, Table 6m-2 gives  $K$  in electrostatic units.

TABLE 6m-2. TABULATED CHARACTERISTICS OF LIQUIDS WITH KNOWN LARGE KERR CONSTANTS\*

Liquid	Symbol†	Kerr constant $K$ , $10^{-7}$ esu ( $\lambda=0.589 \mu\text{m}$ )	Static dielectric constant‡ $\epsilon'$	Melting point, °C Boiling point, °C	$n_D^{20}/H\rho - H\alpha$	Short-wavelength cutoff $\lambda_{co}$ , nm§
Carbon disulfide.....	CS <sub>2</sub>	+3.23	2.6	-108.6/+46.3	1.6295/0.0343	
Acetone.....	C <sub>3</sub> H <sub>6</sub> O	+16.3	21.9	-94.3/+56.1	1.3591/0.0068	
Methyl ethyl ketone.....	C <sub>5</sub> H <sub>8</sub> O	+13.6	18.5	-86.4/+79.6	1.3791/0.0071	
Pyridine.....	C <sub>5</sub> H <sub>5</sub> N	+20.4	12.5	-42/+115.3	1.509/0.0163	3,000
Ethyl cyanoacetate.....	C <sub>5</sub> H <sub>7</sub> NO <sub>2</sub>	+38.8	27.7	-22.5/+206	1.4179/0.0044	3,100
<i>o</i> -Dichlorobenzene.....	C <sub>6</sub> H <sub>4</sub> Cl <sub>2</sub>	+42.6	7.5	-17.6/+179	1.549/0.0176	3,000
Benzenesulfonyl chloride.....	C <sub>6</sub> H <sub>5</sub> ClO <sub>2</sub> S	+89.9	.....	+14.5/+247	.....	3,000
Nitrobenzene.....	C <sub>6</sub> H <sub>5</sub> NO <sub>2</sub>	+326	36.1	+5.7/210.0	1.5520/0.0252	4,600
Ethyl <i>B</i> -aminocrotonate.....	C <sub>8</sub> H <sub>11</sub> NO <sub>2</sub>	+31.0	.....	+33.9/210	.....	
Paraldehyde.....	C <sub>6</sub> H <sub>12</sub> O <sub>3</sub>	-23.0	14.5	+10.5/+124	1.4198/0.0081	3,200
			12.0			
Benzaldehyde.....	C <sub>7</sub> H <sub>6</sub> O	+80.8	18.0	-56.0/+179.5	1.5464/0.0232	4,000
			14.1			
<i>p</i> -Chlorotoluene.....	C <sub>7</sub> H <sub>7</sub> Cl	+23.0	6.4	+7.8/+162.5	1.521/0.0164	3,200
<i>o</i> -Nitrotoluene.....	C <sub>7</sub> H <sub>7</sub> NO <sub>2</sub>	+174	27.4	-4.1/+222.3	1.5462/	
<i>m</i> -Nitrotoluene.....	C <sub>7</sub> H <sub>7</sub> NO <sub>2</sub>	+177	23.8	+15.5/+231	1.5475/	4,600
<i>p</i> -Nitrotoluene.....	C <sub>7</sub> H <sub>7</sub> NO <sub>2</sub>	+222	18.7	+51.3/+238	1.5346/	
Benzyl alcohol.....	C <sub>7</sub> H <sub>8</sub> O	-15.4	13.0	-15.3/+205.8	1.5399/0.0173	3,200
			10.8			
<i>m</i> -Cresol.....	C <sub>7</sub> H <sub>8</sub> O	+21.2	13.0	+10/+202.8	1.540/0.0181	3,400
			5.0			
<i>m</i> -Chloroacetophenone.....	C <sub>8</sub> H <sub>7</sub> ClO	+89.1	18.3	+19.7/+202.3	1.5339/0.0217	3,800
Acetophenone.....	C <sub>8</sub> H <sub>8</sub> O	+85.6	15.9			
Quinoline.....	C <sub>8</sub> H <sub>7</sub> N	+15.0	9.0	-19.5/+239.7	1.6283/0.0312	3,600
Ethyl salicylate.....	C <sub>9</sub> H <sub>10</sub> O <sub>2</sub>	+19.6	8.6	+1.3/+231.5	1.5226/0.0206	3,400
Carvone.....	C <sub>10</sub> H <sub>14</sub> O	+23.6	11.2	<0/+230	.....	4,000
Ethyl benzoylacetate.....	C <sub>11</sub> H <sub>12</sub> O <sub>2</sub>	+16.0	12.8	<0/+270	1.5338/0.0202	
Water.....	H <sub>2</sub> O	+4.0	81	0/100	1.3330/	

\* The data reported here are from ICT except where noted.

† The chemical symbol as shown here is used in the ICT for locating or reference purposes only.

‡ Several values for each liquid are listed in the ICT for several temperatures and frequencies. A single value in this table means that the approximate value is valid at both audio and radio frequencies (10<sup>3</sup> to 10<sup>9</sup>) and at temperatures near 20°C. If two values are quoted, the first refers to audio, the second to radio frequencies.

§ Low-resolution measurements.

6m-3. Elasto-optic Effects in Isotropic Bodies. Elastic stresses generally cause changes of the refractive index proportional to the stress. This index change is different (and usually of opposite sign) for the electric vector of the light wave parallel and normal to the direction of an applied uniaxial stress. Isotropic bodies therefore develop induced uniaxial birefringence, with a retardation

$$\Gamma = \frac{(n_p - n_s)l}{\lambda} - \frac{CTl}{\lambda} \quad (6m-3)$$

where  $l$  is the light-path length in the body,  $\lambda$  the wavelength, and  $T$  the applied tensile stress. Positive value of  $C$  means that the refractive index for the electric vector parallel to an applied tension is the higher one. The customary unit for the stress-optical coefficient  $C$  is 1 Brewster =  $10^{-13}$  cm<sup>2</sup>/dyne =  $10^{-12}$  m<sup>2</sup>/N. See Sec. 6m-4 for absolute values of refractive-index change in some isotropic materials.

TABLE 6m-3. ENGINEERING STRESS-OPTICAL COEFFICIENT  $C$  FOR VARIOUS ISOTROPIC MATERIALS

Material	$C$ , Brewsters ( $10^{-12}$ m <sup>2</sup> , N)	$t$ , °C	$\lambda$ , $\mu$ m	Ref.
Polystyrene (glassy).....	+8 to +10	27	0.546	1
Polymethyl methacrylate:				
Flexiglas II.....	-3.8	27	0.546	1
Lucite 130.....	-2.7	27	0.546	1
Polyphenyl methacrylate.....	+39.8	27	0.546	1
Polycyclohexyl methacrylate.....	+5.9	27	0.546	1
<i>p</i> -Cl Polystyrene.....	+23	27	0.546	1
Benzyl methacrylate.....	+45	20-25	.....	1
Natural rubber.....	2,000	20	.....	2
Gutta-percha.....	3,080	85	.....	2
Polyethylene.....	~2,000	130	.....	2
Polymethylene.....	~1,700	180	.....	2
Celluloid.....	11			
Bakelite.....	53			
Gelatin.....	1,700-14,000			
Fused silica (code 7940)*.....	-3.36	.....	0.633	3
Borosilicate (code 7070)*.....	-4.6	.....	0.633	3
Lead silicate (code 8363)*.....	+1.1	.....	0.633	3

\* Codes refer to Corning designation.

#### References for Table 6m-3

1. Rudd, J. F., and R. D. Andrews: *J. Appl. Phys.* **31**, 818 (1960).
2. Saunders, D. W.: *Trans. Faraday Soc.* **52**, 1414, 1425 (1956).
3. Borelli, N. F., and R. A. Miller: *Appl. Opt.* **7**, 743 (1966).

6m-4. Elasto-optic Effects in Crystals. The optic effects of stress in crystals are in general dependent on the orientation of the stress tensor with respect to the crystal axes. The changes in refraction caused by stress are superimposed on permanent birefringence in all crystal systems except the cubic. The refractive properties of the crystal may be represented by the index ellipsoid. The changes of refraction due to applied stress are then expressed in additive terms in the coefficients of the index ellipsoid equation. Related to orthogonal crystal axes, the ellipsoid equation is

$$a_1x^2 + a_2y^2 + a_3z^2 + 2a_4yz + 2a_5zx + 2a_6xy = 1 \quad (6m-4)$$

For all systems except the monoclinic and triclinic  $a_4 = a_5 = a_6 = 0$ , and  $a_1, a_2, a_3$  are reciprocal to the square of the refractive indices for vibration in the respective axial directions. The additive terms in the index ellipsoid coefficients are related to the stress components by

$$\Delta a_i = q_{ij} T_j \quad (i, j \text{ from } 1 \text{ to } 6, \text{ summation over repeated index}) \quad (6m-5)$$

$\Delta a_1, \dots$  determine changes of magnitude of the refractive indices, while  $\Delta a_4, \dots$  cause either rotation of the index ellipsoid or a reduction of its symmetry from cubic or uniaxial to biaxial. The  $q_{ij}$  are the stress-optic coefficients; there are 36 independent  $q_{ij}$  for an asymmetric crystal. Crystal symmetries introduce equations between some coefficients and make others zero. The matrices of coefficients for all crystal classes have been derived.<sup>1,2</sup> Earlier derivations [quoted for instance by W. P. Mason, *Bell System Tech. J.* **29**, 161-188 (1950)] show additional equalities which are erroneous.

For orthorhombic crystals the matrix is:

$$\begin{array}{cccccc} q_{11} & q_{12} & q_{13} & 0 & 0 & 0 \\ q_{21} & q_{22} & q_{23} & 0 & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{66} \end{array}$$

For crystals of the tetragonal classes  $\bar{4}2m$  and  $4/mmm$  the same matrix holds with the equalities

$$q_{11} = q_{22}, q_{12} = q_{21}, q_{13} = q_{23}, q_{31} = q_{32}, q_{44} = q_{55}$$

For the cubic crystal classes  $\bar{4}3m$  and  $m\bar{3}m$  one finds the additional equalities  $q_{33} = q_{11}$ ,  $q_{13} = q_{31} = q_{12}$ ,  $q_{66} = q_{44}$ , so that only three independent coefficients remain:  $q_{11}$ ,  $q_{12}$ ,  $q_{44}$ . In the less symmetric cubic classes  $m\bar{3}$  and  $23$ , however, there are four independent coefficients, with  $q_{12} = q_{23} = q_{31}$  and  $q_{13} = q_{21} = q_{32}$ , but  $q_{12} \neq q_{21}$ . The matrix for isotropic materials differs from that for the most symmetric cubic class by the additional relation  $q_{44} = q_{11} - q_{12}$ .

The matrix for the trigonal classes  $32$  and  $3m$  is:

$$\begin{array}{cccccc} q_{11} & q_{12} & q_{13} & q_{14} & 0 & 0 \\ q_{12} & q_{11} & q_{13} & -q_{14} & 0 & 0 \\ q_{31} & q_{31} & q_{33} & 0 & 0 & 0 \\ q_{41} & -q_{41} & 0 & q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{44} & 2q_{41} \\ 0 & 0 & 0 & 0 & q_{14} & q_{11} - q_{12} \end{array}$$

The matrix for the hexagonal classes  $6mm$  and  $6/mmm$  follows from the preceding by setting  $q_{41} = q_{14} = 0$ .

The changes of refraction caused by elastic deformation may also be expressed as a function of the strain tensor. The resulting strain-optic coefficients  $p_{ij}$  are related to the stress-optic coefficients by

$$p_{ij} = q_{ik} c_{kj} \quad (6m-6)$$

<sup>1</sup> J. F. Nye, "Physical Properties of Crystals," 2d ed., Clarendon Press, Oxford, 1960.

<sup>2</sup> S. Bhagavantam, "Crystal Symmetry and Physical Properties," Academic Press, Inc., New York and London, 1966.

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The  $p_i$  are dimensionless coefficients generally of magnitude in the  $10^{-2}$  to 1 range. The relations introduced by symmetry are the same as for the stress-optic coefficients except that for the trigonal crystals  $p_{33} = p_{44}$ ; for both trigonal and hexagonal crystals  $p_{66} = (p_{11} - p_{12})/2$ ; also for isotropic bodies  $p_{44} = (p_{11} - p_{12})/2$ .

In piezoelectric crystals the values of some  $q_{ik}$  and  $p_{ik}$  depend on the electric boundary conditions. This effect is more pronounced for the  $q_{ik}$  than  $p_{ik}$ . The listed values may be assumed to relate to the short-circuit (constant  $E$ ) condition.

The optic path difference is found by solving the equation of the modified index ellipsoid for the desired wave propagation direction. The solutions are of the form  $\Gamma = n^3(\Delta a_i - \Delta a_k)\gamma l$ , where  $n$  is the appropriate refractive index, and  $\gamma$  a numerical constant in the order of unity.

TABLE 6m-4a. STRESS-OPTIC CONSTANTS OF CRYSTALS\* ( $10^{-12}$  m<sup>2</sup>/N)

Crystal	Symmetry	$q_{11}$	$q_{12}$	$q_{44}$	$q_{13}$	$q_{21}$	$q_{33}$	$q_{66}$	$q_{14}$	$q_{11}$
KCl	$m3m$	4.75	2.87	-4.32	$q_{12}$	$q_{12}$	$q_{11}$	$q_{44}$	0	0
NaCl	$m3m$	1.27	2.58	-0.84						
LiF	$m3m$	-0.40	1.12	-0.83						
CaF <sub>2</sub>	$m3m$	-0.29	1.16	0.698						
Diamond	$m3m$	-0.43	0.37	-0.27						
ZnS	$43m$			-3.16	$q_{12}$	$q_{12}$	$q_{11}$	$q_{44}$	0	0
Ammonium alum	$m3$	5.5	11.6	-1.15	16.9	$q_{13}$	$q_{11}$	$q_{44}$	0	0
Potassium alum	$m3$	3.7	9.1	-0.63	8.5					
Ba(NO <sub>3</sub> ) <sub>2</sub>	$m3$	$q_{11} - q_{12} = -23.84$		-1.60		$q_{11} - q_{12} = -17.13$				
Pb(NO <sub>3</sub> ) <sub>2</sub>	$m3$	$q_{11} - q_{12} = -19.13$		-1.39		$q_{11} - q_{12} = -11.54$				
NH <sub>4</sub> H <sub>2</sub> PO <sub>4</sub>	$42m$	8.6	7.9	-5.8	-37.3	12.3	-35.7	-12.2	0	0
Calcite	$32/m$	-0.81	0.92	0.35	2.53	1.55	-0.45	$(q_{11} - q_{12})$	-1.11	0.67
$\alpha$ -Quartz	$32$	1.11	2.50	-1.010	1.97	2.77	0.183	$(q_{11} - q_{12})$	-0.097	-0.320
Beryl	$6/mmm$	-0.356	0.592	-2.32	0.739	0.931	-0.426	$(q_{11} - q_{12})$	0	0

Crystal	Symmetry	$q_{11}$	$q_{22}$	$q_{33}$	$q_{44}$	$q_{55}$	$q_{66}$	$q_{12}$	$q_{21}$	$q_{13}$	$q_{31}$	$q_{23}$	$q_{32}$
Barite	$mmm$	-0.079	1.77	2.56	0.20	0.435	1.33	3.81	1.49	1.64	0.44	1.23	1.23
Topaz	$mmm$	-0.514	-0.594	-0.456	0.852	-0.233	-0.750	0.326	0.521	0.226	0.376	0.251	0.222
Rocheite salt	$222$				-0.9	1.7	-1.7						

\* R. Bechmann, "Landolt-Börnstein," New Series, Group III, vol. 1, K.-H. Hellwege, ed., 1966; vol. 2, 1969. Most of data determined at 0.589  $\mu$ m.

TABLE 6m-4b. STRAIN-OPTIC CONSTANTS OF CUBIC CRYSTALS AND SOME GLASSES

Material	Symmetry	$p_{11}$	$p_{12}$	$p_{44}$	$p_{13}$	Wavelength, $\mu\text{m}$	Refs.
Fused silica	Glass	+0.121	+0.270	$\frac{1}{2}(p_{11}-p_{12})$	$p_{12}$	0.633	1
Code 7070*	Glass	+0.113	+0.23	.....	.....	0.633	2
Code 8363*	Glass	+0.196	+0.185	.....	.....	0.633	2
As <sub>2</sub> S <sub>3</sub>	Glass	+0.277	+0.272	.....	.....	0.633	1
As <sub>2</sub> S <sub>5</sub>	Glass	+0.308	+0.299	.....	.....	1.15	1
KCl	m3m	0.215	0.159	-0.024	$p_{12}$	0.589	3, 4
NaCl	m3m	0.137	0.178	-0.011	.....	.....	3, 4
LiF	m3m	0.02	0.130	-0.045	.....	0.589	3, 4
Tl(Br,I) = KRS-5	m3m	$p_{11}-p_{12} = 0.08$	.....	0.157	.....	0.61	.....
MgO	m3m	-0.32	-0.08	-0.096	.....	0.560	4
CaF <sub>2</sub>	m3m	0.056	0.228	0.024	.....	0.589	3, 4
Y <sub>2</sub> Al <sub>2</sub> O <sub>7</sub>	m3m	-0.029	+0.009	-0.062	.....	0.633	1
Y <sub>2</sub> Fe <sub>2</sub> O <sub>7</sub>	m3m	0.025	0.073	0.041	.....	0.633	1
Y <sub>2</sub> Ga <sub>2</sub> O <sub>7</sub>	m3m	0.091	0.019	0.079	.....	0.633	5
SrTiO <sub>3</sub>	m3m	0.15	0.095	0.072	.....	0.633	5
Diamond	m3m	-0.31	+0.09	-0.12	.....	0.589	3, 4
GaAs	$\bar{4}3m$	-0.165	-0.140	-0.072	$p_{12}$	1.15	1
GaP	$\bar{4}3m$	-0.151	-0.082	-0.074	.....	0.633	1
Ammonium alum.	m3	0.378	0.465	-0.009	0.454	0.589	3, 4
Potassium alum.	m3	0.275	0.354	-0.005	0.345	0.589	3
Barium nitrate	m3	$p_{11}-p_{12} = 0.992$	.....	-0.0205	$p_{11}-p_{12} = 0.713$	0.589	3
Lead nitrate	m3	$p_{11}-p_{12} = 0.281$	.....	-0.039	$p_{11}-p_{12} = 0.174$	0.589	3, 4

Crystal	Symmetry	$p_{11}$	$p_{12}$	$p_{44}$	$p_{13}$	$p_{21}$	$p_{33}$	$p_{66}$	$p_{14}$	$p_{41}$	Wavelength, $\mu\text{m}$	Ref.
NH <sub>4</sub> H <sub>2</sub> PO <sub>4</sub>	$\bar{4}2m$	-0.11	-0.15	-0.056	-0.93	0.20	-0.71	-0.077	0	0	.....	3
KH <sub>2</sub> PO <sub>4</sub>	$\bar{4}2m$	0.302	0.246	.....	0.236	0.195	0.263	0.075	.....	.....	0.633	1
		0.291	0.249	.....	0.246	0.225	0.221	0.058	.....	.....	0.560	3
TiO <sub>2</sub>	4/mmm	0.011	0.172	.....	0.168	0.0965	0.055	.....	0	0	0.633	1
Calcite	$\bar{3}2/m$	0.095	0.189	-0.090	0.215	0.309	0.178	$\frac{1}{2}(p_{11}-p_{12})$	-0.006	0.010	0.559	3, 4
$\alpha$ -Al <sub>2</sub> O <sub>3</sub>	$\bar{3}2/m$	0.25	0.038	0.10	$\leq 0.005$	0.032	0.23	$\frac{1}{2}(p_{11}-p_{12})$	<0.02	$\leq 0.01$	0.633	5
$\alpha$ -Quartz	32	0.138	0.250	-0.0685	0.250	0.258	0.008	$\frac{1}{2}(p_{11}-p_{12})$	-0.029	-0.042	0.589	3, 4
LiNbO <sub>3</sub>	3m	0.036	0.072	.....	0.092	0.178	0.088	$\frac{1}{2}(p_{11}-p_{12})$	0.070	0.155	0.633	1, 5
LiTaO <sub>3</sub>	3m	0.0504	0.0804	0.022	0.094	0.086	0.150	$\frac{1}{2}(p_{11}-p_{12})$	0.031	0.024	0.633	1
CdS	6mm	0.142	0.066	-0.054	.....	0.041	.....	.....	0	0	0.633	1
Beryl	6/mmm	0.010	0.175	-0.152	0.191	0.313	0.023	$\frac{1}{2}(p_{11}-p_{12})$	0	0	0.589	3
		.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Measured at  $\lambda = 0.589 \mu\text{m}$

Crystal	Symmetry	$p_{11}$	$p_{12}$	$p_{13}$	$p_{44}$	$p_{33}$	$p_{66}$	$p_{12}$	$p_{21}$	$p_{13}$	$p_{31}$	$p_{23}$	$p_{32}$	Ref.
Barite	mmm	0.21	0.24	0.31	0.002	-0.012	0.037	0.25	0.34	0.16	0.275	0.19	0.22	3, 4
Topaz	mmm	-0.055	-0.120	-0.083	-0.095	-0.031	-0.098	-0.069	0.093	0.052	0.095	0.065	0.085	3
Rochelle salt	222	.....	.....	.....	-0.096	-0.005	-0.016	.....	.....	.....	.....	.....	.....	3

\* Corning Glass Works.

The full matrix is given for the first example of each symmetry. Those labeled 0 are identically zero by symmetry; . . . indicates no value available.

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6m-5. **Linear Electro-optic Effect (Pockels Effect).** A linear interaction between the vectorial quantities, electric field or displacement, and the tensor quantities, stress or strain, is permitted by symmetry in the 20 piezoelectric crystal classes. The symmetry relations are exactly the same as for the piezoelectric effects, specifically the converse piezoelectric effect giving strain as function of applied field. The linear electro-optic effect has in recent years been termed Pockels effect in recognition of F. Pockels who made the first systematic studies in the 1890s. The defining set of equations is

$$\Delta a_i = r_{ij} E_j \quad (6m-7)$$

where the  $\Delta a_i$  are increments of the coefficient of the index ellipsoid as in Sec. 6m-4,  $E_j$  are the components of the electric field vector, and  $r_{ij}$  the electro-optic coefficients. There are 18 in the absence of any symmetry. The values of all electro-optic coefficients depend on the elastic boundary conditions. If the superscripts  $T$  and  $S$  denote respectively the conditions of zero stress (free crystal) and zero strain (clamped crystal), one finds

$$r_{ij}^T = r_{ij}^S + q_{ik}^E e_{jk} = r_{ij}^S + p_{ik}^E d_{jk} \quad (6m-8)$$

Here  $e_{jk} = (\partial T_k / \partial E_j)_S$  and  $d_{jk} = (\partial S_k / \partial E_j)_T$  are the customary piezoelectric coefficients (see Sec. 9f). Note that the order of subscripts in the electro-optic tensor is the reverse of the customary order in piezoelectric coefficients, where  $i$  indicates the electric and  $j$  the elastic variable. The  $r_{ij}^S$  are sometimes referred to as the direct electro-optic effect. It is generally observed at frequencies above the principal elastic resonances of the crystal which are typically in the 100-kHz range. The additional term in Eq. (6m-8) describes the "indirect" electro-optic effect which may have the same order of magnitude as the direct effect. The  $r_{ij}^T$  as well as the  $r_{ij}^S$  may be regarded as special values of a second-order dielectric constant relating dielectric displacement to two electric field components at different frequencies (see Sec. 6n, Nonlinear Optic Effects). One may also express the Pockels effect with dielectric displacement or polarization as independent variable. The coefficients so defined show less temperature dependence than the  $r_{ij}$ , especially in ferroelectric crystals.

The effect of symmetry on the electro-optic matrix for the crystal classes represented in Table 6m-5 is as follows:

Orthorhombic		Tetragonal		Trigonal		Cubic
222	$mm^2$	$4mm$	$\bar{4}2m$	32	3m	$\bar{4}3m$
0 0 0	0 0 $r_{13}$	0 0 $r_{13}$	0 0 0	$r_{11}$ 0 0	0 $-r_{22}$ $r_{13}$	0 0 0
0 0 0	0 0 $r_{23}$	0 0 $r_{13}$	0 0 0	$-r_{11}$ 0 0	0 $r_{22}$ $r_{13}$	0 0 0
0 0 0	0 0 $r_{33}$	0 0 $r_{22}$	0 0 0	0 0 0	0 0 $r_{33}$	0 0 0
$r_{41}$ 0 0	0 $r_{42}$ 0	0 $r_{42}$ 0	$r_{41}$ 0 0	$r_{41}$ 0 0	0 $r_{42}$ 0	$r_{41}$ 0 0
0 $r_{52}$ 0	$r_{51}$ 0 0	$r_{42}$ 0 0	0 $r_{41}$ 0	0 $-r_{41}$ 0	$r_{42}$ 0 0	0 $r_{41}$ 0
0 0 $r_{63}$	0 0 0	0 0 0	0 0 $r_{63}$	0 $-2r_{11}$ 0	$-2r_{22}$ 0 0	0 0 $r_{41}$

The hexagonal class  $6mm$  has the same matrix as  $4mm$ .

As in the elasto-optic case, optic path differences result from the  $\Delta a_i$  of Eq. (6m-7) after multiplication with  $n^3$ . The utility of crystals with about equal Pockels coefficients is therefore strongly dependent on the value of the refractive index. The specific equations for preferred orientation in all crystal classes have been tabulated.<sup>1</sup>

<sup>1</sup> O. G. Vlokh and I. S. Zheludev, *Soviet Phys.—Cryst.* **5**, 368-380 (1960).





TABLE 6m-5. LINEAR ELECTRO-OPTIC (POCKELS) EFFECT (Continued)

Crystal	Symmetry	Wave-length, $\mu\text{m}$	$r_{22}$	$r_{23}$	$r_{33}$	$r_{33} - (n_o/n_e)^3 r_{12}$ $= r_2$	Indices of refraction $n_o, n_e$	Relative dielectric constants $\epsilon_1, \epsilon_3$	Refs.
Tourmaline.....	3m	0.589	0.3	.....	.....	.....	1.70-1.63	8.2, 7.5	18
LiNbO <sub>3</sub> .....	3m	3.39	3.1	.....	.....	16	2.147, 2.081	84, 30	20-22
LiNbO <sub>3</sub> .....	3m	0.633	6.7	9.7	32.6	20.1	2.287, 2.200	84, 30	21-23
LiNbO <sub>3</sub> *.....	3m	0.633	3.4	8.6	28	21.1	2.237, 2.200	41, 29	24, 21, 22
LiTaO <sub>3</sub> .....	3m	0.633	~0.1	7.0	.....	23.5	2.175, 2.180	51, 45	25, 22
CdS.....	6mm	0.633	0	1.1*	3.7†	5.0	2.501, 2.519†	9.35, 10.3	26, 40, 5
Ba <sub>0.1</sub> Na <sub>0.4</sub> Nb <sub>2</sub> O <sub>6</sub> *.....	mm2	0.633	0	7.2	95	( $r_{12} = 79$ )	.....	.....	37
Ba <sub>0.1</sub> Na <sub>0.4</sub> Nb <sub>2</sub> O <sub>6</sub> .....	mm2	0.633	0	18	.....	( $r_{12} = \text{N.A.}$ )	2.326-2.221	245, 50	27
Sr <sub>0.7</sub> Ba <sub>0.3</sub> Nb <sub>2</sub> O <sub>6</sub> .....	4mm	0.633	0	67	42	.....	2.312, 2.299	~3400	25
K <sub>1.2</sub> Li <sub>0.4</sub> Nb <sub>2</sub> O <sub>6</sub> .....	4mm	0.633	0	9	.....	.....	2.277, 2.163	309, 100	28
K <sub>0.4</sub> Sr <sub>0.4</sub> Nb <sub>2</sub> O <sub>6</sub> .....	4mm?	0.633	0	.....	.....	.....	~2.25	~800	29
BaTiO <sub>3</sub> .....	4mm	0.546	0	.....	1,640	.....	2.436, 2.365	2920, 168	30-32
BaTiO <sub>3</sub> .....	4mm	0.633	0	8*	840*§	108	2.336, ~2.325	1970, 110*	31-34
K(Ta,Nb)O <sub>3</sub> [KTN]¶.....	4mm	0.633	0	.....	14,000	1,120	2.318, 2.275	.....	35

\* Values at constant strain.  
 †  $r_{12}$  at constant strain is (from top to bottom) 8.8, 21, and  $5.5 \times 10^{-13}$  m/V.  
 ‡ Measured at 0.589  $\mu\text{m}$ .  
 § Measured at 0.546  $\mu\text{m}$ .  
 ¶ Curie point = 60°C. Constants vary sharply with composition.

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6m-6. Electro-optic Effects in Ferroelectric Crystals. Ferroelectric crystals with high dielectric constants show higher-order electro-optic effects of significant magnitude. In oxide ferroelectrics, it has been found that these effects can be adequately described as quadratic (Kerr effect) if electric polarization  $P$ , instead of field  $E$ , is taken as the independent variable. Since the products of two electric polarization components form a symmetric tensor, the quadratic electro-optic coefficients  $g_{ik}$  have the same symmetry relation as the elasto-optic coefficients, and we have

$$\Delta a_i = g_{ij}(P^2)_j \quad (6m-9)$$

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where the index  $j = 1, \dots, 6$  gives the usual contracted tensor components. For the cubic class  $m\bar{3}m$ , there are three independent coefficients  $g_{11}, g_{12}, g_{44}$ . Above the Curie point (excluding its immediate vicinity),  $P$  is a linear function of  $E$ . In rationalized units, for cubic crystals

$$P = (\epsilon - 1)\epsilon_0 E \approx \epsilon\epsilon_0 E \tag{6m-10}$$

the dielectric constant  $\epsilon$  follows the law

$$\epsilon = \frac{C}{T - T_c} \tag{6m-11}$$

where  $C$  is the Curie constant, and  $T_c$  the Curie temperature. Equations (6m-9) to (6m-11) combine to

$$\Delta a_i = \frac{g_{ij} C^2 \epsilon_0^2 E^2}{(T - T_c)^2} \tag{6m-12}$$

The conventional Kerr constants are obtained by

$$K_{[100]} = \frac{2n^3(g_{11} - g_{12})\epsilon^2}{\lambda} \tag{6m-13}$$

or

$$K_{[110]} = \frac{2n^3 g_{44} \epsilon^2}{\lambda} \tag{6m-14}$$

for electric field in the crystallographic [100] and [110] directions, respectively. The values for crystals which are not cubic above the Curie point refer to quasi-cubic axes [4]. The  $g_{ik}$  show little temperature dependence, and apply also below the Curie point if the resultant of spontaneous polarization  $P_s$  and polarization induced by an applied field is inserted into Eq. (6m-9). The relation to the linear Pockels coefficients for an applied field small compared to the spontaneous polarization is of the form  $r_{33} = 2\epsilon_0 \epsilon_0 g_{11} P_s$ .

TABLE 6m-6. KERR CONSTANTS OF FERROELECTRIC CRYSTALS

Crystal	Symmetry	Wave-length, $\mu\text{m}$	Kerr constants, $\text{m}^4/\text{C}^2$				$n$	Curie temperature, K	Curie constant, K	Refs.
			$g_{11}-g_{12}$	$g_{11}$	$g_{12}$	$g_{44}$				
BaTiO <sub>3</sub>	$m\bar{3}m, 4mm$	0.633	+0.13	.....	.....	.....	2.29	370	170,000	1, 2
SrTiO <sub>3</sub>	$m\bar{3}m$	0.633	+0.14	.....	.....	.....	2.38	.....	.....	1
KTa <sub>0.88</sub> Nb <sub>0.12</sub> O <sub>3</sub>	$m\bar{3}m$	0.623	+0.174	+0.138	-0.038	+0.147	2.29	271	145,000	1, 3
KTaO <sub>3</sub>	$m\bar{3}m$	0.623	+0.16	.....	.....	+0.12	2.24	~4	.....	1, 3
LiNbO <sub>3</sub>	$3m$	0.633?	0.07	0.09 <sub>4</sub>	0.02 <sub>4</sub>	0.06 <sub>4</sub>	2.291-2.201	1468	.....	4, 6
LiTaO <sub>3</sub>	$3m$	0.633?	0.08	0.10	0.01 <sub>7</sub>	0.07	2.175, 2.180	883	.....	4, 5
Ba <sub>0.8</sub> Li <sub>0.2</sub> Nb <sub>0.2</sub> O <sub>3</sub>	$mm2$	.....	0.110	0.155	0.044	.....	2.326-2.221	833	.....	4

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