8j. Particle Accelerators

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8j-1. Introduction. Particle accelerators are devices designed to impart high kinetic energies to charged particles such as electrons, protons, and positive or negative ions of atoms and molecules. In the broadest sense this definition includes any device in which particle acceleration occurs, e.g., electron and ion guns, X-ray machines, cathode-ray tubes, etc. However, the term *particle accelerator* is commonly used only for machines which are capable of generating particle energies in excess of 1 MeV. By definition, 1 electron volt (eV) is the kinetic energy acquired by a particle with charge $e = 1.602 \times 10^{-19}C$ in falling freely across an electric potential difference of 1 volt: 1 eV = $1.602 \times 10^{-19}J$. Kinetic energies of 1 MeV and higher are necessary for the particles to penetrate and probe the structure of the nucleus of the atom or to achieve nuclear disintegration. At particle energies above 150 MeV mesons are created, whereas in the range of a few GeV[1] and higher, other members of the family of elementary particles, such as antiprotons, hyperons, etc., are produced in collisions of the accelerated particles with matter.

The genesis of particle accelerators and the construction of the various types of devices now existing form an exciting and interesting chapter of modern science and technology. The first successful accelerator was a small two-stage linear accelerator (linac) built by R. Wideröe [1] in 1928. It was followed in 1931 and 1932 by the invention of the electrostatic accelerator by R. J. Van de Graaff [2], the cyclotron by E. O. Lawrence [3], and the voltage multiplier by Cockroft and Walton [4], the last mentioned being the first device used for artificial nuclear disintegration. Other important milestones in the history of particle accelerators are: the invention of the betatron in 1928 by Wideröe [1] and the successful construction of the first machine of this kind by Kerst [5] in 1940 and 1941; the invention of the synchrotron principle [6] by McMillan and Veksler in 1945; the proposal of strong focusing by Christofilos in 1950 and, independently, by Courant, Livingston and Snyder in 1952 [7]; and the proposal of sector-focusing by L. H. Thomas in 1938 [8]; which led to the development of the isochronous cyclotron after the principle was tested in 1949 to 1956 at the Lawrence Radiation Laboratory [9].

During the four decades of accelerator development the energies of accelerated particles have increased on the average by roughly a factor of 10 every six years [10], the largest machines presently operating being the 33-GeV synchrotrons at Brookhaven National Laboratory and the European Nuclear Research Center (CERN) at Geneva, and the 70-GeV synchrotron at the Serpukhov in the USSR. A 500-GeV synchrotron is being built at the National Accelerator Laboratory, Batavia, Illinois.

The following review of the various types of accelerators and their design principles

[1] GeV = 1 billion electron volts = 10^9 eV; in the United States the term BeV is frequently used for 10^9 eV.

is organized in a topical rather than chronological order. After a general survey of the fundamental concepts and a classification of the various types of accelerators some useful general formulas are presented. This is followed by a review of the basic theory and mode of operation of each type of accelerator. Only the fundamental equations and major results of the theory are discussed. Important relations are presented in a numerical form suitable for calculations in practical units.

For a comprehensive and detailed account of particle accelerators several books are available; also one volume of the "Encyclopedia of Physics" is devoted to this topic. In addition, the proceedings of several international and national conferences on particle accelerators present a collection of valuable review and technical papers describing the developments during the last two decades. They are listed in the bibliography at the end of this section.

8j-2. Acceleration Principles and Types of Accelerators. The simplest method of acceleration is to let the particles cross a gap between two electrodes, one of which is at a high electrostatic potential with respect to the other. Electrical breakdown sets an intrinsic upper limit to the potential differences that can be achieved between electrodes, and, consequently, such single-gap or potential-drop accelerators are feasible only at low energies corresponding to potentials of less than about 10 MV. To get above this limit the energy must be accumulated in many steps by directing the beam through a series of gaps (or by multiple traversal of one gap), in which case time-varying electric fields must be employed. The "electrodes" are formed by conducting tubes which are separated by small gaps and connected to a rf power source. After passing through a gap, the particles travel through the field-free interior region of a tube. By the time they enter the gap on the other end of the tube, the electric field has reversed its polarity and the particles are accelerated again. As the velocity increases, the tube sections between gaps have to be longer to assure that the particles are in synchronism with the applied rf fields in the gaps. This is the principle of the linear accelerator (linac).

Since the length of a linear accelerator increases with increasing energy, a limit is set by the sheer physical size of a machine (the largest linac so far is 2 miles long!). This drawback is to a large extent avoided in circular or cyclic accelerators where a magnetic field is employed to force the particles into cyclic orbits, during which they pass many times through one or several rf acceleration gaps. The classical machine of this type is the cyclotron, and all circular accelerators are based on this principle.

Figure 8j-1 presents a classification of particle accelerators which divides the various types of machines into three main groups: d-c accelerators, linear accelerators, and circular accelerators. The first group, where only machines capable of acceleration to energies above 1 MeV are mentioned, can be subdivided into voltage multipliers and electrostatic accelerators. Linear accelerators can be grouped into proton or heavy-ion linacs and electron linacs.

Circular accelerators are divided into two branches, one representing the betatron, and the second branch comprising all the other circular machines. The latter are grouped according to the type of magnetic field employed for focusing the particles. In conventional cyclotrons, microtrons, and synchrocyclotrons, the magnetic field is axially symmetric, $B = B(r)$, and a constant negative gradient, $dB/dr < 0$, provides the focusing forces for the particles. In the isochronous cyclotron B is a function of both radius r and azimuth angle ϕ; the radial gradient of the average field $\bar{B}(r)$ is positive, i.e., $d\bar{B}/dr > 0$, and focusing is provided by the azimuthal variation (sector focusing). The constant-gradient synchrotrons use a negative field gradient for focusing like conventional cyclotrons; but, in addition the magnetic field is pulsed, $B = B(r,t)$, the time dependence being programmed to keep the orbit radius constant during acceleration. Finally, in the AG synchrotrons the magnetic ring consists of sectors with alternating gradients: a sector with $dB/dr < 0$ is followed by one where

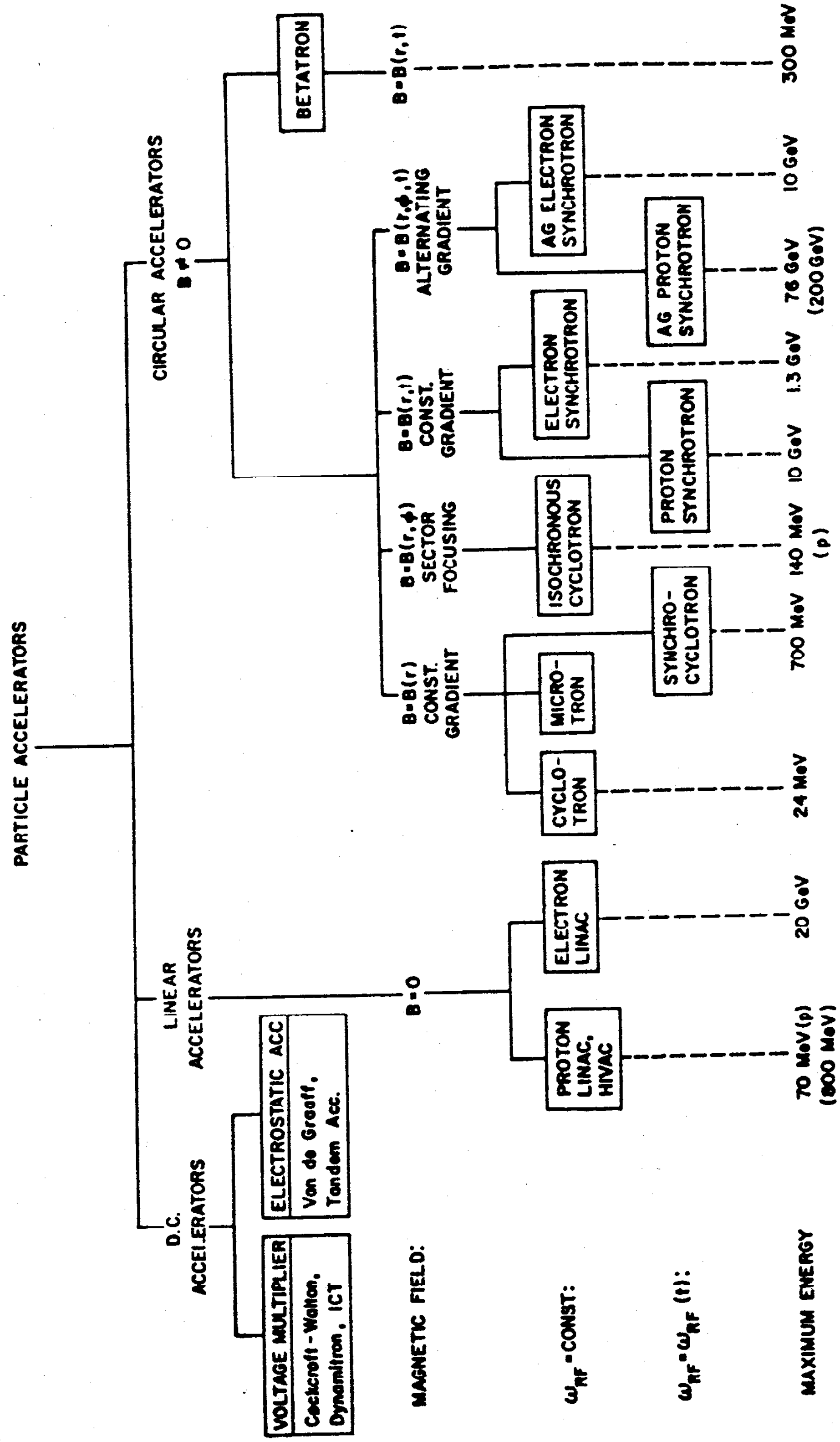

FIG. 8j-1. Classification of particle accelerators.

$dB/dr > 0$, which results in an overall strong focusing effect similar to the combination of focusing and defocusing lenses in optics.

Within each of these four groups further distinction is made between machines where the frequency of the accelerating rf fields is constant and those where it is varying with time.

Below each type of linear and circular accelerator the maximum energy achieved in an existing machine is given; figures in parenthesis indicate design goals for machines presently planned or under construction.

8j-3. General Relations and Beam Characteristics. In most accelerators particle velocities v reach values where relativistic effects are significant. This section presents a number of useful general relations in relativistic form, followed by general parameters and definitions which are commonly used to characterize the properties of an accelerator beam. Practical units (m, kg, sec, V, A, etc.) are used except where noted.

The relativistic mass increase is given by

$$\frac{m}{m_0} = \frac{E}{E_0} = \gamma = (1 - \beta^2)^{-\frac{1}{2}} = 1 + \frac{E_k}{E_0} \qquad \text{(8j-1)}$$

where $E_0 = m_0c^2$ = rest energy
$E = E_0 + E_k$ = total energy
E_k = kinetic energy,
$\beta = v/c$

It follows that

$$\gamma^2 = \frac{1}{1 - \beta^2} \qquad \text{(8j-2)}$$

$$\beta^2 = \frac{\gamma^2 - 1}{\gamma^2} \qquad \text{(8j-3)}$$

$$\beta = \frac{(E^2 - E_0^2)^{\frac{1}{2}}}{E} = \frac{[(E_k/E_0)^2 + 2E_k/E_0]^{\frac{1}{2}}}{1 + E_k/E_0} \qquad \text{(8j-4)}$$

Figure 8j-2 shows β versus kinetic energy for several particles.

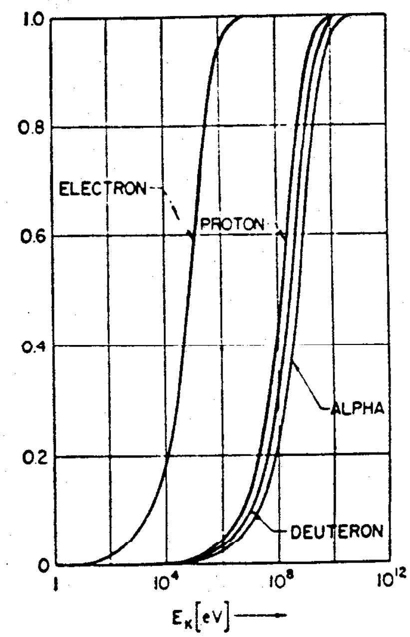

FIG. 8j-2. v/c of electrons, protons, deuterons, and α particles as a function of kinetic energy.

The relativistic relation between momentum and energy is

$$p = mv = \frac{1}{c}(E^2 - E_0^2)^{\frac{1}{2}} \qquad \text{(8j-5)}$$

The orbit radius of charged particles with momentum p and charge q in a magnetic field of strength B (Wb/m²) along the orbit is determined by the relation

$$R = \frac{p}{qB} = \frac{1}{qBc}(E^2 - E_0^2)^{\frac{1}{2}} = \frac{E_0}{qBc}\left[\left(\frac{E_k}{E_0}\right)^2 + 2\frac{E_k}{E_0}\right]^{\frac{1}{2}} \qquad \text{(8j-6)}$$

The radian or cyclotron frequency is

$$\omega = \frac{qB}{m} = \frac{qBc^2}{E} = \frac{qB}{m_0\gamma} = \frac{qBc^2}{E_0\gamma} \qquad \text{(8j-7)}$$

with the revolution period being $T = 2\pi/\omega$.

With $E_e = 0.511$ MeV being the rest energy of an electron, the rest energy of an ion can be calculated to good approximation as follows:

$$E_0 = MA - Z \cdot E_e(\text{MeV}) \qquad \text{(8j-8)}$$

where $A = 931.481$ MeV represents the atomic mass unit based on $^{12}C = 12$ (exactly); M is the mass number of the element, Z the number of electrons removed from the atomic shell (the binding energy of the removed electrons can be neglected). The rest energy of a number of ions of light elements which are of practical interest for accelerators are given in Table 8j-1.

TABLE 8j-1. REST ENERGIES OF SOME ISOTOPES AND IONS

Isotope	M, amu	Rest energy, MeV	Ion	Rest energy, MeV
^1H	1.007825	938.7698	$^1H^+$	938.2588
^2H	2.01410	1876.0959	$^2H^+$	1875.5849
^3He	3.01603	2809.3746	$^3He^+$	2808.8636
			$^3He^{2+}$	2808.3526
^4He	4.0026	3728.3459	$^4He^+$	3727.8348
			$^4He^{2+}$	3727.3238
^6Li	6.01512	5602.96999	$^6Li^+$	5602.458999
			$^6Li^{3+}$	5601.43698
^{12}C	12.0	11,177.772	$^{12}C^{3+}$	11,176.23898
			$^{12}C^{6+}$	11,174.70597
^{14}N	14.00307	13,043.59365	$^{14}N^+$	13,043.08265
			$^{14}N^{7+}$	13,040.01662
^{16}O	15.99491	14,898.9548	$^{16}O^+$	14,898.4438
			$^{16}O^{8+}$	14,894.8667
^{20}Ne	19.99244	18,622.578	$^{20}Ne^+$	18,622.06699
			$^{20}Ne^{10+}$	18,617.46796

Magnetic fields are usually measured in units of gauss (G) or kilogauss (kG) rather than in the unit of the practical international system which is variously denoted with webers per square meter (Wb/m²) or tesla, where 1 Wb/m² = 10 kG. The "rigidity" of a charged particle in a magnetic field can be expressed by the formula

$$RB = 3.33564 \times 10^{-2} \frac{(2E_0E_k + E_k^2)^{\frac{1}{2}}}{Z} \tag{8j-9}$$

Solving for the kinetic energy yields

$$E_k = (E_0^2 + 898.755\ R^2B^2Z^2)^{\frac{1}{2}} - E_0 \tag{8j-10}$$

where R is in meters, B in kG, and the energies in MeV.

Figure 8j-3 shows a plot of the rigidity versus kinetic energy for electrons, protons, and deuterons (on a logarithmic scale). The interesting feature is that at extremely relativistic energies (10 GeV and above) the rigidity is the same for all three particles. Also note that in this region RB increases to good approximation linearly with E_k, as follows from Eq. (8j-9) if $E_k \gg E_0$. At low, nonrelativistic energies $(E_k < E_0)RB$ is proportional to $\sqrt{E_k}$, and one can use the formula

$$RB = 1.44 \frac{(ME_k)^{\frac{1}{2}}}{Z} \tag{8j-11}$$

where again R is in m, B in kG, and E_k in MeV.

For the orbital frequency $f = 2\pi/\omega$ of the particles one obtains the relations

$$f = \frac{1.43041 \times 10^3 \times Z \times B}{\gamma E_0} = \frac{1.53563 \times Z \times B}{M\gamma} \tag{8j-12}$$

where f is in MHz, B in kG, and E_0 in MeV. Specifically,

$$f\,(\text{MHz}) = \frac{1.52454B(\text{kG})}{\gamma} \quad \text{for protons} \tag{8j-13}$$

and

$$f\,(\text{GHz}) = \frac{2.79922B(\text{kG})}{\gamma} \quad \text{for electrons} \tag{8j-14}$$

The characteristics and quality of a particle beam obtained from an accelerator is usually described in terms of intensity, kinetic energy, time structure, energy spread, and phase-space area or emittance. The various parameters associated with this description of beam properties are explained in Figs. 8j-4 and 8j-5. All beams from linacs, cyclotrons, and synchrotrons are composed of short bursts spaced at intervals determined by the oscillation period $T = 2\pi/\omega_{rf}$ of the accelerating r-f (or microwave) fields. The width of the individual bursts τ (which may be defined as full width at

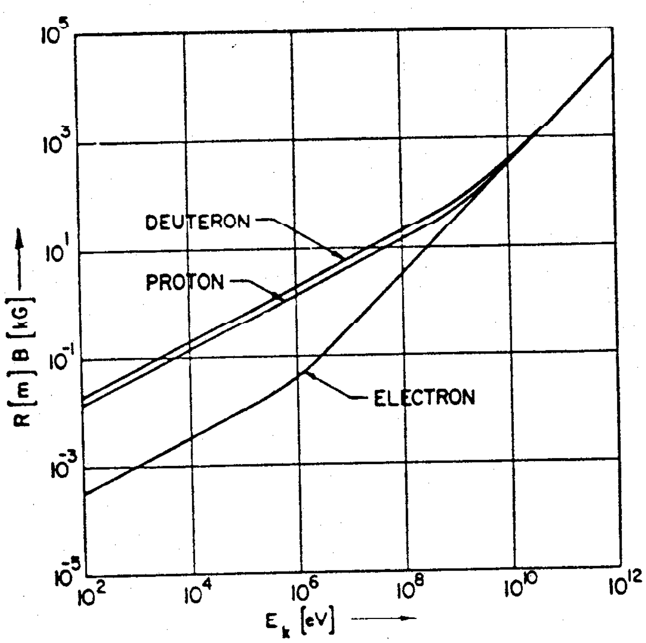

FIG. 8j-3. Rigidity RB of electrons, protons, and deuterons versus kinetic energy.

half maximum) is usually much smaller than T, and the ratio $\delta = \tau/T$ is known as the (*microscopic*) *duty factor*. In cyclotrons the "pulse train" of ion bursts is continuous (Fig. 8j-4*b*). Synchrocyclotrons and synchrotrons, where ω_{rf} is modulated, and linacs operating in pulsed schemes to avoid excessive power problems, generate beams of a pulsed nature, as shown in Fig. 8j-4*c*. The repetition period T_m is determined by the modulation frequency or repetition rate. The width τ_m of the macroscopic pulses, which may contain many thousands of "microbursts," defines what is known as the *macroscopic duty* factor $\delta_m = \tau_m/T_m$. If I_0 denotes peak current, then the time-averaged current is roughly $I = I_0\delta$ in Fig. 8j-4*b* and $I = I_0\delta\delta_m$ in Fig. 8j-4*c*.

The state and history of a beam during the acceleration process can be described, in principle, by the space and momentum coordinates of all the particles in six-dimensional phase space and the phase-space particle density as a function of time. If the mutual interaction of the particles can be neglected, Liouville's theorem states that the volume occupied by the beam in phase space remains constant, even though the shape of this volume may change considerably during acceleration. Let x denote the distance from the center line or axis of the beam, and p_x the transverse momentum of a particle in the x direction and a given time, as illustrated in Fig. 8j-5. If the external electric and magnetic forces acting on the particles in the three directions are not coupled, or in the absence of such forces, the area occupied by the beam in two-dimensional phase space, $\int dp_t\, dx$, remains a constant. Under ideal conditions this

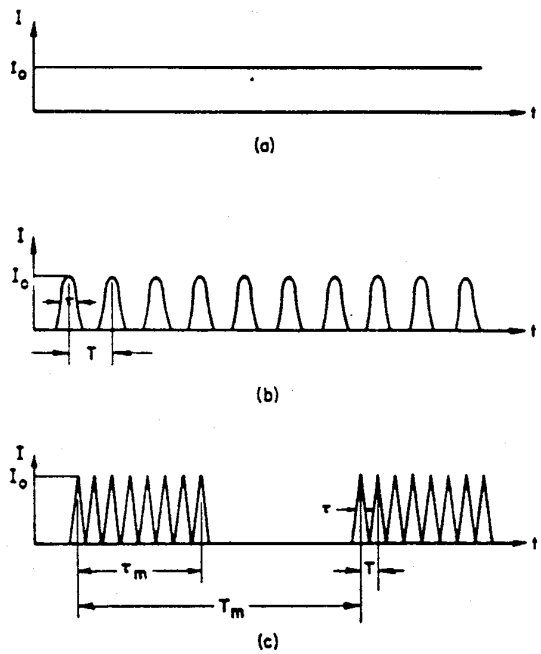

FIG. 8j-4. Typical time structure of accelerator beams. (a) Continuous beam from d-c accelerators. (b) Continuous rf beam (cyclotron); microscopic duty factor $\delta = \tau/T$. (c) Pulsed rf beam (pulsed linacs synchrocyclotrons, etc.); macroscopic duty factor $\delta_m = \tau_m/T_m$; microscopic duty factor $\delta = \tau/T$.

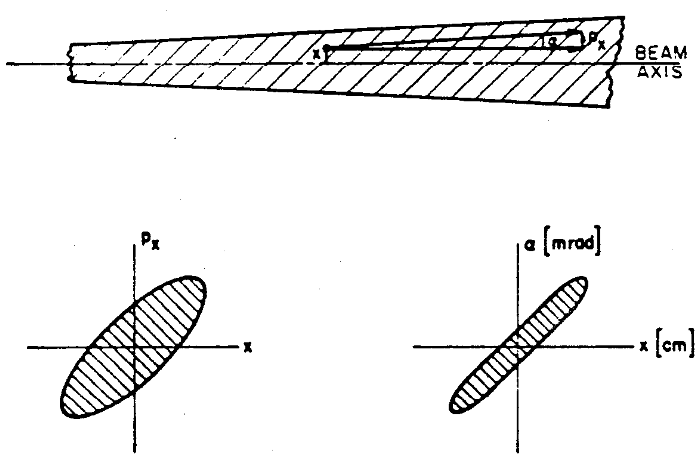

FIG. 8j-5. Transverse phase space or emittance of accelerator beam.

area has the shape of an ellipse, as depicted in Fig. 8j-5. However, in practice this shape may be quite irregular and "filamented," which is an indication that nonlinear effects and other factors have caused a deterioration of beam quality. For practical purposes it is more convenient to express the beam quality in terms of the angle of divergence α; in most cases this angle is very small so that $\alpha = p_r/p$, where p is the total momentum of the particles. The area filled by the beam in $\alpha - x$ "phase space" is called the *emittance* and usually measured in cm mrad (see Fig. 8j-5) If the beam has no axial symmetry, the emittance in the other transverse direction must also be given to fully describe the beam quality. In contrast to $\int dp_x\, dx$, the emittance $\int d\alpha\, dx$ is not constant but is inversely proportional to the momentum. For comparison of different accelerators or beams of different energy it is, therefore, better to define the emittance in terms of cm · mrad $\text{MeV}^{-\frac{1}{2}}$.

The energy spread of an accelerator beam is commonly defined as $\Delta E/E$, where ΔE represents the full width at half maximum of the intensity-versus-energy curve.

8j-4. D-C Accelerators. *Voltage Multipliers.* Voltage-multiplying circuits had been developed by Schenkel in 1919 and Greinacher in 1921 [11]. Cockroft and

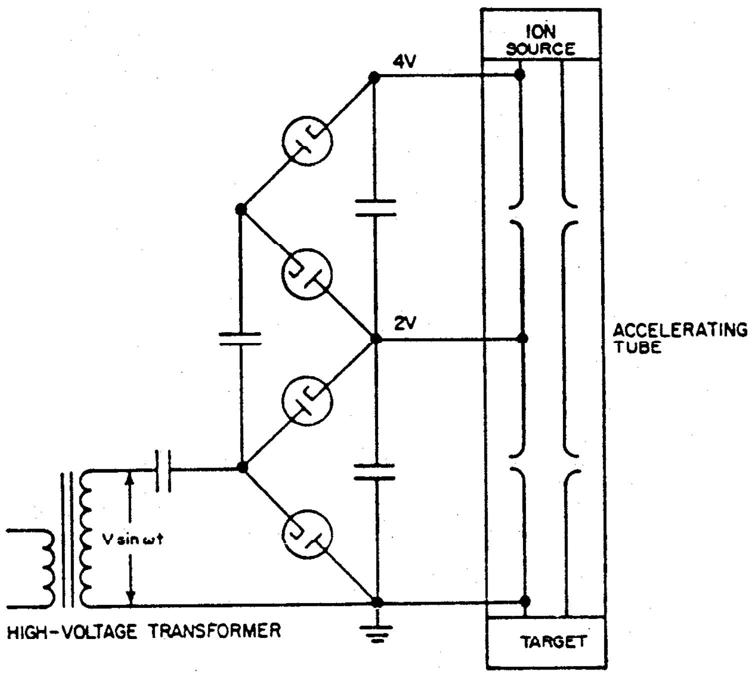

FIG. 8j-6. Schematic diagram of Cockcroft-Walton accelerator.

Walton adapted the Greinacher scheme for their famous accelerator which achieved the first nuclear disintegration in 1932 [4]. Accelerators based on this principle of voltage multiplication are now commonly called Cockroft-Walton machines. The circuit, which achieves voltage multiplication through a cascade-rectification scheme, is shown in Fig. 8j-6. The ac or r-f voltage across the secondary winding of the high-voltage transformer is applied to a multistage combination of capacitors and rectifiers. With no load current the rectified output voltage is equal to the total number of capacitors times the peak voltage of the transformer. Cockroft-Walton used a capacitor-rectifier system with a voltage-multiplication factor of 4 and an output voltage of about 700 kV.

Under load conditions the voltage is reduced below the ideal maximum by an amount which is proportional to the current and inversely proportional to the frequency input voltage.

Voltage-multiplier systems of the Cockroft-Walton type are used in many laboratories, as high-voltage power supplies, generators for intense neutron beams, or injectors for linear accelerators. A special high-frequency version of the Cockroft-Walton is the Dynamitron, which was developed by Radiation Dynamics, Inc. and generates voltages up to 3 MV with beam currents up to 10 mA.

Another type of voltage-multiplication accelerator is the insulating-core transformer (ICT) developed by the High-Voltage Engineering Corporation. This machine is basically a transformer with a magnetic circuit consisting of insulated segments. The outputs of the secondary windings of each segment are rectified and connected in series, yielding direct voltages up to 4 MV with maximum beam currents in the range of 10 mA.

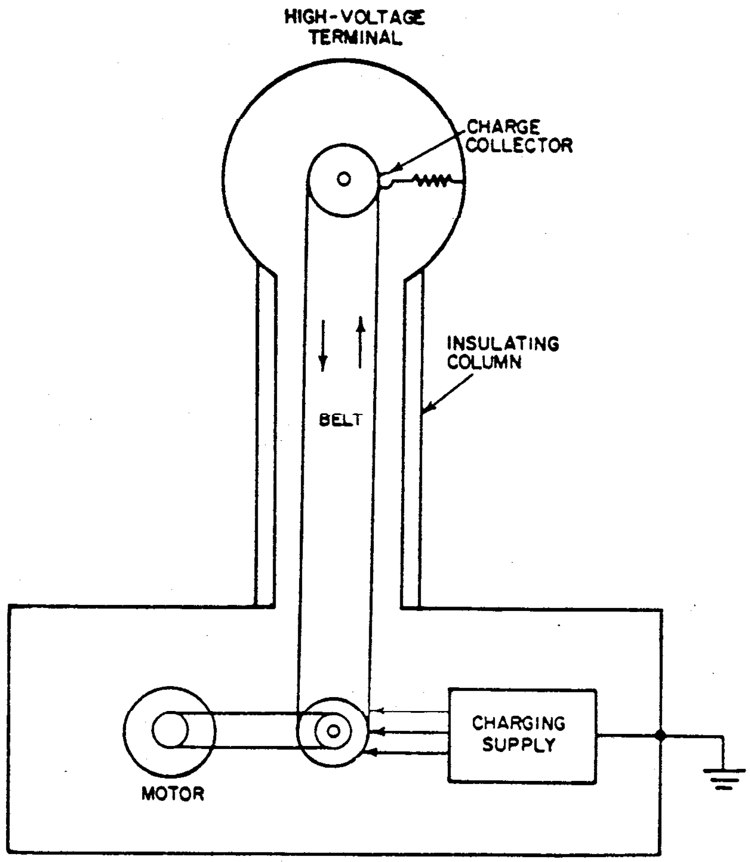

Fig. 8j-7. Schematic diagram of Van de Graaff accelerator.

The Electrostatic Accelerator. The most successful dc accelerator is the electrostatic generator developed by R. J. Van de Graaff in 1930. The "Van de Graaff," as it is commonly called, is relatively simple in concept but involves a sophisticated technology in coping with the numerous engineering problems, primarily the fundamental limitations due to electrical breakdown. The principle of operation is illustrated schematically in Fig. 8j-7. The main structure is a large metal sphere, called the high-voltage terminal, which is supported by an insulating column and charged to high potentials through a moving belt. At the grounded end an electric power supply with 20 to 30 kV voltage produces a corona discharge from a row of sharp metallic needles pointed at the motor-driven belt. A charge of the desired polarity is sprayed on the belt, which carries it to the high-voltage terminal where it is removed by a collecting device. As the charge deposited on the sphere is increased, the potential rises until the charging current is balanced by the accelerated ion beam and leak currents to ground (or until breakdown occurs).

The electron gun or ion source is housed inside the terminal, and the beam is accelerated to ground through the accelerating tube which may be in the same insulating column as the charging belt or, as in early models, is located in a separate column.

Early Van de Graaff generators operated in air where voltages up to 2 MV could be achieved. However, at higher voltages where air-insulated machines become unreasonably large in size, pressurized tanks with gas mixtures of nitrogen and carbon dioxide are used for insulation. With gas pressures of several hundred pounds per square inch, voltages as high as 8 to 10 MV have been achieved in such pressurized models. Pressure-tank Van de Graaffs have been built as vertical as well as horizontal machines and for acceleration of electrons as well as ions.

In recent years a multistage electrostatic generator, the so-called "tandem accelerator," has been developed by the High Voltage Engineering Company. In the simplest model, the two-stage tandem, the positive high-voltage terminal is in the center of a horizontal cylindrical pressure tank with accelerating columns on either side. Both ion source and target are at ground potential. Negative ions produced in a special source are injected into one of the accelerating tubes and accelerated to the terminal in the center. Inside the terminal they pass through a gas cell where two or more electrons are stripped, leaving the ions in a positive-charge state. These positive ions are then accelerated again through the second acceleration tube and arrive at the other end at ground potential with an energy twice that available from a conventional single-stage Van de Graaff of the same terminal voltage. By adding another two-stage tandem with opposite terminal potential, even higher final energies can be achieved. In one version the particles are injected as neutral atoms, unaffected by the electric field, until they reach the negative terminal where an electron is added in a special gas canal. Then acceleration occurs to the other end at ground, and from there into the positive terminal of the second tank. At this point electrons are stripped, and the positive ions are further accelerated to ground, the final energy being three times that of a single-stage generator. In yet another version, the positive ions are returned to the first stage through bending magnets and accelerated to the negative terminal where they hit the target with an energy equivalent to four times the terminal voltage. With terminal voltages of 8 MV, energies between 20 and 30 MeV (for single-charged particles) can thus be obtained from such multistage tandems.

Many hundreds of electrostatic accelerators have been built throughout the world. Most of them are used as high-precision tools for nuclear reaction studies and nuclear spectroscopy at low energies.

8j-5. Linear Accelerators. *Proton and Heavy-ion Linacs.* The concept of the type of linear accelerator originally proposed by Wideröe is easily explained with the help of Fig. 8j-8*a*. The accelerating structure consists of a straight-line array of hollow cylindrical metal tubes of increasing length through which the beam is traveling. The tube sections are separated by small gaps, and alternate tubes are connected to opposite terminals of an ac generator; thus a time-varying sinusoidal electric field is produced across the gaps. To obtain continuous acceleration the electric potential of the tubes must change polarity while the particles coast through the field-free interior of the so-called drift tubes. As the particle velocity v increases, with the frequency f of the supplied alternating voltage being fixed, the length L of the tube sections has to be increased to maintain "resonance" between particles and electric field. If L_n is the length of the nth gap-plus-tube section, τ the period, and λ the wavelength of the ac field, then

$$L_n = v_n \frac{\tau}{2} = \frac{v_n}{2f} = v_n \frac{\lambda}{2c} = \frac{\beta_n \lambda}{2} \qquad \text{(8j-15)}$$

Under nonrelativistic conditions $v_n \approx (2E_{kn}/m_0)^{\frac{1}{2}}$, and if the energy is gained in equal increments of ΔE_k, we can write

$$L_n \approx \frac{\lambda}{2c}\left(\frac{2E_{kn}}{E_0}\right)^{\frac{1}{2}} = \frac{\lambda}{2}\left(\frac{2n\,\Delta E_k}{E_0}\right)^{\frac{1}{2}} \qquad (8j\text{-}16)$$

This shows that the length of the sections must increase roughly as the square root of the section number n. If, specifically, n denotes the last section, then the final kinetic energy, $E_{kn} = (2/\lambda^2)L_n{}^2E_0$ is fixed, being determined only by particle rest mass, wavelength λ, and the geometry. Thus the linear accelerator does not permit a variation of energy, as is possible with dc machines. In the Wideröe-type linac the wavelength λ of the applied field is large compared to the length of a tube section L_n ($\beta_n \ll 1$, $\lambda \gg L_n$), so that each drift tube is essentially at a (spatially) constant

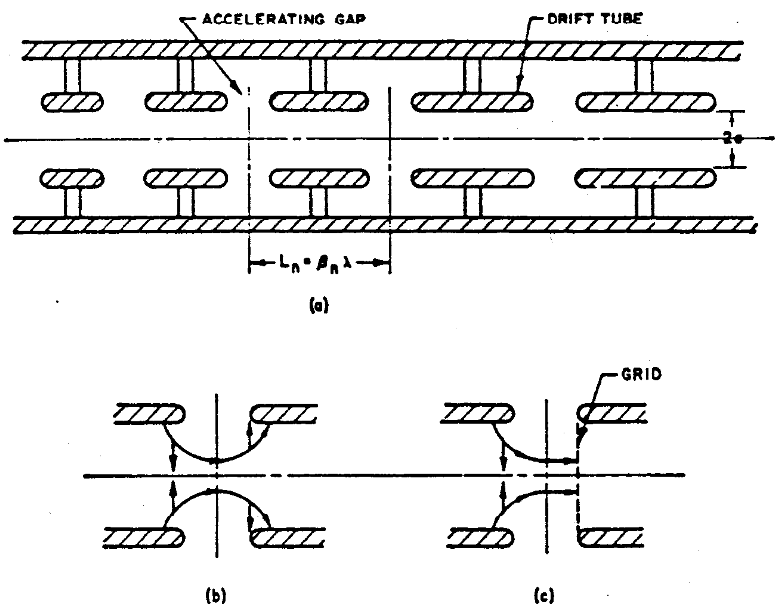

FIG. 8j-8. Linear accelerator. (a) Acceleration system of a proton linac. (b) Electric force lines in the accelerating gap. (c) Accelerating gap with focusing grid.

potential. On the other hand, the total length of a linac is proportional to λ, which implies that such an accelerator becomes excessively large if it is designed for low-frequency operation.

All modern linacs differ from the original Wideröe scheme in that they are designed as short-wave cylindrical resonant cavities or waveguides in either a standing-wave or a traveling-wave mode of operation. The mode of operation depends on the particle velocities. Electrons are injected at sufficiently high energies that v is equal to the speed of light, which permits the use of traveling waves, as will be described in the next section. With protons and heavier ions, on the other hand, velocities usually remain far below c, and a standing-wave mode has to be employed. Because of this fundamental difference, electron linacs generally operate at frequencies of several thousand MHz while proton accelerators usually work with frequencies in the range of 100 to 200 MHz.

The present form of the standing-wave linac for protons and heavy ions was first worked out by L. W. Alvarez in 1945 and built at Berkeley in 1946 and 1947 [12]. In this design the cylindrical tank enclosing the drift tubes forms a resonant cavity which is excited in the TM_{010} mode with the wavelength of the longitudinal electric field given by $\lambda = 2.61a$ (where $2a$ = diameter of tank). The nodes of the standing field waves are at the center of the drift tubes; the maxima at the gaps. Thus the electric field has the same direction in each gap at any given instant of time. If the particles are to be continuously accelerated, the drift time inside the tubes between consecutive gap crossings must be equal to a full rf cycle. The resonance condition for an Alvarez-type linac is therefore

$$L_n = v_n \tau = \beta_n \lambda \tag{8j-17}$$

The energy gain per gap, neglecting the finite transit time, is given approximately by

$$\Delta E_k = E_z g \cos \phi \tag{8j-18}$$

where E_z is the average electric field on the axis, g the gap width, and ϕ the phase of the field at the moment of particle transit. A particle is synchronous with the rf field if it crosses each gap at the same phase ϕ_s. A fundamental requirement for a resonance accelerator like the linac is the existence of phase stability which assures that nonsynchronous particles (ϕ different from ϕ_s) are not lost during the acceleration process. Simple consideration shows that this is achieved only if ϕ_s lies in the phase interval where E_z is increasing, i.e., $-\pi/2 < \phi_s < 0$. In this case particles crossing the gaps at phases different from ϕ_s are forced into phase oscillations about ϕ_s, provided the starting phase is within certain limits.

Unfortunately this principle of phase stability is incompatible with the requirements of transverse focusing. The electric fields in the gaps constitute electric lenses which are focusing only if the transverse field components in the entrance region (see Fig. 8j-8*b*) are stronger than the defocusing components in the exit half of the gap, i.e., if $0 < \phi < \pi/2$. This dilemma was solved in earlier designs by the use of grids (Fig. 8j-8*c*), reducing the defocusing field components, and/or the use of solenoidal magnetic lenses incorporated in the drift tubes. The latest linacs, however, generally employ magnetic quadrupole lenses as proposed by Blewett [13], which provide the most effective means of focusing resulting in only negligible beam loss.

One of the greatest drawbacks of any linear accelerator is the high power required, which is in the range of megawatts. It is therefore necessary to operate linacs in a pulsed scheme with relatively low (macroscopic) duty factor to keep power losses at a manageable level.

The largest existing proton linac in the world is the 100-MeV injector for the Serpukhov 75-GeV synchrotron. (The 70-MeV linac at Minnesota which was the largest machine in the past, has recently been shut down.) A 800-MeV proton machine is being built at Los Alamos.

Electron Linacs. In the electron linac the electron velocity is practically equal to the speed of light, which permits operating the cavity as a waveguide in a traveling-wave mode. No drift tubes are necessary since the electrons are riding on the crest of the wave, being continuously accelerated to full energy. However, the phase velocity of a traveling wave in an empty waveguide is greater than the speed of light, and to reduce it to the value c, it is necessary to load the cavity with disk-shaped irises spaced at intervals of $\lambda/4$.

The first successfully operating electron linacs were developed in 1945 to 1947 by W. W. Hansen and collaborators at Stanford and D. W. Fry and coworkers at the Telecommunications Research Establishment in England [14].

In contrast to the proton linac, transverse focusing poses no great problem in electron linacs. This is due to the fact that, at the much higher frequencies of the electron linacs, the azimuthal magnet field B_θ resulting from the time-varying electric field produces a focusing force vB_θ which is comparable to the defocusing qE_r term.

The net defocusing force is proportional to $1 - \beta^2$ and thus goes to zero as β approaches 1.

The largest electron accelerator in the world is the 2-mile linac at Stanford which is designed for electrons of 20-GeV energy.

8j-6. The Conventional Cyclotron. The cyclotron, invented by Lawrence in 1930, was the first successful circular accelerator. It is based on the fact that a magnetic field B forces charged particles into circular orbits with angular frequency $\omega = qB/m$ (Eq. 8j-7) and orbit radius $R = v/\omega$ (Eq. 8j-6). During each revolution the particles

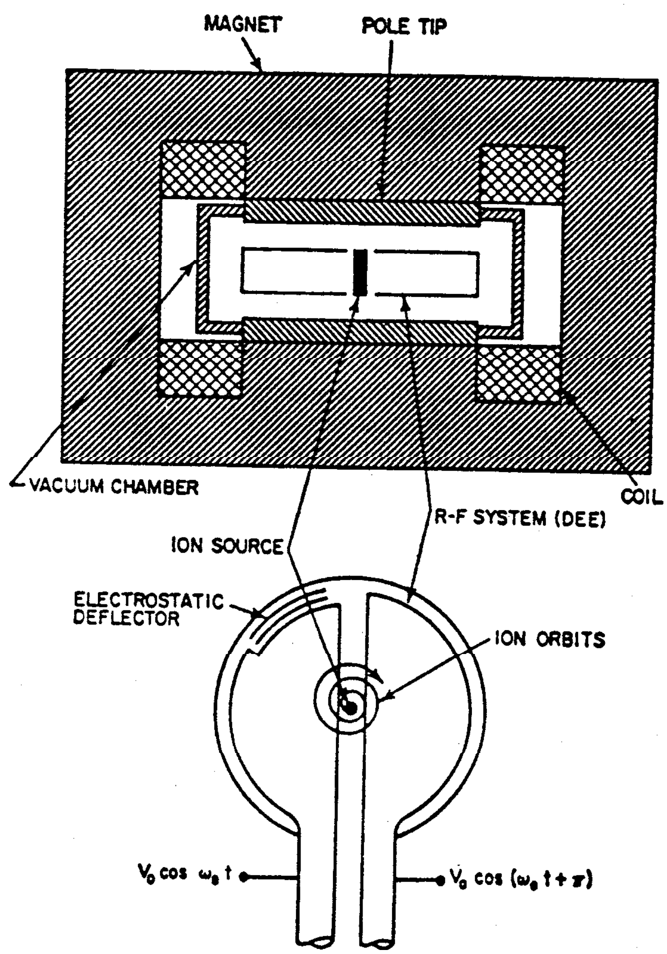

FIG. 8j-9. Cyclotron.

pass through an acceleration gap across which a r-f voltage $V = V_m \cos \omega_e t$ is maintained. When the radio frequency is in "resonance" with the circulating ions, i.e., when $\omega_e = \omega$, continuous acceleration occurs, and the ions travel on an expanding spiraling orbit from the center of the magnetic field ($R = 0$) to some maximum energy and radius determined by the size of the pole shoes of the magnet. The r-f system consists of a large pillbox-type structure split into two halves, which are shaped like a D and therefore called "dees" (see Fig. 8j-9). Each dee is part of a quarter-wave resonance system. The two resonators are oscillating in a push-pull mode (180 deg out of phase); i.e., $V_1 = V_0 \cos \omega_e t$, $V_2 = V_0 \cos (\omega_e t + \pi)$, and the peak voltage across the acceleration gap is $V_m = 2V_0$. The dees are located in the gap between the poles of an electromagnet and are enclosed by a vacuum chamber. The ions are produced in a low-pressure gas-discharge tube at the center of the magnetic field. Ion source design and technology have been considerably improved over the years; the "open-arc source" of the early days was replaced by the "hooded" structure, and today most cyclotrons use the "chimney"-type source with reflector developed by R. S. Livingston and R. J. Jones at Oak Ridge [15].

The magnetic field in the conventional cyclotron must decrease slightly with radius to produce the required force component toward the median plane, which serves to focus the beam during the many revolutions from center to maximum radius. The equation of motion for the z direction (perpendicular to the median plane) is

$$\ddot{z} = \frac{q}{m} v B_r \tag{8j-19}$$

Using the linear term of the Taylor expansion, $B_r = (\partial B_r/\partial z)z$, and div $B = 0$, or $\partial B_r/\partial z = \partial B_z/\partial r$, we can write

$$\ddot{z} = \frac{q}{m} v \frac{dB}{dr} z = \frac{qB}{m} \frac{\omega}{r} \frac{r}{B} \frac{dB}{dr} z \tag{8j-20}$$

where $B = B_z(r)$ is the field in the median plane. Introducing the "field index" $n = -(r/B)dB/dr$, and azimuth angle $\theta = \omega t$, Eq. (8j-20) can be written in the form

$$\frac{d^2z}{d\theta^2} + \nu_z^2 z = 0 \tag{8j-21}$$

where

$$\nu_z^2 = n \tag{8j-22}$$

If $n > 0$, or $dB/dr < 0$, the particles perform stable oscillations about the median plane. These oscillations are known as *betatron oscillations* because they were first investigated in connection with the betatron [16]. The parameter ν_z measures the number of betatron oscillations per revolution.

A similar equation can be derived for the radial motion, yielding for the radial betatron frequency the relation

$$\nu_r^2 = 1 - n \tag{8j-23}$$

with the stability condition $n < 1$.

Thus simultaneous stability in both vertical and radial directions can exist only if

$$0 < n < 1 \tag{8j-24}$$

The focusing requirement of $dB/dr < 0$ implies that the orbital frequency $\omega = qB/m$ of ions is not a constant, but decreases with radius. As a result, the resonance condition $\omega_e = \omega$ is violated, particles get out of step with the r-f voltage, and after a certain number of turns the phase slip is large enough so that deceleration occurs. This dilemma is still further enhanced by the relativistic mass increase which, unfortunately, goes in the same direction as a radially decreasing field. In practice, the electric frequency is set to correspond with the orbital frequency at some intermediate radius r_c. Inside this radius the phase shift per turn, $\Delta\phi = 2\pi(\omega_e - \omega)/\omega$, is negative ($\omega > \omega_e$); the ion phase ϕ with respect to the radio frequency reaches a minimum $\phi_{min} < 0$ at r_c, then increases in the region $r > r_c$ (where $\Delta\phi > 0$), goes through the peak-voltage phase $\phi = 0$, and reaches a maximum value ϕ_{max} close to $\pi/2$ at the final radius. At this point, just before deceleration would occur, the particles have reached the maximum energy attainable and enter an electrostatic deflector which extracts them out of the magnet for bombardment of an external target.

The maximum energy attainable in this type of cyclotron depends on the number of revolutions, which is inversely proportional to the peak dee voltage. The largest conventional machine is the 86-in. cyclotron at Oak Ridge National Laboratory, which accelerates protons to 24 MeV (with a dee-to-ground voltage of 250 kV, or

$V_m = 500$ kV!). With the development of the sector-focusing cyclotrons, conventional cyclotrons are no longer built, and many existing machines are being converted or shut down.

8j-7. The Microtron. Cyclotrons are only capable of accelerating protons or heavy ions since the orbital frequencies of these particles are low enough to permit the use of quarter-wave resonators where the wavelength λ is substantially larger than the magnet pole diameter. Electron frequencies for magnetic fields between 10 and 20 kG are in the range of several thousand MHz, and quarter-wave resonance systems are impractical (λ is too small). Besides, the relativistic mass increase of electrons begins at much lower energies (1 percent increase at 5 keV!) than that for protons.

The microtron, or electron cyclotron, first proposed by Veksler [6] is a cyclotron-type device for the acceleration of electrons. It employs a small microwave cavity near the periphery of the magnetic field, through which the electron beam passes once per revolution. The orbits form a family of circles of increasing radius with a common tangent at the point where they intersect the cavity gap. Resonance acceleration occurs if the electrons cross the gap at the same voltage phase in each revolution. The rotation period of the electrons is given by

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi E}{eBc^2} = \frac{2\pi E_0}{eBc^2}\gamma = \tau_0\gamma \tag{8j-25}$$

Resonance exists if the electron rotation period on the first revolution τ_1 and the difference $\Delta\tau = \tau_{n+1} - \tau_n$ between consecutive orbits are each equal to some integral multiple of the r-f period τ_{rf}, i.e.,

$$\tau_1 = \tau_0\gamma_1 = \tau_0\left(1 + \frac{E_{k1}}{E_0}\right) = k\tau_{rf} \tag{8j-26}$$

$$\Delta\tau = \tau_{n+1} - \tau_n = \tau_0\Delta\gamma = \tau_0\frac{\Delta E_k}{E_0} = m\tau_{rf} \tag{8j-27}$$

where k and m are positive integers.

If $E_{k1} = \Delta E_k$, we find by elimination of τ_{rf} the relation

$$\frac{\Delta E_k}{E_0} = \frac{m}{k - m} \tag{8j-28}$$

In contrast to the cyclotron, the energy gain in a microtron cannot be arbitrary but must occur in fractions of the rest energy E_0 as determined by the choice of the integers k and m in Eq. (8j-28). Note also that k must be larger than m (minimum k value is 2) in order for ΔE_k to be finite and positive. For $k = 2$, $m = 1$ the energy gain must be equal to the rest energy.

It should be pointed out that the magnetic field in a microtron is practically uniform, $2\pi E_0/eBc^2 = \tau_0 =$ const, which implies that there is little or no vertical focusing. However, if the number of orbits is not too large, this is not a serious problem. Microtrons for electron energies between 1 and 30 MeV have been built at many places. However, very little is known about operating experience and performance characteristics.

8j-8. The Synchrocyclotron. In 1945 McMillan and Veksler independently proposed the synchrotron and synchrocyclotron [6] which made it possible to get beyond the energy limits of the conventional cyclotron. The two basic ingredients in this new accelerator concept are (1) the modulation of the electric frequency (and in the synchrotron also the magnetic field) with time to achieve synchronism between radio frequency and circulating particles; (2) the existence of phase stability which assures the continuous acceleration of nonsynchronous particles within certain limits.

The synchrocyclotron employs a cyclotron-type r-f system with frequency ω_e modulated by the use of a rotating capacitor, tuning fork, or other means, such that ω_e is a function of time, decreasing in synchronism with the orbital frequency of the ions. After a group of ions is accelerated to full energy, the radio frequency returns to its starting value and begins another cycle of acceleration. The major drawback of this scheme is that beam intensities are down by a factor of 10^2 to 10^4 compared to those of the fixed-frequency cyclotrons. On the other hand, the substantial increase in particle energy more than outweighs this disadvantage. Many synchrocyclotrons were built throughout the world, the largest machines producing protons of more than 600 MeV.

The time variation of the electric frequency in a synchrocyclotron is determined by the rate of change of the orbital frequency of the synchronous particle:

$$\frac{d\omega_s}{dt} = -\frac{\omega_s^2 KqV \cos\phi_s}{2\pi E_s} \tag{8j-29}$$

where

$$K = 1 + \frac{n}{(1-n)\beta_s^2} \tag{8j-30}$$

and $qV \cos\phi_s$ is the energy gain per turn, and E_s the total energy of the synchronous particle. A particle which passes the acceleration gap at a phase ϕ different from ϕ_s will gain a different amount of energy, and, therefore, its orbital radius will be slightly different from that of the synchronous particle. If $\Delta p/p$ is the fractional difference in momentum between nonsynchronous and synchronous particles, then the corresponding difference in revolution time is given by the relation

$$\frac{\Delta T}{T} = \frac{1}{\alpha} - \frac{1}{\gamma^2}\frac{\Delta p}{p} \tag{8j-31}$$

where $\alpha = 1 - n$, $\gamma = E/E_0$.

In synchrocyclotrons the values of α and γ are such that $\Delta T/T$ has the same sign as $\Delta p/p$. This implies that phase stability exists only in the phase interval $0 < \phi_s < \pi/2$ where the voltage falls: A particle crossing the gap at a phase $\phi > \phi_s$ gains less energy, i.e., $\Delta p < 0$; as a result has a shorter revolution time than the synchronous particle; and arrives, therefore, earlier at the next gap crossing. A similar argument can be made if $\phi < \phi_s$. In both cases the phase ϕ oscillates about the synchronous phase ϕ_s. The differential equation for these phase oscillations is

$$\frac{d}{dt}\left(\frac{E_s}{\omega_s^2 K}\frac{d\phi}{dt}\right) = \frac{qV}{2\pi}(\cos\phi - \cos\phi_s) \tag{8j-32}$$

Ion capture takes place only during a small time interval Δt at the beginning of each modulation cycle. With a few simplifying assumptions Bohm and Foldy derived the expression [17]

$$\Delta t = \frac{4}{\omega_s}\sqrt{\frac{\pi E_s}{KqV}}\frac{L(\phi_0,\phi_s)}{\cos\phi_s} \tag{8j-33}$$

where $L(\phi_0,\phi_s)$ is a function of starting phase ϕ_0 and synchronous phase ϕ_s. As was first pointed out by McKenzie [18], the beam current that can be accelerated in a synchrocyclotron is space-charge limited. If $I_{\text{sp.ch.}}$ denotes the maximum (direct) current that can pass through the available beam space within the dees under space-charge conditions, the captured average current in a synchrocyclotron is given by

$$I = I_{\text{sp.ch.}}\frac{\Delta t}{T_m}\frac{\Delta\phi_0}{2\pi} = I_{\text{sp.ch.}}\,\Delta t f_m \frac{\Delta\phi_0}{2\pi} \tag{8j-34}$$

$\Delta\phi_0/2\pi$ is the microscopic, $\Delta t/T_m$ the macroscopic duty factor (see Fig. 8j-4). $I_{\text{sp.ch.}}$ is roughly proportional to the voltage V and the square of the vertical focusing frequency, ν_z^2 in the center [10,20]. Since the repetition rate f_m is proportional to V, and Δt is proportional to $V^{-\frac{1}{2}}$, the beam current in a synchrocyclotron is in this crude approximation proportional to $V^{\frac{3}{2}}$ or some similar power of V. In all existing synchrocyclotrons the dee voltage is very small (5 to 20 kV) to minimize r-f power losses. The low voltage also necessitates the use of an open-arc source since ions would not be able to clear the chimney-type structure of the type of ion source used in fixed-frequency machines.

All these factors explain the very low internal beam currents (down by a factor 10^2 to 10^3 compared to fixed-frequency cyclotrons), poor beam quality, and poor extraction efficiency (a few percent compared with typically 40 to 90 percent in FF cyclotrons). After the successful development of sector-focusing cyclotrons, several synchrocyclotrons are being modified and improved to remain competitive with the new type of machines. All these synchrocyclotron conversion programs (a survey is given by Blosser in [21]) involve an increase of the dee voltage and thus the repetition rate, the installation of a "chimney"-type ion source used in other cyclotrons, and an improvement of the vertical focusing in the center through use of magnetic bumps or sectors.

8j-9. Sector-focusing (Isochronous) Cyclotrons. In 1938 L. Thomas had shown in a theoretical study that it should be possible to build a cyclotron with constant ion frequency ω by employing a magnetic field which varies sinusoidally with azimuth angle. The average magnetic field increases with radius to compensate the relativistic mass increase, thus keeping $\omega - qB/m$ a constant, while at the same time vertical focusing is provided by the azimuthal field variation (called "flutter"). Because of World War II and the invention of the synchrotron, this idea was not acted upon until 1950 when a group at the Lawrence Radiation Laboratory began a study and built an electron model which proved the feasibility of the new cyclotron concept [9]. Similar studies were soon started at other places in the United States and Europe, and since then a large number of sector-focusing cyclotrons have been built and are now in operation.

Details of the theory, design, and performance characteristics of sector-focusing cyclotrons can be found in J. R. Richardson's monography [22] and in the proceedings of several international conferences [23].

Most sector-focusing cyclotrons employ a wedge-shaped rather than a sinusoidal variation in azimuth. Besides, in most cases the pole-shoe sectors or "hills" are spiral-shaped rather than straight, which provides additional focusing, as was first proposed by the MURA group (Midwestern Universitities Research Association) in 1955 [24]. In this general case the median-plane magnetic field is of the form

$$B(r,\theta) = \bar{B}(r)\left[1 + \sum_n f_n(r) \cos n\,(\theta - \phi_n(r))\right] \quad (n = N, 2N, 3N, \text{etc.}) \qquad \text{(8j-35)}$$

The number of sectors or periods N is 3 or 4 in most existing cyclotrons. The average magnetic field $\bar{B}(r)$ increases with radius according to the relativistic mass change:

$$\bar{B}(r) = B_0\left(1 + \frac{E_k}{E_0}\right) = B_0(1 - \beta^2)^{-\frac{1}{2}} = B_0\left(1 - \frac{r\omega_0^2}{c}\right)^{-\frac{1}{2}} \qquad \text{(8j-36)}$$

Then

$$\omega = \omega_0 = \frac{qB_0}{m_0} = \text{const} \qquad \text{(8j-37)}$$

Calculation of the betatron frequencies for such a sector field leads to rather complicated analytical expressions. (For high accuracy, numerical orbit integration by computer is required.) Neglecting a number of less important terms, first-order theory gives the following approximate results:

$$\nu_r^2 = 1 + k \qquad \text{(8j-38)}$$

$$\nu_z^2 = -k + \frac{N^2}{N^2 - 1}F(1 + 2\tan^2\alpha) \qquad \text{(8j-39)}$$

where

$$k = \frac{r}{\bar{B}}\frac{d\bar{B}}{dr} = -n \qquad F \approx \frac{1}{2}\sum_n f_n^2 \qquad \text{(8j-40)}$$

α is the (effective) spiral angle defined by

$$\tan \alpha = r\, d\phi/dr \tag{8j-41}$$

More accurate formulas are given in the literature ([22,24] for example).

Equation (8j-38) for the radial frequency is identical with Eq. (8j-23) except that in this case $\nu_r \geq 1$ as k is positive. With respect to the vertical frequency (Eq. 8j-39), the spiral angle α and flutter amplitude F must be large enough to compensate for the defocusing average field and, in addition, provide a net focusing effect such that $\nu_z > 0$ (in most cases ν_z is between 0.1 and 0.2). At small radii, sector focusing ceases to be effective since the azimuthal field amplitude, measured by $F(r)$, goes to zero as $(r/g)^N$, where g is the magnet gap width, and N the number of sectors. To achieve good focusing at small radii, the number of sectors should be small, i.e. three or four (fields with fewer than three sectors are unstable for the radial motion). The problem can be further alleviated by utilizing electric focusing through careful programming of the particles' phase history with respect to the radio frequency [25] and, if necessary, employing a small magnetic bump with negative k. Improved central-region design (source position, beam optics, space-charge compensation, defining slits, etc.) is one of the main reasons for the excellent beam quality in sector-focusing cyclotrons [26]. It is also possible to control the pulse width to a certain extent and achieve microscopic duty factors of 10 to 20 percent and higher or get narrow nanosecond pulses for time-of-flight experiments by employing phase-selection slits in the center.

Sector-focusing cyclotrons are limited in energy by resonances in the radial motion which arise whenever the betatron frequency ν_r passes through certain critical values. Under the condition of isochronism,

$$k = \gamma^2 - 1 = (1 + E_k/E_0)^2 - 1 \tag{8j-42}$$

and hence, approximately,

$$\nu_r = \gamma = 1 + \frac{E_k}{E_0} \tag{8j-43}$$

Thus ν_r starts at unity and increases linearly with energy. According to the theory of resonances in sector fields, a stop band occurs in the radial motion whenever $\nu_r = N/2$, where N is the number of sectors. A two-sector field is therefore intrinsically unstable. According to Eq. (8j-43), in a three-sector cyclotron the stop band $\nu_r = \frac{3}{2}$ occurs at a proton energy of 469 MeV, while $N = 4$ ($\nu_r = 2$) leads to a limit of 938 MeV. If terms neglected in Eqs. (8j-38) and (8j-43) are taken into account, the stop-band energy limits are found to be considerably lower than these values. The resonance problem as well as practical considerations such as achieving the desired field shapes put an upper limit for isochronous cyclotrons at a proton energy of about 800 to 1000 MeV.

The largest sector-focusing machine built so far is the isochronous cyclotron at the University of Maryland, which is capable of accelerating protons to a maximum energy of 140 MeV. At energies above about 200 MeV, new design concepts must be invoked. Several projects in this category are presently under study or construction: A 500-MeV sector-focusing "meson factory" designed as a ring accelerator with a 70-MeV isochronous cyclotron as injector, is presently under construction at Zurich, while a somewhat similar ring machine for 200-MeV protons is being designed at Indiana University. The concept of a separated-orbit cyclotron (SOC) has been studied at Oak Ridge National Laboratory, and a negative-hydrogen cyclotron (H^-) for 500 MeV, called TRIUMF, is being built at Vancouver, Canada.

One of the outstanding features of most existing isochronous cyclotrons is the variability of energy and the possibility of accelerating different types of particles. The r-f system can be tuned over a wide range of frequencies, and the desired magnet-field profiles at various excitation levels are achieved by means of a system of trimming coils.

Extraction of the beam out of the cyclotron [27] is generally accomplished by inducing a coherent radial oscillation at the $\nu_r = 1$ resonance, which occurs at the transition from the isochronous field to the fringe field. In traversing the resonance, the radial amplitude, and thus separation between consecutive turns, is increased sufficiently so that the beam can enter an electrostatic deflector (or a combination of electric and magnetic deflector channels) which bends it into the external beam pipe. Extraction efficiencies of 40 to 90 percent have been achieved in existing machines for proton currents between 10 and 100 μA. The energy spread of the extracted beam is in the range of 0.1 to 0.3 percent while the emittance in radial and vertical direction is typically between 10 and 30 mm mrad. Sector-focusing cyclotrons are, therefore, excellent tools for nuclear-structure physics in the intermediate-energy range of 10 to 200 MeV for light nuclei.

8j-10. Constant-gradient Synchrotons. For acceleration of protons to energies above 1 GeV, linacs and cyclotrons are impractical, as the size of such machines would become prohibitively large. The only type of accelerator that has been capable so far of generating protons in the billion-volt energy range is the synchrotron, which is based on the principle of phase-stable synchronous acceleration proposed by Veksler and McMillan. Fundamentally the synchrotron is an extension of the synchrocyclotron, the main difference being that the orbit radius is kept constant, and the guiding magnetic field is provided by a number of individual magnets placed along the orbit. The particles are first preaccelerated in a Van de Graaff, Cockcroft-Walton, or linac, and then injected into the synchrotron ring. To keep the orbit radius constant in the synchrotron, the magnets are pulsed such that $B = B(t)$ increases from a minimum value at injection to the maximum given by the final energy of the particles. Orbit stability is provided by constant-gradient focusing as in cyclotrons. Magnets and pole shoes have to be designed carefully to keep the field index $n = -(r/B)dB/dr$ within acceptable limits over the entire range of variation of the magnetic field.

The orbital frequency of the particles is determined by the radius of curvature R in the magnets and the length l of the straight drift section between the magnets. With N straight sections, the circumference of an orbit is $L = 2\pi R + Nl$, and substituting Eq. (8j-4) for $v = \beta c$, one gets

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{L} = \frac{2\pi c}{L}\left(\frac{E^2 - E_0^2}{E^2}\right)^{\frac{1}{2}} \qquad \text{(8j-44)}$$

or, in view of Eq. (8j-6)

$$\omega = \frac{2\pi c}{L}\frac{BRqc}{[(BRqc)^2 + E_0^2]^{\frac{1}{2}}} \qquad \text{(8j-45)}$$

The particles are accelerated by r-f resonators located in the straight sections between magnets. From Eq. (8j-6) the rate of energy increase dE/dt is determined by dB/dt:

$$\frac{dE}{dt} = \left(\frac{E^2 - E_0^2}{E^2}\right)^{\frac{1}{2}} qRc\,\frac{dB}{dt} \qquad \text{(8j-46)}$$

The corresponding energy gain per turn $\Delta E = qV \cos\phi = (\omega/2\pi)dE/dt$, is then obtained from Eqs. (8j-44) and (8j-46), and is given by

$$\Delta E = L\,qR\,\frac{dB}{dt} \qquad \text{(8j-47)}$$

Since accurate timing of the magnet pulse is exceedingly difficult at such high power levels, no predetermined time schedule can be set up for the variation of B, ΔE, and ω_{rf} with time. Instead, ω_{rf} and ΔE are controlled electronically to follow the rate of change of the magnetic field. A pickup loop in the magnetic field supplies a signal proportional to dB/dt, from which B is obtained at any given time through electronic integration. A computer solves Eq. (8j-45), and the values for ω and V are sent to the control circuits of the r-f oscillators. The required frequency bandwidth of the oscillators is porportional to the range of velocities between injection and full energy; it is the smaller, the higher the energy of the preaccelerator.

As in the synchrocyclotron, phase stability in the constant-gradient synchrotron is obtained when the phase of the synchronous particle lies in the interval of decreasing voltage amplitude.

Electron synchrotrons differ from proton machines in several aspects. Because of the smaller rest mass, electron velocities at energies above a few MeV are essentially equal to the speed of light. The orbital frequency is thus higher than for the proton machines but practically constant, so that frequency modulation is unnecessary. In addition, energy losses due to electromagnetic radiation of the accelerated electrons are substantially higher than for the protons where they are practically negligible. However, these losses are automatically compensated for by the mechanism of phase stability: A decrease in momentum due to radiation losses causes a shrinkage of the orbit radius, so that the electron arrives earlier at the acceleration gap and thus gains additional energy which compensates for these losses.

Historically the electron synchrotron preceded the proton synchrotron by several years. The largest constant-gradient electron machine is the 1.3-GeV synchrotron at Cornell University, while the largest proton synchrotron with constant gradient is the 10-GeV accelerator at Dubna.

The focusing forces in constant-gradient synchrotrons are inherently weak, and, consequently, the amplitudes of the betatron oscillations are relatively large. This necessitates the use of magnets with large gap dimensions to contain the beam, and makes an accelerator of this kind prohibitively expensive if the energy exceeds more than a few GeV. (The magnets for the 10-GeV "Synchrophasotron" at Dubna weigh 36,000 tons!) The invention of the alternating-gradient or strong-focusing principle was, therefore, a major breakthrough in high-energy accelerator design. Alternating-gradient synchrotrons can be built with smaller magnets and have better beam quality and higher beam intensities than constant-gradient machines.

8j-11. Alternating-gradient Synchrotrons. The principle of strong focusing was independently discovered first in 1949 by Christofilos, whose work was not published then, and shortly after that by Courant, Livingston, and Snyder in 1952. This new concept is most easily understood in terms of its well-known optical analog, the combination of focusing and defocusing lenses. If two lenses of focal lengths f_1 and f_2 are combined, with a separation d between them, the focal length F of this system is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \tag{8j-48}$$

In the special case of a converging and diverging lens of equal, but opposite, strength, one has $f_2 = -f_1$, and hence

$$F = \frac{f_1^2}{d} \tag{8j-49}$$

The focal length of such a two-lens system is thus always positive (focusing). The application of this idea to synchrotrons implies the combination of strongly focusing and defocusing magnets. According to the theory of betatron oscillations, Eqs. (8j-20) to (8j-24), a magnet with negative gradient, $dB/dr < 0$, is focusing vertically while defocusing radially if $n > 1$. A radially increasing field ($n < 0$), on the other hand, focuses the particles only in the radial direction and is defocusing with respect to the vertical motion. The alternating-gradient synchrotron ring consists of a succession of magnets arranged in such a way that a magnet with large positive gradient is followed by one with a negative gradient of equal strength. The absolute values of n are typically in the range of 200 to 300, as compared to 0.5 in the conventional weak focusing machines. Consequently, the frequencies of the corresponding radial and vertical oscillations are between one and two orders of magnitude larger than in constant-gradient accelerators.

The strong focusing forces reduce the required beam space and the size of the magnets, and thus result in substantial reduction of costs and in improvement of beam quality.

With regard to synchrotron oscillations and phase stability, the alternating-gradient machines are distinctly different from the weak focusing accelerators. The theory shows that the parameter α in Eq. (8j-31) is always greater than 1, and hence $1/\alpha < 1$, in contrast to the constant-gradient machines. At low energies, where $\gamma^2 < \alpha$, an increase in momentum causes a decrease in revolution time. This implies that stability exists if the synchronous particle crosses the accelerating gaps when the voltage is rising. As γ increases, a critical transition energy occurs where $\gamma^2 = \alpha$. Above that energy ($\gamma^2 > \alpha$), particles behave as in the synchrocyclotron and constant-gradient synchrotron; i.e., the synchronous phase must be in a region of falling voltage. This means that in AG synchrotrons provisions must be made to shift the phase of the accelerating voltage at the point where the particles pass through the transition energy. If the injection energy is, however, higher than the transition energy, this difficulty can be avoided.

A major problem in the design of strong focusing synchrotrons is the existence of resonances which occur whenever the values of the betatron frequencies are integers or integral fractions. The operating point must be carefully chosen, taking into account the effects of misalignments and space-charge forces. The electric and magnetic self-fields of the circulating beam produce a net defocusing force which is equivalent to an effective change of the field index n given by

$$\Delta n = \frac{qQR}{4\pi^2\epsilon_0 a^2 E}\,\frac{1-\beta^2}{\beta^2} = \frac{qQR}{4\pi^2\epsilon_0 a^2 E_0}\,\frac{1}{\gamma(\gamma^2-1)} \tag{8j-50}$$

where Q is the total charge, and R is the major and a the minor radius of the toroidal ring of circulating beam.

If Δn denotes the maximum tolerable change in field index (to stay away from a resonance or avoid defocusing), then the maximum number of particles N_{lim} which can be contained in the ring is given by

$$N_{\text{lim}} = 2.18 \times 10^{15}\,\frac{a^2}{R}\,E_0\gamma(\gamma^2-1)\,\Delta n \tag{8j-51}$$

where E_0 is in MeV, a and R in m. (A detailed analysis of space-charge effects, including image effects in surrounding walls, was made by Laslett [28].) For electrons at extremely relativistic energies ($\gamma \gg 1$) the total current $I_{\text{lim}} = qN_{\text{lim}}v/2\pi R$ contained in the ring is given by the relation

$$I = 8{,}500\left(\frac{a}{R}\right)^2\gamma^2\,\Delta n \tag{8j-52}$$

By proper choice of the n values and careful alignment of the magnets it was possible to overcome the difficulties imposed by resonances and space-charge effects. Several AG synchrotrons are now operating successfully. The presently largest proton accelerator in the world is the 70-GeV alternating-gradient synchrotron at Serpukhov (U.S.S.R.) which went into operation during 1968. In second and third place follow the 33-GeV AGS at Brookhaven (U.S.A.) and the 28-GeV proton synchrotron at CERN, the European Nuclear Research Center at Geneva, Switzerland.

The largest alternating-gradient synchrotron for the acceleration of electrons is the 10-GeV accelerator built at Cornell University. Other large electron machines are the 7-GeV synchrotron (DESY) at Hamburg, Germany, which began operation in 1964; the 6-GeV machine at Cambridge, Massachusetts, operating since 1962; and a 6.5-MeV accelerator in the Soviet Union.

Although synchrotron radiation losses put an upper limit in the range of 10 GeV to electron synchrotrons, proton machines with energies up to 1000 GeV appear to be within the reach of technical feasibility. Preliminary studies of a 1000-GeV accelerator have been carried out in the United States and are in progress in the U.S.S.R. At the National Accelerator Laboratory at Batavia, Illinois, a 200-GeV alternating-gradient synchrotron is being constructed, with provisions to extend the energy to 500 GeV at a later time. Acceleration in this project will take place in three stages: A 200-MeV linac will inject the beam into a 10-GeV booster synchrotron (diameter of 150 m), from which the particles are steered into the main ring (diameter of 2,000 m) for acceleration to full energy.

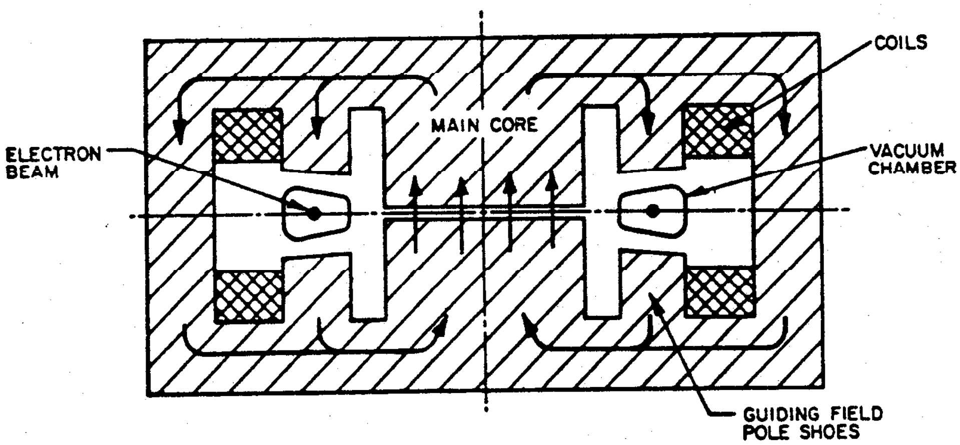

FIG. 8j-10. Betatron.

8j-12. Betatrons. The betatron differs from other circular accelerators in that the electromotive force for accelerating the particles is generated by the time variation of the magnetic flux. It is only suitable for the acceleration of electrons. The magnet structure of a betatron (Fig. 8j-10) resembles that of a cyclotron; the major difference is in the design and shape of the core part with the pole shoes. As in a synchrotron, the orbit radius of the circulating electron beam is kept constant throughout the acceleration process. This implies that the increase in energy due to the changing magnetic flux linked by the circulating electrons must be precisely in step with the increase of the magnetic field strength at the orbit radius. The accelerating electric field E along the circular orbit is determined by Maxwell's second equation

$$\int \mathbf{E} \cdot d\mathbf{L} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{S} \tag{8j-53}$$

In cylindrical coordinates, if $\mathbf{B} = B\mathbf{a}_z$ and increasing in time, $\mathbf{E} = -E\mathbf{a}_\phi$, and thus

$$2\pi RE = \frac{\partial}{\partial t} \int_0^R 2\pi BR\, dR = \pi R^2 \frac{d\bar{B}}{dt} \tag{8j-54}$$

$\bar{B}$ is the average magnetic field inside the circular orbit with constant radius R. The rate of change of momentum of the electron is given by

$$\frac{d}{dt}(mv) = eE = \frac{d}{dt}(eBR) = eR\frac{dB}{dt} \tag{8j-55}$$

Elimination of E from Eqs. (8j-54) and (8j-55) gives the fundamental betatron relation

$$\frac{d\bar{B}}{dt} = 2\frac{dB}{dt} \tag{8j-56}$$

which says that the change in the space-averaged field inside the orbit $\bar{B}$ must equal twice the change in the field at the orbit $B(R)$. If both the average core field and the field at the orbit are zero when the acceleration process starts, as is usually the case, integration of (8j-56) gives

$$\bar{B} = 2B \tag{8j-57}$$

The average core field must thus be twice as high as the field at the orbit ("two-to-one" rule) which explains the shape of the magnet core and pole shoes in Fig. 8j-10.

The magnet is driven with an ac power supply which generates a sinusoidally varying current at a frequency in the range of 30 to 60 Hz. To minimize eddy currents the magnet structure is laminated. The electrons are injected from an electron gun close to the equilibrium orbit, with a starting energy between 10 and 100 keV. The acceleration process then takes place during the quarter cycle during which the field rises from the value (close to zero) that corresponds to the injection energy to the peak value, where the electrons have reached the maximum energy.

Radial and axial stability of the beam during acceleration is maintained by constant-gradient focusing; i.e., the field near the orbit is decreasing with radius such that the index n has values between 0 and 1. In fact, the resulting oscillations are known as *betatron oscillations* because the theory of gradient focusing was first developed in connection with the betatron by Kerst and Serber.

At the end of each acceleration cycle the electron beam is displaced from the equilibrium orbit by a perturbation in the magnetic field. This is accomplished by additional coils which disturb the "two-to-one rule," resulting in an increase of the orbit radius and thereby forcing the beam to hit the internal target or deflecting it out of the magnetic field for external use. Most betatrons are used primarily for production of hard X rays from internal targets.

Electromagnetic radiation emitted by the circulating electrons sets an upper-energy limit to betatron-type acceleration. In the relativistic electron-energy range above a few MeV, the rate of energy loss due to radiation is proportional to the fourth power of the kinetic energy and inversely proportional to the orbit radius,

$$\Delta E_{rad} = 8.8 \times 10^{-8} \frac{E_k^4}{R} \tag{8j-58}$$

where ΔE_{rad} is in electron volts per revolution, E_k in MeV, and R in meters.

The betatron was invented by Wideröe, but the first successful machine was built by Kerst. Today a large number of betatrons are in operation in hospitals, for industrial applications as well as for scientific use. The largest betatron is the 300-MeV machine at the University of Illinois, Urbana.

8j-13. New Developments. Accelerator technology is advancing at a rapid rate in many areas. New design concepts have been proposed or are being investigated, and in all likelihood new types of accelerators will be built in the future. It is impossible to survey all these developments, but below a few examples will be discussed briefly to illustrate major present trends.

Heavy-ion Accelerators. In principle all the existing types of accelerators with the exception of the betatron and microtron are capable of accelerating ions of heavy elements. The main problem in practically every instance is that a high charge state is either required to facilitate acceleration or desired to obtain a sufficiently high energy per nucleon. Most ion sources, however, which are utilizing a gas discharge produce ions with only a few electrons removed (typically 1 to 5). However, to accelerate heavy ions ($M > 20$) with low charge state ($Z < 4$) in a cyclotron, for example, the wavelength of the r-f system would have to be impractically large, or operation at a very high harmonic, $\omega_{rf}/\omega = N \gg 1$, would be necessary, which again is not feasible. There are basically two solutions to this problem: One is to develop new types of ion sources which yield higher charge states; the other approach is to accelerate ions with low charge state to some intermediate energy, then remove more electrons by stripping in a foil or gas cell, and accelerate further. Thus a negatively charged heavy ion can be injected into a tandem where stripping takes place in the positive-voltage terminal, followed by several steps of acceleration and stripping until the ions with various charge states and energies arrive at ground potential. If desired, one ion component can then be injected into a cyclotron for acceleration to even higher energies. Similar possibilities exist with a multistage linear accelerator or combination of linac and synchrotron. Various schemes of this kind are discussed in the Proceedings of the 1969 Accelerator Conference in Washington, D.C.

High-energy Cyclotrons. Several sector-focusing cyclotron projects in the 200- to 500-MeV range are under construction (Indiana, Zurich, Vancouver), and should come into operation in the 1970s. In addition, the improvement of existing synchrocyclotrons is of great interest as currents in such converted machines should be close to those achieved in isochronous cyclotrons. The 600-MeV synchrocyclotron at CERN is being improved by a change of the rf system (higher dee voltage and repetition rate) and of the ion source and central region. The 385-MeV synchrocyclotron of Columbia University, New York, is being converted into a 500-MeV machine by changing rf voltage, repetition rate, and central region, as in the CERN case, but also adding sector focusing in the magnetic field. These modifications should increase internal beam currents by a factor of 10 to 20 and external beams by 100.

The Collective-ion or Electron-ring Accelerator (ERA). First proposed by Veksler in 1956 [29], the ERA involves an entirely new acceleration concept which holds great promise for the acceleration of protons to superhigh energies. The basic idea involves the formation of a relativistic high-density electron ring (typically 10^{13} to 10^{14} particles, major radius 5 cm, minor radius 1 mm, energy 20 to 25 MeV) in a strong magnetic field. After formation of the ring, gas is admitted, the ions formed by collisions with the electrons are trapped in the deep potential well of the electron cluster, and the ring with ions is subsequently accelerated to high energies. Since the ions travel with the same speed as the electrons, their final kinetic energy is substantially larger than that of the electrons. If M_ic^2 is the rest energy of the ions, E_{e0} the total energy of the ring electrons before and E_{ef} after acceleration, the final total ion energy is given by $E_{if} = (E_{ef}/E_{e0})M_ic^2$. Thus to obtain a proton energy of 1 GeV, requires $E_{ef} \approx 2E_{e0}$, and if the initial energy of the ring electrons is $E_{e0} = 25$ MeV, an additional amount of 25 MeV must be added by acceleration of the ring. If the energy is gained at a rate of 40 keV/cm, the accelerator needs only a length of a little more than 6 m to produce the 1-GeV protons. The size of a multi-GeV proton accelerator would therefore be substantially smaller than that of a synchrotron, which explains the attractiveness of the electron-ring accelerator concept. At the same time the ERA holds great promise also as an accelerator for heavy ions. The various design problems and prospects of the ERA are discussed in the proceedings of a symposium in Berkeley [30]. Compressed electron rings in a pulsed magnetic field were obtained during 1968 in experiments at Dubna, Berkeley [31], and the University of Maryland [32]. A promising alternative to a pulsed system is the formation of the electron ring in a static magnetic field [33]. For further information on the ERA see the article by D. Keefe in the journal *Particle Accelerators* [34].

Other interesting developments in the accelerator field, such as *storage rings, superconducting linacs*, and the *racetrack microtron*, are reviewed in the proceedings of the latest accelerator conferences listed in the general bibliography.

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