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QUANTUM MECHANICS AND RADIOACTIVE DISINTEGRATION[1]

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ABSTRACT

Application of quantum mechanics to a simple model of the nucleus gives the phenomenon of radioactive disintegration. The statistical nature of the quantum mechanics gives directly disintegration as a chance phenomenon without any special hypothesis. §1 contains a presentation of those features of quantum mechanics which are here used and gives a simple calculation of the disintegration constant. §2 discusses the qualitative application of the model to the nucleus. §3 presents quantitative calculations amounting to a theoretical interpretation of the Geiger-Nuttall relation between the rate of disintegration and the energy of the emitted α-particle. In getting this relation one arrives at the rather remarkable conclusion that the law of force between emitted α-particle and the rest of the nucleus is substantially the same in all the atoms even where the decay rates stand in the ratio 10^{22}. §4 calls attention to the natural way in which the paradoxical results of Rutherford and Chadwick on the scattering of fast α-particles by uranium receive explanation with the model here used. §5 discusses certain limitations inherent in the methods employed.

THE study of radioacitivity itself together with the application of it as a working source of high speed helium nuclei and electrons has played a fundamental role in the development of quantum physics. The scattering experiments of Rutherford and his associates gave the picture of the nuclear atom on which all of the success of modern atomic theory depends. Bohr's formulation of quantum postulates to be applied to such a model was a great step in the extension of knowledge of atomic structure and finally culminated in 1925 in the discovery by Heisenberg and by Schrödinger of a reformulation of mechanical laws which has subsequently proved extremely powerful in handling atomic structure problems. In this development of the last fifteen years little advance has been made on the problem of the structure of the nucleus.

It seems, however, that the new quantum mechanics has had sufficient success to justify the hope that it is competent to carry out an effective attack on the problem. The quantum mechanics has in it just those statistical elements which would seem appropriate to an explanation of the phenomenon

[1] An account of this work was first published in Nature for September 22, 1928. In a number of the Zeitschrift für Physik (**51**, 204, 1928) received here two weeks ago there appears a paper by Gamow who has arrived quite independently at the same basic idea as was presented in our letter and which is here treated in detail. Reports of this paper were also given at the Schenectady meeting of the National Academy of Sciences on November 20, 1928 and at the Minneapolis meeting of the American Physical Society on December 1, 1928.

of radioactive decay. This is the feature of the general problem with which we are concerned in this paper. We believe that the results provide at last an interpretation of nuclear disintegration which in its fundamental points is very close to the truth although it is necessarily quite incomplete.

The outstanding difficulties in the way of a good theoretical treatment of nuclear structure at present are mainly bound up with our lack of understanding of the quantum mechanics of the magnetism of the fundamental particles. This question has been much advanced this year by Dirac's extension of Pauli's theory of the spinning electron[2] but this remains essentially a theory of the behavior of one electron in an electromagnetic field. Not only is it apparently still unsatisfactory as such but this limitation must necessarily be disposed of in principle before the many body nuclear problem can be approached. And with that done there will remain the inevitable analytical difficulties.

Enough is known, however, to teach us that probably the magnetic interaction is not to be handled simply by an alteration of a potential energy function depending solely on the coordinates of the several interacting particles. This tends to detract from the value of arguments based simply on the use of quantum mechanics with the positional coordinates of the nuclear constituents. Nevertheless we shall restrict ourselves to the use of such methods in the discussion of the instability or capacity for spontaneous disintegration of a very much simplified nuclear model. The simplification to be made will consist in supposing that we can discuss the behavior of any one constituent by applying the quantum mechanics to it as a single body moving in a force field due to the rest of the nucleus.

The difference between quantum mechanics and classical mechanics which is here made responsible for the disintegration process is easily stated. In classical mechanics the orbit of a particle is entirely confined to those points in space at which its potential energy is less than its total energy. This is not true in quantum mechanics. Classically if a particle be moving in a basin of low potential energy and have not as much total energy as the maximum of potential energy surrounding the basin, it must *certainly* remain there for all time, unless it acquires the deficiency in energy somehow. But in quantum mechanics most statements of certainty are replaced by statements of probability. And the above statement must now be altered to read "· · · it may remain there for a long time but as time goes on the probability that it has escaped, even without change in its total energy, increases toward unity."

In §1 of this paper the detailed development of the argument leading to the conclusion of the preceding paragraph is given. In §2 we discuss its qualitative application to the nuclear disintegration problem. §3 is devoted to semi-quantitative estimates of the rates of decay.

1. Coupling of Motions of Equal Energy

Consider a particle of mass μ. It is sufficient to consider one degree of freedom; let the coordinate of the particle be x and let the forces be measured by the potential energy function $V(x)$.

[2] Dirac, Proc. Roy. Soc. **A117**, 610; **A118**, 351 (1928).

In classical mechanics the equations of motion possess the energy integral

$$p^2/2\mu + V(x) = W \tag{1}$$

(p = momentum) which, for values of x such that $W - V(x) < 0$, can only be satisfied by p pure imaginary. Therefore, classically, one had the result that a particle could only be where $W - V(x) \geqq 0$. An important consequence of this was that if there were several ranges of x for which $W - V(x) \geqq 0$ separated by ranges where $W - V(x) < 0$, then there were several different motions possible with the energy level W, each of which was wholly confined to one of these separate ranges. Thus in Fig. 1 for the energy level indicated there would be two distinct types of motion of the same energy W; one is a libration in the range I, and the other a libration in the range II.

These results are modified considerably by the new quantum mechanics. In the first place, Eq. (1) loses its validity and is replaced by an integral theorem, as Born[3] has shown, in which there is no longer a definite correlation between simultaneous value of position and momentum as (1) implies. The quantum mechanical form of (1) is, if $\psi(x)$ is Schroedinger's wave function

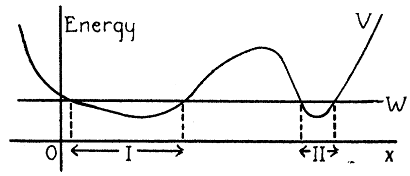

Fig. 1.

$$W = \int_{-\infty}^{+\infty} \left(\frac{h^2}{8\pi^2\mu} \frac{d\psi}{dx} \frac{d\bar{\psi}}{dx} + V(x)\psi\bar{\psi} \right) dx \tag{1a}$$

The lack of a precise correlation has been much emphasized by Heisenberg and by Bohr,[4] and is a general characteristic of quantum mechanics. From the new standpoint, one has to consider the behavior of Schrödinger's equation for the problem

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2\mu}{h^2}(W - V(x))\psi = 0. \tag{2}$$

As is well known, in some problems there are solutions $\psi(W, x)$ for certain values of W which are finite and continuous everywhere. These are the "allowed" values of quantum theory. For the $\psi(W, x)$ which comes out of (2) as a by-product, Born has shown that its square may be satisfactorily interpreted as giving the probability that the particle lies between x and $x + dx$ when it is in the state of energy W. This is really the ground for requiring that ψ remain finite. For an energy level, such that $\psi(W, x)$, does not remain finite as $x \rightarrow \pm\infty$, the probability that it is not "at infinity" is vanishingly small, and therefore these states do not exist physically. Adopting the probability interpretation of $\psi(W, x)$ one has at once the result that there is a finite probability of being outside the range of the classical motion of that energy.

[3] Born, Zeits. f. Physik **38**, 806 (1926).

[4] Heisenberg, Zeits. f. Physik **43**, 172 (1927); Bohr, Nature, April 14, 1928.

A simple case is the lowest state of the harmonic oscillator, which has the energy $h\nu/2$. The $\psi(W, x)$ for this state is $e^{-1/2(x/a)^2}$ so $\psi^2 = e^{-(x/a)^2}$ where a is the classical amplitude of motion associated with this energy. The probability of being outside the classical range is therefore

$$\frac{2\int_a^{\infty} e^{-(x/a)^2}dx}{\int_{-\infty}^{+\infty} e^{-(x/a)^2}dx} = 0.157$$

or more than 15 percent.

When one studies the behavior of $\psi(W, x)$ from (2) for a $V(x)$ somewhat like the one in Fig. 1, he finds that, if the W is one for which ψ is finite everywhere, then ψ approaches zero very rapidly (exponential decrease) as $x \rightarrow \pm\infty$. In the neighborhood in which $W - V(x)$ is small, the function takes on appreciable values and has oscillatory character where $W - V(x) < 0$, and non-oscillatory character elsewhere. Cases like that of Fig. 1 have been discussed by Hund[5] in connection with his studies of molecular spectra.

An important case is that in which the potential energy curve consists of a single "obstacle" or barrier as in Fig. 2, and the motion is one of insufficient energy, W, to clear the obstacle. In such cases there are *two* finite solutions $\psi_1(W, x)$, and $\psi_2(W, x)$ associated with each energy level, W, and so an arbitrary linear combination of them is also a solution of (2). Born has shown that there is always a combination of them which depends on x as $e^{+i\sigma x}$ and represents a pure left-to-right progressive wave motion as $x \rightarrow +\infty$. Such a solution for x large and negative can then be said to represent an incident left-to-right wave coming from the left side and a reflected wave which is not as strong as the incident wave. The interpretation is that the incident beam of particles is partly reflected and partly transmitted. In the range where $(W - V) < 0$ the de Broglie wave-length h/p becomes imaginary, and so gives rise to an exponential behavior of ψ whose nearest analogue is, perhaps, in optics in the slight penetration of a refracted ray into a rarer medium even beyond the angle of total reflection where the refracted angle is imaginary. In this way, one can find the probability that a particle coming up from the left will get through the wall and escape to the right. The case illustrated in Fig. 3a for which

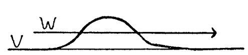

Fig. 2.

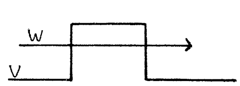

Fig. 3a.

$$V(x) = 0 \qquad x < -a,$$
$$V(x) = V \qquad -a < x < 0,$$
$$V(x) = 0 \qquad x > 0,$$

and for $0 < W < V$, is a simple one with which to illustrate the nature of the calculation. For a given energy level, W, there are two ψ functions satisfying

[5] Hund, Zeits. f. Physik **40**, 742 (1927); Wentzel, Zeits. f. Physik **38**, 518 (1926).

the requirements of finiteness everywhere and of continuity for the ordinates and slopes at the discontinuities in $V(x)$.

These are readily found to be

$$\psi_1(W,x)=\begin{cases}\cosh\sigma_2 a\cdot\cos\sigma_1(x+a)-(\sigma_2/\sigma_1)\sinh\sigma_2 a\,\sin\sigma_1(x+a) & (x<-a)\\ \cosh\sigma_2 x & (-a<x<0)\\ \cos\sigma_1 x & (0<x)\end{cases}$$
$$\psi_2(W,x)=\begin{cases}-(\sigma_1/\sigma_2)\sinh\sigma_2 a\cdot\cos\sigma_1(x+a)+\cosh\sigma_2 a\,\sin\sigma_1(x+a) & (x<-a)\\ (\sigma_1/\sigma_2)\sinh\sigma_2 x & (-a<x<0)\\ \sin\sigma_1 x & (0<x)\end{cases} \tag{3}$$

where $\sigma_1=(2\pi/h)(2\mu W)^{1/2}$ and $\sigma_2=(2\pi/h)[2\mu(V-W)]^{1/2}$. To find the ψ function corresponding to a beam of particles incident from the left which is partly transmitted and partly reflected, one has to add these together in such a way that to the right of the obstacle there is only the pure left-to-right flow, i.e., one must take

$$\psi_1(W,x)+i\psi_2(W,x)$$

To the left of the obstacle, the ψ function represents the superposition of a left-to-right, or incident beam

$$\psi_{inc}=\left[\cosh\sigma_2 a-\frac{i}{2}\left(\frac{\sigma_1}{\sigma_2}-\frac{\sigma_2}{\sigma_1}\right)\cdot\sinh\sigma_2 a\right]e^{i\sigma_1(x+a)}$$

and a reflected beam

$$\psi_{ref}=\frac{i}{2}\left(\frac{\sigma_1}{\sigma_2}+\frac{\sigma_2}{\sigma_1}\right)\sinh\sigma_2 a e^{-i\sigma_1(x+a)} \qquad x<-a \tag{4}$$

The transmitted beam is simply

$$\psi_{tr}=e^{i\sigma_1 x} \qquad x>0 \tag{5}$$

These expressions have, of course, the conservation property

$$(\psi\bar{\psi})_{inc}=(\psi\bar{\psi})_{ref}+(\psi\bar{\psi})_{tr}.$$

The probability that a particle coming up to the wall shall get through to the other side is simply $(\psi\bar{\psi})_{tr}\div(\psi\bar{\psi})_{inc}$ which for $e^{\sigma_2 a}\gg 1$ is clearly equal to

$$P_a(W)=16(W/V)(1-W/V)e^{-2\sigma_2 a}. \tag{6}$$

The controlling factor is the exponential term except when W/V is very near to 0 or 1.

For application to a theory of the pulling of electrons out of metals by electric fields Fowler and Nordheim[6] have derived the probability expression by similar methods for the curve of Fig. 3b, i.e.

$$V(x)=0 \qquad x<0$$
$$V(x)=C-Fx \qquad x>0$$

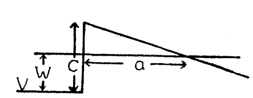

Fig. 3b.

[6] Fowler and Nordheim, Proc. Roy. Soc. **A119, 1** (1928).

The probability that a particle of energy W get through the wall they find to be

$$P_b(W) = 4[(W/C)(1-W/C)]^{1/2}\exp(-4k(C-W)^{3/2}/3F)$$

from their Eq. (18) p. 178. The term in the exponent can be written

$$4k(C-W)^{3/2}/3F = 4\sigma_2/3a \quad (k^2 = 8\pi^2\mu/h^2) \tag{7}$$

to exhibit the similarity with the case of the square wall. Here a is the positive value of x for which $V(x) = W$, and σ_2 is defined as

$$\sigma_2 = (2\pi/h)[2\mu(C-W)]^{1/2}.$$

The exponents in each of these cases can be written in the form

$$(4\pi/h)\int [2\mu(V-W)^{1/2}dx$$

the integration extending across the barrier, the limits being the two places where $V(x) - W = 0$.

Application of the method of approximate integration of Schrödinger's wave equation which was first used in quantum mechanics by Wentzel[5] indicates that such a result is quite general. The probability of getting through the wall at a single approach is governed essentially by the factor

$$\exp\left\{-(4\pi/h)\int [2\mu(V-W)]^{1/2}dx\right\} \tag{8}$$

being equal to it except for a factor of the order of magnitude of unity.

We have next to consider the case of a potential energy curve of the type shown in Fig. 4. According to classical mechanics there are two modes of motion associated with energy levels below the maximum such as W in the figure. One is a periodic motion in the range I while the other is an aperiodic motion in the range II. By the Bohr-Sommerfeld rule the periodic motions would give a discrete spectrum of allowed energy levels which would overlie the continuous spectrum associated with the aperiodic motions. On the quantum mechanics every energy level is allowed with the essential difference that *there are no energy levels with which two types of motion are associated*. With each energy level there is associated just one wave function $\psi(W, x)$ whose square gives the relative probability of being at different parts of the possible range of x. The $\psi(W, x)$ functions do show traces of the discreteness of the energy levels which the Bohr-Sommerfeld rule associates with the perodic motions in I, in an interesting way. The $\psi(W, x)$ for every W show sinusoidal oscillations as $x\rightarrow\infty$ and also oscillate in the range I. For most energies the amplitude of the oscillations in the range II is overwhelmingly large compared to that in range I, the ratio being of the order of $\exp\{(2\pi/h)\int[2\mu(V-W)]^{1/2}dx\}$ the integration extending across the barrier. This situation is just reversed however for little ranges of W values near those given by the old quantization rules. For these the amplitude in I is large compared to that in II in the same ratio. These then are the "allowed"

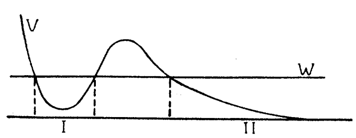

Fig. 4.

energy levels. It is not a stationary state for the particle to be in range I and remain there. But for certain energy levels there is an extraordinarily large probability of being in unit length of range I relative to unit length of range II.

We have to find the mean time which a particle remains in the range I before "leaking through" to the outer range II. This can be obtained from the following simple consideration. When the particle is at a place of x large and positive, $V(x)=0$ (Fig. 4) so the energy is all kinetic and the speed is therefore $(2W/\mu)^{1/2}$. The amount of time which the particle spends in unit length for x large is therefore $(\mu/2W)^{1/2}$. The time spent in a range of length a is therefore $a\ (\mu/2W)^{1/2}$. Now according to the wave-functions the probability of being in unit length of range I for one of the quasi-discrete energy values relative to the probability of being in unit length of range II is of the order $\exp\{(4\pi/h)\int[2\mu(V-W)]^{1/2}dx\}$. Therefore since the motion is aperiodic and the particle escaping from range I will in the mean only go through unit length of II once, the time T which must be spent in range I before getting through to range II is of the order of

$$T\sim a(\mu/2W)^{1/2}\exp\left\{(4\pi/h)\int[2\mu(V-W)]^{1/2}dx\right\}$$

where a is of the order of the breadth of range I.

Like all of the results of quantum mechanics this is to be interpreted as a probability result. So that if we start with a number of particles in the same allowed energy level in identical regions similar to range I, the number which leak out in time dt is governed by

$$dN=-N\lambda dt$$

which gives the usual exponential law of decay $N(t)=N_0e^{-\lambda t}$ where

$$\lambda=1/T. \tag{9}$$

The expression for T may be arrived at in a somewhat different way. One can think of the particle as executing its classical motion in range I, but as having at each approach to the barrier the probability of escaping to range II given by expression (8) above. The frequency of the periodic motion in I, which represents the number of approaches to the barrier in unit time, is of the order $a(\mu/2W)^{1/2}$ so the mean time of remaining in range I before excape comes out as the quotient of these two quantities as before. The reader will find it of interest to examine Oppenheimer's formula[7] for the pulling of electrons out of hydrogen atoms by an electric field. His formula for the mean time required for dissociation of the atom by a steady electric field splits naturally into a factor which is the classical frequency of motion in the Bohr orbit multiplied by an exponential probability factor of the type of expression (8) used in this paper.

2. Application to Radioactive Disintegration

After the exponential law in radioactive decay had been discovered in 1902, it soon became clear that the time of disintegration of an atom was

[7] Oppenheimer, Proc. Nat. Acad. Sci. **14**, 363 (1928).

as independent of the previous history of the atom as it was of its physical condition. One could not for example suppose that an atom at its birth begins to lose energy by radiation and that its instability is the result of the drain of energy from the nucleus. On such a view it would be expected that the rate of decay would increase with the age of the atoms. When later it was observed that the number of atoms breaking up per second showed the fluctuations demanded by the laws of probability it became clear that the disintegrating depended solely on chance. This has been very puzzling so long as we have accepted a dynamics by which the behaviour of particles is definitely fixed by the conditions. We have had to consider the disintegration as due to the extraordinary conjunction of scores of independent events in the orbital motions of nuclear particles. Now, however, we throw the whole responsibility on to the laws of quantum mechanics, recognizing that the behaviour of particles everywhere is equally governed by probability.

From what was said in the preceding section it is clear that the property of the nucleus which we need to know in order to apply the theory is its potential energy curve; and this happens to be a property which we know fairly definitely. Outside a nucleus whose net charge is given by the atomic number we should expect to find a Coulomb inverse-square field of the appro-

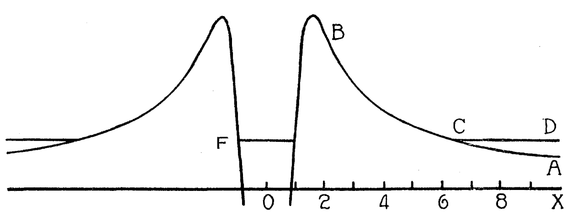

Fig. 5. The unit of abscissas is 10^{-12} cm. The horizontal line gives the energy of the α-particle emitted by uranium, 6.5×10^{-6} ergs.

priate strength. And it is well known that in experiments on the scattering of alpha particles from heavy nuclei the proper inverse-square field is found to extend through the whole accessible region. In Fig. 5 the curve AB is a plot of the potential energy of an alpha particle in this field against the distance from the centre of a nucleus of atomic number $Z=90$. To provide the attractive field which holds alpha particles in the nucleus it has long been recognized that the potential energy curve must turn over in the way shown in Fig. 5. And it has been shown, for example by Enskog,[8] that curves of this type may be obtained by giving the particle a magnetic moment.

In order to explain the ejection of a particle one has hitherto supposed that the particle in the internal region received energy sufficient to raise it over the potential barrier. The suggestion that this energy was obtained by absorption of some ultra-penetrating radiation from outside never received wide acceptance. But it was necessary on classical mechanics to suppose that the emitted particle had received energy, if not from outside then from the other nuclear particles. Now the potential barrier which confines particles in the nucleus, i.e. the area under the curve in Fig. 5, is a region where

[8] Enskog, Zeits. f. Physik **45**, 852 (1927).

the total energy would be less than the potential energy. And since the quantum mechanics endows particles with the new property of being able to penetrate such regions, this gives us at last a nucleus which can disintegrate without the absorption of energy.

We see that a mere qualitative application of the principles of quantum mechanics seems to account for the principal properties of radioactive atoms, most of which have been familiar for nearly thirty years. We have now to consider the question: How can nearly similar nuclei have periods of decay of anything from a small fraction of a second to over 10^9 years? It has been shown above that in coupling the possible motions of a particle on either side of a potential barrier, the probability of transmission through the barrier is extremely sensitive to the area of the barrier; in fact the relation to it is exponential. In this way we shall show that we can obtain all rates of decay up to practical stability, and that from atoms whose potential curves are almost identical.

3. Quantitative Application

If the height of CD above OX in Fig. 5 gives the energy of the alpha-particle emitted, we have to consider the coupling of the motion along CD with the motion along EF inside the nucleus. We clearly do not have much choice in the area of the potential barrier we may take, since both the point C is fixed and the curve passing through C. For the purposes of numerical calculation we will compare radium A which has a period of 4.4 minutes (half-value period 3.05 minutes) with the extreme cases of uranium and

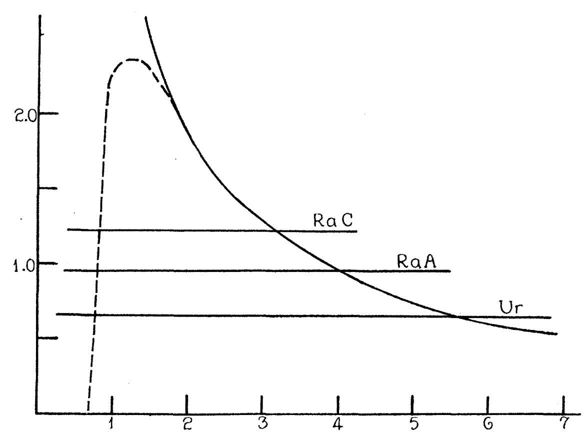

Fig. 6. The unit of ordinates is 10^{-5} ergs, and the unit of abscissas 10^{-12} cm.

radium C′, which have decay periods of about six thousand million years and a millionth of a second respectively.

An alpha-particle emitted by an element of atomic number Z escapes through the Coulomb field corresponding to (Z-2). Hence the potential energy for $Z=82$, which has been plotted in Fig. 6 is appropriate to radium A. The three horizontal lines in Fig. 6 give the energies of the alpha-particles

[9] Blackett, Proc. Roy. Soc. **A107,** 369 (1925).

from Ra A, Ra C′, and Uranium, which are 1.22×10^{-5}, 9.55×10^{-6}, and 6.5×10^{-6} ergs respectively. It is clear that the factor $(V\text{-}W)$, which occurs in the expression (9), given above for the rate of decay, is simply the vertical distance between the horizontal line for W and the potential energy curve for the proper value of Z-2. In Fig. 7 is plotted a curve derived from Fig. 6 giving the value of $(4\pi/h)\ [2\mu(V-W)]^{1/2}$ for Ra A as a function of the radius. The upper curve is for uranium and the lower for Ra C′, derived from curves for the proper atomic numbers.

From these curves we can at once find how large a barrier we have to take in order to obtain any observed rate of decay. For the integral occurring in the exponent of expression (9) is merely the area that we will take under the curve in Fig. 7. Since the unit of abscissas taken is 10^{-12} cm and the unit of ordinates 10^{13} cm^{-1}, each of the squares in Fig. 7 has the dimensionless value 10; so that for an element whose potential barrier has an area of one

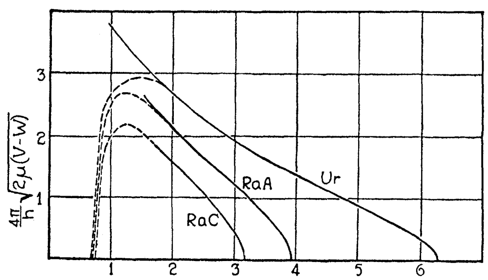

Fig. 7. Ordinates give the value of $(4\pi/h)[2\mu(V-W)]^{1/2}$ the unit being 10^{13} cm^{-1}. The unit of abscissas is 10^{-12} cm.

square on this diagram we should employ the factor e^{-10}. The broken line in Fig. 6 has been drawn so as to give for Ra A in Fig. 7 an area of approximately the value 53.7. For substituting this value in expression (9) together with $W=9.55\times10^{-6}$ and $a=10^{-12}$ we obtain the decay constant $8.45\times10^{20}\times e^{-53.7}=3.8\times10^{-3}$ sec.$^{-1}$, or the decay period $1/\lambda$ is 4.4 minutes in agreement with observation. In the expression for λ the precise value of the first factor is obviously unimportant, for if it were taken five times larger or smaller this would only alter the area of the required barrier by about 1 percent. The general size of the potential barrier in Fig. 6 that we have had to take seems to be a very reasonable one.

Now we reach an unexpected result. In drawing the areas for uranium and Ra C′ in Fig. 7 the continuous lines were predetermined, and the broken lines have been derived from the curve in Fig. 6, already used for Ra A. The values of the two areas are found to be 34.4 and 90, though the exact values depend on how the broken line is made to join the Coulomb potential curves. On substituting the values 34.4 and 90 in the expression for T we obtain for Ra C′ and uranium decay periods of the order of 10^{-6} sec. and 10^{10} years respectively, in agreement with observation.

It was already clear from Fig. 6 that we should obtain for all elements some qualitative agreement with the Geiger-Nuttall relation: the higher the energy of the alpha-particle the greater the rapidity of decay. But now we have found the unexpected result that the agreement is almost quantitative; that we do not have to choose a different potential energy inside the nucleus for each alpha-particle but having taken one potential curve for the whole series, it is the energy of the emitted alpha-particle which determines its own rate of decay. The mere fact that the velocity of the alpha-particle from Ra A, 1.69×10^9 cm per sec. is a little greater than the 1.4×10^9 cm per sec. of uranium, and a little less than the 1.92×10^9 cm per sec. of Ra C′, gives Ra A a decay period 10^{15} times as short as that of uranium and 10^8 times as long as that of Ra C′, in agreement with observation. Questions raised by this agreement with the factor of Geiger and Nuttall will be discussed in the last section of this paper. The radius 2×10^{-12} cm, at which we have taken the deviation from the inverse-square law, seems to be of the magnitude which our knowledge of the nucleus would lead us to expect.

Further we see at once why it is that no slow alpha-particles have been discovered. Although particles of ranges between 2.5 and 7 cm are plentifully distributed, no alpha-particles of energy less than 6.5×10^{-6} ergs have been found. But we now see from Figs. 6 and 7 that for particles of lower energy the area of the potential barrier increases very rapidly; so that for particles of range 2 cm or less the exponential factor would reduce the rate of decay of the element to a value at which its manifestation of radioactivity would be beyond the limits of detection.

Beta-ray disintegration.—It has been customary to assign the central core of the nucleus as the habitat of the nuclear electrons, with a potential energy curve of the type shown in Fig. 8. The outer slope AB again represents the Coulomb inverse-square field, as in Fig. 5. But since the charge of the electron is $-e$ instead of $+2e$ the potential energy is reversed in sign, and of half the magnitude of that in Fig. 5. There is nothing new in this assumed curve, although it looks somewhat artificial; this type of curve for the nuclear electron was obtained for example by Enskog in the paper referred to above.[8] What is new is the suggestion that an electron in the internal region again has a certain chance of penetrating the barrier, and of escaping at any time along CD with kinetic energy given by the height of CD above the axis.

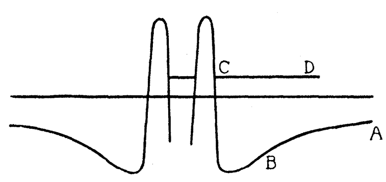

Fig. 8.

If we have alpha and beta-particles both with this chance of escaping from the nucleus, it might be thought that every radioactive element should be found to disintegrate part with expulsion of alpha-particles and part with beta-particles. But we would repeat that the chance of escape is extremely sensitive to the height to which the potential energy curve rises above the energy-level in question; and that if the size of this potential barrier be increased by a small factor the probability of escape may be decreased more than a *million-fold*. There seems then no reason why there should not be the three types of disintegration: that in which the probability of escape is much greater for an alpha-particle than for an electron; that in which it is much greater for an electron than for an alpha-particle; and that in which the probabilities of escape are comparable. The last gives the branching type of disintegration as shown by Ra C, of which 99.97 percent emits beta-particles, and 0.03 percent alpha-particles. By taking this view of the disintegration process, we have raised the question: Does any radioactive element have a unique mode of disintegration, or does it merely appear unique in most cases because the secondary mode is a million times less frequent and escapes detection? The present discussion certainly favours the latter alternative. It need not surprise us then that so few cases of branching disintegration have so far been discovered, since it is unlikely (so far as we know) that the areas of the potential barriers will in many nuclei happen to have just that relative size which will give for alpha and beta-particles comparable probabilities of escape.

Artificial disintegration.—Blackett's cloud-chamber photographs[9] of artificial disintegration in nitrogen showed that the impinging alpha-particle was caught and retained by the nucleus. One is tempted to apply the present theory, using again the fact that the impinging alpha-particle may penetrate the barrier of potential, this time from the outside, instead of passing over the top as required by classical theory. But when we do this we are at once confronted by the fact that instead of approaching the barrier 10^{20} times per second, like a nuclear particle, our alpha-particles will only make one impact apiece. So it would seem that the capture of the alpha-particle could not be due to penetration. There is, however, another consideration; and that is that if the impinging alpha-particle have an energy very near that of an allowed but unoccupied nuclear energy-level, the chance of its penetrating the barrier at a single impact approaches unity. This property has already been referred to in section 1.

4. Experimental Evidence for the Penetration of Potential Barriers

The essential basis of the present theory is the assumed power of particles to pass through regions where their total energy would by classical mechanics be less than their total energy. For this property there is no direct experimental evidence in physics, although it follows from the laws of quantum mechanics. But in applying this to the nucleus we have found that we can actually *obtain* direct experimental evidence. Though on classical mechanics the passage of a particle through such a forbidden region was a manifest absurdity, it was found in 1925 by Rutherford and Chadwick[10] that that is exactly what the alpha-particles from uranium appear to do.

Consider the alpha-particle which the uranium nucleus emits during its disintegration. The alpha-particle will gain energy in escaping through the repulsive Coulomb field outside the nucleus. This energy is given on classical theory as $2Ze^2/r$. Even if the alpha-particle leaves its place in the nucleus

[10] Rutherford and Chadwick, Phil. Mag. **50**, 889 (1925).

with no initial velocity, its energy cannot be less than this amount. The energy with which the alpha-particles leave the disintegrating Uranium atom is observed experimentally to be 6.5×10^{-6} ergs. On referring to Fig. 5, which was drawn for $Z=90$, we see that this energy corresponds to $r=6.3 \times 10^{-12}$ cm and if any of the energy was initial energy and not acquired through falling through the repulsive field, the value of r would have to be greater than this value.

It was concluded that the inverse-square law of repulsive field could not possibly hold within this value of r. Consequently if we fire at the uranium nucleus an alpha-particle having slightly more energy than the 6.5 10^{-6} ergs, it should penetrate its structure to where the Coulomb law no longer holds; while still faster particles should penetrate, even when not fired directly at the nucleus. It was therefore disconcerting when, on examining the scattering of fast alpha-particles fired at uranium, Rutherford and Chadwick could find no indication of any departure from the inverse-square laws. The Coulomb field was found to hold inside the radius from which the uranium alpha-particle appeared to come. That is to say, the uranium alpha-particle appeared to emerge from a region where its kinetic energy was negative. To escape this conclusion Rutherford[11] supposed that the uranium alpha-particles before ejection are electrically neutral, having been neutralised by two electrons which they leave behind when they are ejected. This hypothesis succeeded in circumventing the paradox. But if we abandon classical mechanics, the paradox disappears, yielding us direct experimental evidence in favor of the phenomenon of quantum mechanics in which we are interested.

5. Discussion of Limitations

It must be clearly understood that although the Coulomb part of the potential curve outside the nucleus, represented by AB in Fig. 5 is necessarily common to all particles, the internal part is merely intended to represent the potential energy of a particular alpha-particle. And it must not be taken to represent a general central field common to many particles, such as we are so accustomed to in atomic structure. There is no reason why the internal field should be necessarily symmetrical about the center of the nucleus as drawn in Figs. 5 and 8. In fact, Rutherford[12] has suggested that the nucleus may have something analogous to a crystalline structure. If this caution is lost sight of, difficulties are encountered.

For the atom of each radioactive element contains within its nucleus not only the alpha-particle which it will itself emit, but also the alpha-particles destined to be emitted by its successors in the radioactive series. Now if the velocity of escape of the alpha-particles from each element were always less than that of those emitted by its predecessors, there would be no serious difficulty; for from an atom loaded with alpha-particles in various allowed energy levels, the particle in the highest level would have the

[11] Rutherford, Phil. Mag. **4,** 580 (1927); Proc. Phys. Soc. **39,** 370 (1927).

[12] Rutherford, Jour. Franklin Inst. **198,** 743 (1924).

greatest probability of escape. This however is the opposite of what is observed; and we have to account for the subsequent emission of particles of higher energy than that emitted by the parent substance. We may do this by supposing either (a) that the alpha-particles of higher energy have in the parent element been confined by correspondingly high barriers; or (b) by supposing that the alpha-particles in the nucleus are not permanently in the high energy levels from which they emerge, but are temporarily raised up from lower levels. The latter seems to be a retrograde step, for the principle advantage of the present theory is that it has offered an escape from such processes.

If, however, we accept the former supposition (a), we see that the emission of one alpha-particle must profoundly modify the potential barrier which confines the alpha-particle destined to be emitted next. As we have shown, the Geiger-Nuttall relation seems to require that the barrier through which this alpha-particle emerges be approximately the same in all elements of the series. But until we know how this comes about, it seems inadvisable to discuss the Geiger-Nuttall relation in greater detail. In speaking of the energy of one particle in the nucleus, it must not be forgotten that we are making use of the simplification mentioned in the introduction: that of discussing one nuclear constituent alone.

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November 20, 1928.