Undergraduate Relativity Experiment

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The primary electrons in a γ-ray detector crystal are typically rather relativistic having kinetic energies of the order of their own rest mass. A technique is described which enables the student to observe the relativistic behavior of these electrons and compare his results with the classical and relativistic predictions. It is shown that it is possible to unambiguously determine the electron's momentum by knowing the Compton edge and photopeak energies of a γ-ray energy spectrum.

Almost every undergraduate laboratory has some kind of gamma-ray spectrometry experiment. The most common type is the NaI scintillator coupled to a photomultiplier. In well-equipped laboratories there are multichannel analyzers to sort the pulses from the PM tube; more economical laboratories use a single channel analyzer with a scalar to determine the pulse height spectrum. The most economical setup uses a Polaroid camera mounted on an oscilloscope to take time exposure photographs of the PM tube pulses. In any case, whether one uses a state-of-the-art lithium drifted germanium crystal or the more conventional NaI scintillator, the students are expected to construct a pulse height spectrum resulting from the detection of one or more γ-ray lines. A pulse height spectrum is the counting rate plotted as a function of the pulse height from the PM tube.

The light pulses from the scintillator (NaI) or the charge pulses from the semiconductor (Li–Ge) are proportional to the energy given to a primary electron in the detector by the incident γ-ray photon. Within the resolution of the detector, whatever energy is given to this electron appears as the height of an output pulse. Energies of γ-rays are typically in the MeV region and are thus of the order of one to several electron rest masses (0.511 MeV). Therefore, the primary electron is highly relativistic and exhibits relativistic behavior. Besides doing the ordinary things one is expected to do on a γ-ray experiment, the student can also test relativistic dynamics versus the classical predictions for these electrons.

There are three principal ways in which a γ ray can give up energy to the primary electron: photoelectric absorption in which all the γ-ray energy is taken up by the electron; Compton scattering in which the electron absorbs only a portion of the available γ-ray energy; and pair production in which an electron-positron pair are formed.

Photoelectric absorption is responsible for the events in the photopeak of the spectrum whereas Compton scattering accounts for the Compton edge and the bulk of the pulses below that value. Pair production is relatively unimportant for γ-ray energies less than about 3 MeV (in NaI). All of the γ rays we use in our experiment have energies less than 1.3 MeV.

The positions of the photopeaks for several isotopes serve to calibrate the energy scale on the spectra and the energy of the Compton edge can thus be unambiguously determined. The Compton edge corresponds to the maximum energy which can be given to a Compton-scattered electron by a photon which has an energy corresponding to the photopeak. That is, it is the result of a head-on collision between the photon and (stationary) electron. What is left of the photon is directly backscattered out of the crystal detector without further incident. There are, of course, multiple events in which the Compton scattered photon is photoelectrically absorbed but such an event would contribute to the photopeak, not to the Compton edge. The pulse height spectrum will give us E_γ (photon energy) and E_e, the Compton edge electron energy. These two quantities serve to reveal the relativistic behavior of the scattered electron.

From classical studies on radiation pressure we know the relationship between a photon's energy and its momentum[1];

$$P_\gamma = E_\gamma / c.$$

In a head-on collision with an electron, momentum will be conserved. Because the direction of the reflected photon will be opposite to the direction of the incident photon,

$$P_\gamma = P_e - P_\gamma' \text{ (scalar magnitudes)};$$
$$P_e = P_\gamma + P_\gamma' = (E_\gamma + E_\gamma')/c.$$

Because of energy conservation,

$$E_\gamma = E_e + E_\gamma'.$$

Combining these two equations we can eliminate E_γ',

$$cP_e = 2E_\gamma - E_e,$$

and determine the electron's momentum from the measured quantities E_γ and E_e (photopeak and

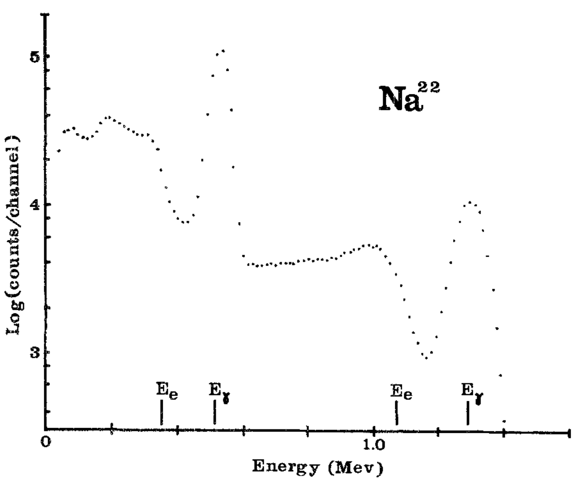

FIG. 1. A typical pulse-height (energy) spectrum from a γ-ray source emitting photons at two discrete energies (0.511 and 1.276 MeV). The detector is a NaI scintillator. A channel is a pulse-height (energy) interval. The Compton edge energy is measured half way up the edge when the plot is linear.

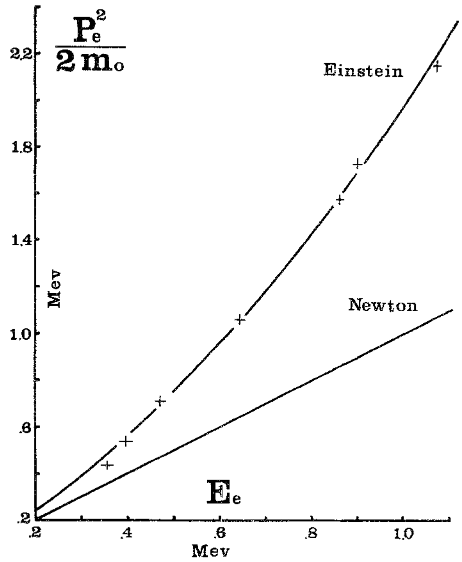

FIG. 2. The classical and relativistic predictions for energetic electrons. For primary electrons in the Compton edge of a pulse-height spectrum, $P_e = (2E_\gamma - E_e)/c$. The measured points are from the gamma sources: Na^{22}, Bi^{207}, Cs^{137}, Mn^{54}, Zn^{65}.

Compton edge energies). We have not invoked relativity and the derivation is quite generally valid.

Classically, the electron energy is related to its momentum by:

$$E_c = (P_e^2/2m_0) = (P_e^2c^2/2m_0c^2)$$
$$= (2E_\gamma - E_e)^2/1.02 \text{ MeV}. \qquad (1)$$

Therefore, if $P_e^2/2m_0$ is plotted versus E_e, classical physics predicts the result should be a straight line. However, the relativistic prediction is a little more complicated and noticeably diverges from the classical in this energy region. In order to determine the relativistic prediction, consider the energy-momentum relationship for a relativistic particle:

$$(E_e + m_0c^2)^2 = P_e^2c^2 + m_0^2c^4,$$

where the left hand side is the particle's total energy squared. Squaring the left hand side we get

$$E_e^2 + 2E_em_0c^2 = P_e^2c^2,$$

which reduces to the expression for the relativistic prediction:

$$P_e^2/2m_0 = E_e + E_e^2/1.02 \text{ MeV}. \qquad (2)$$

The students are asked to prepare a graph (Fig. 2) showing Einstein's prediction [Eq. (2)], Newton's prediction, and their measured values from Eq. (1). They are also asked to show that the correct form of the (kinetic) energy-momentum relationship is:

$$E_e = P_e^2/(m+m_0).$$

From the relativistic energy-momentum equations, the students can measure the observed mass increase and compare it with the theoretical prediction:

$$E_t^2 = P^2c^2 + m_0^2c^4 = m^2c^4;$$

$$(m^2 - m_0^2)c^2 = (mc^2 - m_0c^2)(m+m_0)$$

$$= E_e(m+m_0) = P^2$$

$$mc^2 = (P^2c^2/E_e) - m_0c^2$$

$$\gamma = \frac{m}{m_0} = \frac{mc^2}{m_0c^2} = \frac{P^2c^2}{E_e m_0 c^2} - 1 = \frac{(2E_\gamma - E_e)^2}{0.511 E_e} - 1.$$

The right hand side of this last equation contains all measured quantities so that γ, the relativistic mass increase ratio, can be measured for electrons from each measured Compton edge and plotted versus E_e. The theoretical prediction is given by

$$mc^2 = E_e + m_0c^2,$$

$$\gamma = 1 + E_e/0.511 \text{ MeV}.$$

The method used here to determine the momentum of a relativistic electron is superior to the more usual techniques employing a magnetic field, β-ray source since end-point energies do not have to be measured, and focusing is no problem. The energy and momentum of the electron are unambiguous and easily measured since the energy calibration is a straight-forward procedure. The measurement uncertainty arises because of the resolution of the detection system. If a germanium detector is used, the measurement precision is greatly increased.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1963), Vol. 1, p. 34–11.