son[6] had also observed "photo-electron lines" using NaI(Tl). His results show considerably less detail than those presented here.

* This work received partial support from a U. S. Army Signal Corps Contract and from the joint program of the ONR and AEC.

[1] See also the accompanying paper by R. Hofstadter and J. A. McIntyre, Phys. Rev. **78**, 619 (1950).

[2] Design to be reported by L. W. Hamner.

[3] The double-crystal arrangement of the accompanying paper gives a unique spectrum of gamma-ray lines and avoids the difficulties described in connection with the single crystal.

[4] Pringle, Standil, and Roulston, Phys. Rev. **77**, 841 (1950).

[5] P. R. Bell and J. M. Cassidy, Phys. Rev. **77**, 409 (1950).

[6] S. A. E. Johannson, Nature **165**, 396 (1950).

Measurement of Gamma-Ray Energies with Two Crystals in Coincidence*

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April 13, 1950

THE known properties of the Compton effect may be used in connection with scintillation counters to measure accurately and without confusion the energies of gamma-rays. A method by which this may be accomplished is shown in Fig. 1.

The line, labeled $h\nu$ in Fig. 1, represents a collimated beam of incident gamma-rays whose energy it is desired to measure. In a Compton encounter involving $h\nu$ and crystal X, the scattered gamma-ray $h\nu'$ and recoil electron β appear simultaneously. A fraction of the total number of $h\nu'$ gamma-rays engage in further Compton encounters in "detector" crystal D, or in photoelectric encounters in this crystal. In either case crystal D produces a light flash due to $h\nu'$ while crystal X produces a simultaneous light flash due to β. These coincident light flashes are the ones selected for study. If the pulse in D is used as a gate to trigger a single-channel (or multi-channel) discriminator, the resulting pulse size distribution in X (β-pulses) provides the energy of the incident gamma-ray beam. This follows from the fact that pulse height is proportional to the energy of the recoil electron. If more than one energy is present in the incident beam, each energy provides in X a unique pulse distribution appropriate to this energy.

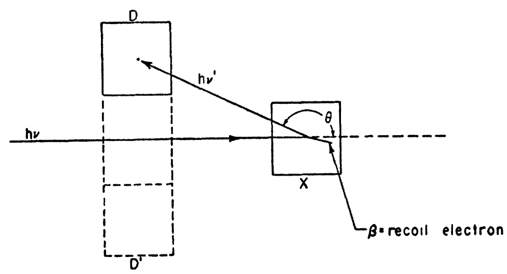

FIG. 1. Schematic of coincidence method for determining gamma-ray energies.

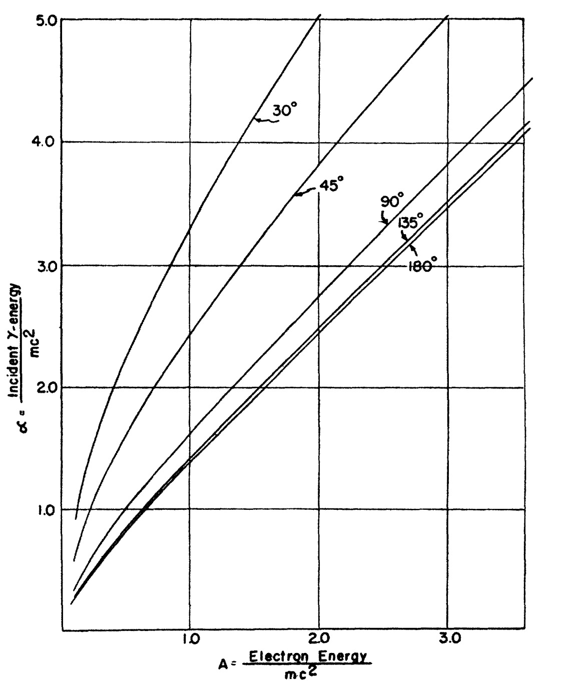

FIG. 2. Energy of incident gamma-ray plotted against electron recoil energy with angle θ as parameter. Energies are expressed in mc^2 units.

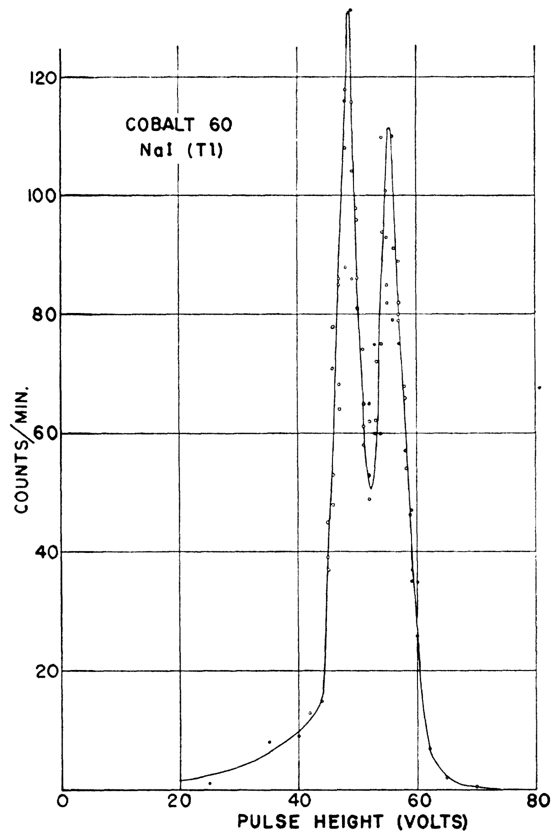

FIG. 3. The spectrum of Co^{60} obtained with the coincidence method.

Using the energy-momentum equations of the Compton effect the energy of the original gamma-ray beam can be calculated from the energy of the recoil electron. Let $h\nu=\alpha(mc^2)$ and β-energy $=A(mc^2)$. Then

$$\alpha=\tfrac{1}{2}A\{1+[1+2/(A\,\mathrm{hav}\theta)]^{\frac{1}{2}}\}. \quad (1)$$

Figure 2 shows a set of curves of α *versus* A obtained from Eq. (1) for various values of θ. It may be observed that between 135° and 180° the dependence on θ is extremely small. This corresponds to the well-known fact that a quantum of approximate energy $\frac{1}{2}mc^2$ is scattered in the back hemisphere for a large range of values of $h\nu\geq mc^2$. Hence a large solid angle for detector D may be employed without sacrificing much energy resolution. To gain still higher efficiencies of detection the ring counter DD' may be used. We are now using the single block D shown in Fig. 1 at an approximate angle of 150° at a center-to-center distance of crystals of 1.5 in.

With this arrangement, a single-channel discriminator, and clear NaI(Tl) crystals, we have studied the gamma-rays of Co^{60} at 1.17 and 1.33 Mev. The results are given in Fig. 3. A 15-millicurie source was used with about 1° collimation. The choice of 1° collimation is accidental and we believe 5° or 10° collimation sufficient. The latter type of collimation and a ring counter (DD′) should allow use of considerably less than one-millicurie source strength to obtain similar results. A multi-channel discriminator should further reduce the required source strength. The radium C spectrum and Sb^{124} gamma-spectrum have also been examined with results agreeing closely with those obtained by Latyshev[1] and the Indiana group.[2] Further details will be published in due course.

* This work received partial support from a U. S. Army Signal Corps Contract and the joint program of the ONR and AEC.
[1] G. D. Latyshev, Rev. Mod. Phys. **19**, 132 (1947).
[2] Kern, Zaffarano, and Mitchell, Phys. Rev. **73**, 1142 (1949). C. S. Cook and L. M. Langer, Phys. Rev. **73**, 1149 (1949).

A Short Method for Evaluation of the Townsend Integral for Electron Avalanche Formation*

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April 6, 1950

IT was observed by C. G. Miller[1] of this laboratory that the starting potentials of the burst pulse corona with coaxial cylindrical geometry in certain gases decreased more slowly than inversely proportional to pressure. Since Loeb[2] has given the relation for this threshold as

$$\beta f \exp\left(\int_a^r \alpha dx\right)=1,$$

it was of importance to show that if the dominant term,

$$\exp\left(\int_a^r \alpha dx\right),$$

was to remain constant in setting the threshold, the potential was required to decrease more slowly than the pressure decreased. The establishment of this conclusion on the basis of functional reasoning led to a much simplified procedure for calculating the integral for coaxial cylindrical geometry and one which will be applicable to other geometries. The integral

$$\int_a^r \alpha dx \tag{1}$$

is called the Townsend integral for avalanche formation, with dx a line element in the gap, α the first Townsend coefficient, $a=x$ at the high field electrode, and $r=x$ at a point where α, in virtue of lowered field intensity, becomes negligible.

Knowing the field along the line of integration, $E=\Delta Vg(x)$, where ΔV is the gap potential, the variable of integration can be changed to the quotient $(E/\Delta V)$ by the transformation $dx=h(E/\Delta V)d(E/\Delta V)$. Then if the electrode system used is such that the function $h(E/\Delta V)$ is homogeneous, $h(Kt)=K^n h(t)$, the ΔV can be transferred outside the integral and the integrand further transformed to a function of the quotient of the field divided by the gas pressure, p,

$$\int_a^r \alpha dx=(p/\Delta V)^{n+1}\int_{g(a)\Delta V/p}^{E(r)/p} \alpha h(E/p)d(E/p).$$

But the relationship between α, p, and E has the form

$$\alpha/p=f(E/p), \tag{2}$$

so that

$$\int_a^r \alpha dx=-p(p/\Delta V)^{n+1}\int_{E(r)/p}^{g(a)\Delta V/p} f(E/p)h(E/p)d(E/p). \tag{3}$$

In Eq. (3) the integrand is independent of both the gas pressure and the gap potential and, therefore, can be set up for a particular gap directly from the general relation (2) for the gas used. The variables p and ΔV appear only in the external factor and in the upper limit of integration (at the high field electrode), the lower limit being zero or some preassigned constant. Thus, the dependence of the value of the Townsend integral on p and ΔV can be more easily observed from Eq. (3) than from the expression (1) alone. Conversely, if the variation of the value of the Townsend integral with, say, gas pressure is known for a particular discharge phenomenon, the dependence of the gap potential producing this phenomenon on pressure can be determined.

For example, consider a coaxial circular cylindrical electrode system having inner and outer radii a and b, respectively. At a distance x from the axis,

$$E=\Delta V/[x\ln(b/a)],$$

and

$$dx=\{-1/[(E/\Delta V)^2\ln(b/a)]\}d(E/\Delta V).$$

Hence $n=-2$ and we have the result

$$\int_a^r \alpha dx=[\Delta V/\ln(b/a)] \times\int_0^{(\Delta V/p)/a\ln(b/a)} (E/p)^{-2}f(E/p)d(E/p). \tag{4}$$

(In this case the integrand is also independent of the electrode dimensions.) It can be seen from Eq. (4) that for a phenomenon such as burst pulse onset, where the Townsend integral has essentially a constant value, the gap potential must decrease as the gas pressure decreases, but $\Delta V/p$ must increase.

* This work was done under ONR contract.
[1] C. G. Miller, doctoral dissertation, University of California at Berkeley (September, 1949).
[2] L. B. Loeb, Phys. Rev. **73**, 798 (1948).

Nuclear Magnetic Moments and Shell Structure

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THE magnetic moments of certain nuclei of intermediate mass, $34<A<132$, provide further evidence for the existence of proton shells at values of $Z=20$ and 50.[1] Table I gives the values of the magnetic moments and spins of pairs of isotopes which differ by two neutrons.[2] It will be seen that for $Z<20$ and $Z>50$, the addition of two neutrons decreases the value of $|\mu|$, while for $20<Z\leqslant 50$, $|\mu|$ is increased instead; this is particularly noticeable at $Z=50$. This general behavior is found regardless of whether the magnetic moment is to be ascribed to an odd proton or to an odd neutron. For all these nuclei for which $Z\leqslant 50$, with the exception of $_{37}Rb^{87}$ (for which $N=50$), the spin is unaffected by the addition of the two neutrons.

By comparison of the actual moments with those predicted by the one-body model for the two limiting cases in which the odd particle's spin is parallel or antiparallel to the orbital angular momentum, i.e., $l=I\mp\frac{1}{2}$, it has been found that the actual state function for the odd particle is generally a mixture of these two possibilities.[1] The addition of the two neutrons will, in general, change the proportions of the mixture and shift the actual magnetic moment toward one or the other of the limits. If we now examine the moments in Table I with this in mind, we find for $20<Z\leqslant 50$ (with the exception of Ag) the proportion of the state with lower orbital angular momentum is increased so that parallel orientation of spin and orbital angular momentum is favored. For $Z<20$ and >50, the opposite is true, the states with larger l and hence antiparallel orientation being favored instead.

As Nordheim points out, the nuclear electric charge will be distributed throughout the volume of the nucleus and will produce a quasi-elastic repulsive force on the odd proton which would tend to favor higher angular momentum orbits. Since this repulsive force is proportional to the mean charge density, the addition of two neutrons, by increasing the nuclear volume without changing the charge, will decrease this force and thereby favor the states with lower angular momentum as is observed for $20<Z\leqslant 50$. This would be the case for a proton inside the nucleus;