$$\frac{\lambda_{\beta^-}}{\lambda} = 0.37 \qquad \text{for } \beta^- \text{ decay to } Zn^{64}$$

$$\frac{\lambda_{\beta^+}}{\lambda} = 0.18 \qquad \text{for } \beta^+ \text{ decay to } Ni^{64}$$

$$\frac{\lambda_{\varepsilon}}{\lambda} = 0.45 \qquad \text{for electron capture to } Ni^{64}$$

The isotope Zn^{65} is produced by the reactions $Cu^{65}(d,2n)Zn^{65}$, $Cu^{65}(p,n)Zn^{65}$, or by neutron capture from Zn^{64}. Because of the long half life of 250 days, it is a very convenient source for laboratory purposes, obtainable from the United States Atomic Energy Commission. Like all odd A radioisotopes, Zn^{65} decays only in one direction, into Cu^{65} by β^+ emission or electron capture. The decay scheme is shown in Figure 23.2. In addition to positrons, gamma rays and X rays are emitted.

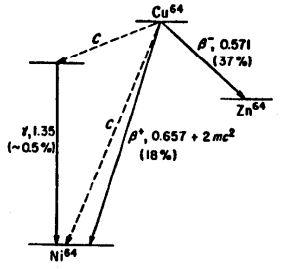

Fig. 23.1. Decay scheme of Cu^{64}. Vertical distances: differences of atomic masses.

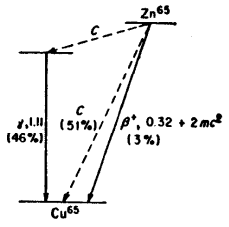

Fig. 23.2. Decay scheme of Zn^{65}.

Furthermore, one always observes a distribution of Compton electrons produced by the gamma rays in the source and the backing unless extreme precautions are taken. The same effect occurs, obviously, with Cu^{64}, but it is unimportant there because of the low intensity of the gamma radiation.

CHARGED PARTICLE SPECTRA

DETERMINATION OF THE ENERGY DISTRIBUTION

The sign and the energy distribution of the charged particles emitted by the source are determined with the aid of a semicircular spectrometer. Figure 23.3 shows a setup suitable for a crude experiment. The baffle system shown is brought between the pole pieces of a small electromagnet, making it necessary for the beta-particles to describe semicircular trajectories in order to reach the counter. For an electron of momentum $p = mv$, the radius ρ of the path in a magnetic field perpendicular to the direction of motion is given by

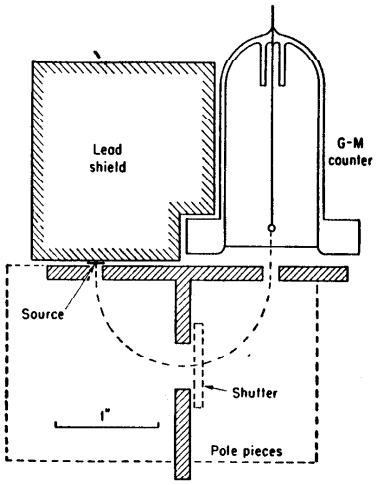

Fig. 23.3. Beta-ray "spectrometer."

$$\frac{mv^2}{\rho} = \frac{evH}{c} \tag{23.1}$$

$$H\rho = \frac{cp}{e} \tag{23.2}$$

with $H\rho$ called the "magnetic rigidity" and measured in gauss-cm. (Note the inconsistency in symbol and unit, which is due to tradition. The symbol $B\rho$ or else the unit oersted-cm would be preferable.) The momentum, measured in units of mc, is

$$P = \frac{p}{mc} = \frac{eH\rho}{mc^2} = \frac{H\rho}{1{,}704} \tag{23.3}$$

The total energy, in units of mc^2, is

$$W = \frac{E}{mc^2} = \sqrt{P^2 + 1} = \sqrt{\left(\frac{H\rho}{1{,}704}\right)^2 + 1} \tag{23.4}$$

This is the most useful formula for obtaining the energy from $H\rho$. The kinetic energy, then, is simply

$$E_{kin} = mc^2(W - 1) = 0.511(W - 1) \text{ (Mev)} \tag{23.5}$$

The spectral distribution may be given by the number $N(W)dW$ of electrons with energies between W and $W + dW$, or by the number $N(H\rho)dH\rho$ with $H\rho$ values between $H\rho$ and $H\rho + dH\rho$. Since the same number of particles is expressed as $N(W)dW$ and $N(H\rho)dH\rho$, one finds

$$N(W) = N(H\rho)\frac{dH\rho}{dW} = N(H\rho)\frac{W}{H\rho}(1{,}704)^2 \tag{23.6}$$

For a given field H in the spectrograph, an intensity $I(H)$ is measured, proportional to the number of particles having radii of curvature between certain limits $\rho - \frac{\Delta\rho}{2}$ and $\rho + \frac{\Delta\rho}{2}$.

$$I(H) = \text{const } N(H\rho)\Delta(H\rho) = \text{const } N(H\rho)H\rho\frac{\Delta\rho}{\rho} \tag{23.7}$$

Since $\frac{\rho}{\Delta\rho}$, the resolving power of the instrument, is constant, the distributions can be calculated from the measured intensity by the formulas

$$N(H\rho) = \text{const}\frac{I(H)}{H\rho} \tag{23.8}$$

$$N(W) = \text{const}\frac{I(H)W}{(H\rho)^2} \tag{23.9}$$

THEORETICAL SHAPE OF THE BETA SPECTRA[1]

According to the Fermi theory, the shape of an allowed spectrum—i.e., for cases where the spin of the nucleus changes by not more than 1ħ, with conservation of parity—is given by the formula

$$N(W) = \text{const } PW(W_0 - W)^2F(Z,W) \tag{23.10}$$

The distribution is determined principally by the so-called statistical factor $PW(W_0 - W)^2$, which is proportional to the density of final states for an electron of energy W and a neutrino of energy $W_0 - W$. Since it is low if the momentum of one of the particles is small, the intensity drops to zero both near $W = 1$ and $W = W_0$. The factor $F(Z,W)$ gives the influence of the Coulomb field on the wave function of the electrons at the nucleus and therefore on the decay probability which depends essentially on the square of the wave function. For $Z = 0$, $F(Z,W) = 1$. It increases for β^- particles with increasing nuclear charge and decreasing energy (attraction), whereas it decreases with Z and decreasing energy for positrons (repulsion). Obviously Z is the charge number of the *residual* nucleus in whose field the escaping beta particle moves. The shape of the distributions for different values of Z is shown in Figure 23.4. Extensive graphs of $F(Z,W)$ have been given by Moszkowski.[2]

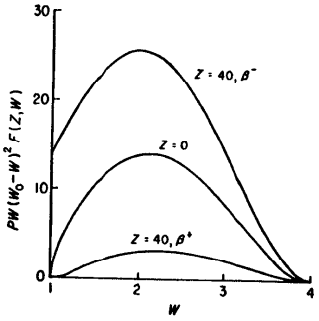

Fig. 23.4. Shape of allowed beta spectra for $W_0 = 4$ (maximum kinetic energy: 1.53 Mev) and $Z = 40$, 0, and "−40" (i.e., positron emission).

FERMI-KURIE PLOT

The direct determination of the upper limit W_0 of the spectrum is rather difficult because the intensity goes quadratically to zero near W_0. If one plots, however,

$$y = \left[\frac{N(W)}{WPF(Z,W)}\right]^{1/2} \tag{23.11}$$

as a function of W, one should get a straight line,

$$y = \text{const } (W_0 - W) \tag{23.12}$$

intersecting the abscissa at $W = W_0$ (Figure 23.5). This plot is called the Kurie plot after F. N. D. Kurie, who first developed the method, or the Fermi plot after E. Fermi, who is responsible for the theory. In addition to allowing an accurate determination of the upper limit, it provides a good check on the shape of the distribution. A deviation from formula 23.10 would reveal itself in a curvature of the plot.

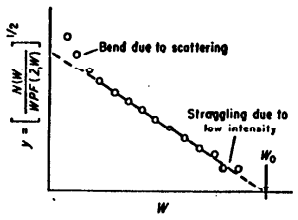

Fig. 23.5. Typical Fermi-Kurie plot.

Since for a spectrometer measurement $N(W)$ is given by the equation 23.9, the ordinate of the Fermi plot is

$$y = \left[\frac{I(H)}{(H\rho)^3 F(Z,W)}\right]^{1/2} \tag{23.13}$$

It is obviously permissible to omit any constant factor, e.g., the factor ρ^3 in the denominator, and to use

$$y = \left[\frac{I(H)}{H^3 F(Z,W)}\right]^{1/2} \tag{23.14}$$

INTENSITIES

The relative intensities of the β^- and the β^+ spectra are obtained by plotting $N(H\rho) = \frac{I(H)}{H\rho}$ vs. $H\rho$ and integrating, for example, using a planimeter.

ELECTRON CAPTURE

IDENTIFICATION OF K CAPTURE

During the process of K capture no detectable radiation is emitted, since the energy is carried away by a neutrino. The hole left in the K shell, however, will be filled by an electron from an outer shell, preferentially the L shell. In a certain fraction of cases—given by the so-called fluorescence yield, Υ—the transition energy is emitted in the form of a K X ray characteristic of the daughter element. In the other cases it is used for the ejection of another electron (Auger electron) from one of the outer shells, no K radiation being emitted. The fluorescence yield Υ increases strongly with increasing atomic number (Figure 23.6).

K capture can be recognized and measured, therefore, by observing either Auger electrons or K-X rays (making sure that they do not follow internal conversion). Since the Auger electrons emitted by Ni have energies of less than 10 kev, they are difficult to observe except in gases. The nature of the X rays, on the other hand, can be determined easily with the method of critical absorption. Table 23.1 lists the absorption coefficients of different materials for the K-X rays of Ni, Cu, and Zn. Since the absorption is mainly due to photoeffect, it drops suddenly,

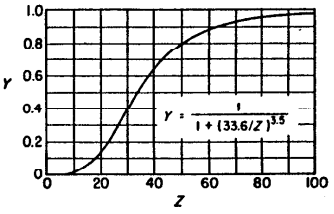

Fig. 23.6. Fluorescence yield.[3]

when, with increasing Z of the absorber, the binding energy of the K shell increases beyond the energy of the X ray in question. Thus the Cu K radiation ($h\nu$ = 8.1 kev) is able to eject a K electron from Fe and Co (E_K = 7.1 and 7.7 kev), but not from Ni (E_K = 8.3 kev). The mass absorption coefficient, therefore, drops sharply between Co and Ni.

Table 23.1

Mass Absorption Coefficients of K-X Rays,[4] in cm²/g

Emitter	Absorber						
	H	C	A	Fe	Co	Ni	Cu
Ni	0.5	5.5	144	400	54	58	61
Cu	0.5	4.5	116	328	358	48	51
Zn	0.5	3.7	93	270	290	310	41

DETERMINATION OF THE INTENSITY OF THE K RADIATION

A rough determination of the intensity of the X radiation is possible if the solid angle and the sensitivity of the counter are known. The probability that a photon hitting the counter will be counted is essentially given by the probability that it will be absorbed in the gas, within the sensitive volume. In contrast to gamma rays, the emission of secondary electrons from the walls is unimportant because of their extremely short range. For an end window counter, with the source