SCATTERING OF SLOW ELECTRONS AND POSITRONS BY COMPLEX ATOMS

1. The Ramsauer and Townsend effects

As was shown in Chap. XVI, § 5, Born's first approximation is not applicable to low-velocity collisions of electrons with atoms, and the experimental results obtained in this region show clearly that a more elaborate theory is required.

In a preliminary investigation in 1921 of the free paths of electrons of very low energy (0.76 to 1.1 eV) in various gases, Ramsauer found the free paths of these electrons in argon to be very much greater than that calculated from gas-kinetic theory. The extension of these observations to a wider range of velocities revealed a surprising variation in the cross-section. It was found that the effective cross-section (proportional to the reciprocal of the free path) of argon atoms increased with decreasing velocity until the electron energy becomes less than 10 eV. For electron energies below this value it decreases again to the low values found in the preliminary measurements. Independently, Townsend and Bailey examined the variation of free path with velocity for electrons with energies between 0.2 and 0.8 eV by a different method, and showed that a maximum of the free path occurs at about 0.39 eV. This was confirmed by much later work of Ramsauer and Kollath.

Since these classical experiments were carried out, the behaviour of a large number of gases and vapours has been examined over a wide voltage range. The results obtained are illustrated in Fig. 89 for some monatomic gases and vapours. In these figures the variation of effective cross-section with velocity is illustrated. This is proportional to the reciprocal of the mean free path. For purposes of comparison the gas-kinetic cross-section is indicated on the figures.

The striking feature of these cross-section-velocity curves is their wide variation in shape and size and also the marked similarity of the effect was a diffraction phenomenon. Thus Bohr suggested in general terms how the minimum cross-section observed in the rare gases near 0.7 eV could be explained. The field of a rare gas atom falls off very much more rapidly with distance than that of any other atom, and might be expected to behave in much the same way as a spherical potential well, discussed in Chap. II, § 4. It was shown there that the well may be deep enough to introduce within its range one or more

† In conversation with Professor R. H. Fowler.
complete wave-lengths of zero angular momentum without affecting waves of higher angular momentum appreciably. An observer at a great distance from the atom will then fail to observe any scattering.

![Graph showing angular distribution of scattered electrons](image)

**Fig. 90.** Observed angular distributions of electrons scattered elastically by argon atoms.

Strong experimental evidence of the wave nature of the phenomena was afforded by measurements of the angular distributions of the elastically scattered electrons. These were first carried out for electrons of energy between 4 and 40 eV by Bullard and Massey for argon.†


Instead of the curves characteristic of Born's first approximation, showing a monotonic decrease of intensity with angle of scattering, the curves obtained by Bullard and Massey exhibit maxima and minima. In Fig. 90 a series of curves is given illustrating the variation in form of these curves, for argon, as the electron energy increases from 1-1 eV to 780 eV; they show the gradual transition to the curves predicted by Born's first approximation. The higher energy measurements (42–780 eV) are due to Arnot,† and at the lowest energies (1-1 and 2-8 eV) to Ramsauer and Kollath.‡ These experiments have been carried out for a large number of gases over a wide range of electron energies, and it is found that in the majority of cases maxima and minima occur in some energy range.§ For light gases, such as hydrogen and helium, this range is small (up to 16 eV in helium and 6 eV in hydrogen), while for mercury pronounced maxima and minima are observed up to the highest energies for which observations have been taken (800 eV). From these experiments it is obvious that the wave nature of the electron is important over a wider range than is apparent from the observation of effective cross-sections.

† *Ibid. A*, 133 (1931), 615.
‡ *Ann. der Physik*, 12 (1932), 539.
It is of interest to examine the evidence from the angular distribution measurements as to the validity of the theoretical result (Chap. V, § 1.1) that the function $2\pi F(\theta) \sin \theta$, giving the number of electrons scattered per unit angle by a gas atom, tends to zero as $\theta$ tends to zero. In Fig. 91 two experimental curves representing scattering per unit angle are illustrated, and it is seen that the evidence is in favor of the theoretical conclusion.

2. The theory of the elastic scattering of low-velocity electrons

2.1. Effective scattering potential

In order to develop a theory of the scattering of low-velocity electrons by atoms, we must refer to the general theory of Chapter XII. The wave function $\Psi$, which in this case represents the system of atom + incident electron, was expanded in the form

$$ \Psi = \left( \sum_n + \int \right) \phi_n(r_a) F_n(r), \quad (1) $$

where $\phi_n(r_a)$ is the wave function representing the $n$th excited state of the atomic system. It was then shown that the function $F_n(r)$ satisfies the equation

$$ (\nabla^2 + k^2) F_n(r) = \frac{2m}{\hbar^2} \int V(r, r_a) \Psi(r_a, r) \phi_n^*(r_a) \, dr_a, $$

where $V(r, r_a)$ is the interaction energy between the incident and atomic electrons, and $k$ is the wave number of the outgoing electron wave, equal to $\hbar w_a / \hbar$.

If we neglect electron exchange, the elastic scattering is completely determined by the function $F_n(r)$ which satisfies the equation

$$ (\nabla^2 + k^2) F_n(r) = \frac{2m}{\hbar^2} \int V(r, r_a) \Psi(r_a, r) \phi_n^*(r_a) \, dr_a. \quad (2) $$

To solve (2) we must substitute some approximate form for $\Psi$ on the right-hand side of (2). For instance, in obtaining Born's approximation in Chapter XII we neglected all the scattered waves and replaced $\Psi$ by $\phi(r_a) \exp(ikr_a)$. A less drastic approximation is to neglect all but the elastically scattered wave, and thus set on the right-hand side of (2)

$$ \Psi = \phi_n(r_a) F_n(r). $$

We thus obtain

$$ \left\{ \nabla^2 + k^2 - \frac{2m}{\hbar^2} V_n(r) \right\} F_n(r) = 0, \quad (3) $$

where

$$ V_n(r) = \int V(r, r_a) \phi_n(r_a) \, dr_a. $$

This is the equation which represents the motion of the incident electron in the stable field of the atom, $V_n$ being just the potential of this field.

In fact the effect of the terms neglected in (3) can be represented by adding to $V_n$ in (3) an interaction of non-local type as shown in Chap. XII, § 5.6. Furthermore, within a certain energy range the non-local interaction may be replaced approximately by an equivalent local interaction which may be energy dependent (Chap. VIII, § 1.42). The combination of this interaction with $V_n$ is the so-called optical model potential (Chap. XII, § 4.1), which is real when the electron energy is below the threshold for inelastic scattering and complex otherwise. We may therefore survey the data on the elastic scattering of electrons with energy below this threshold by assuming an effective central potential $V$ which, while generally similar to $V_n$, must not be expected to be equal to it.

This will be the first procedure we shall adopt. Having shown how the experimental data on total and differential elastic cross-sections may be understood in terms of this concept of an effective scattering potential we shall consider some cases, particularly those of helium, neon, argon, and sodium, in more detail, isolating the contribution from $V_n$ and examining those which arise from electron exchange and atomic excitation or polarization.

The method required for this calculation is described in Chap. II, § 1. Since we shall have no further occasion to consider the inelastic scattering in this chapter, we shall drop the suffix 0 in $F_n$. If we expand the function $F$ in the form

$$ F = \sum_n F_n(r) P_n(\cos \theta), $$

and substitute in (3), the function $F_n(r)$ satisfies the equation

$$ \frac{d^2}{dr_a^2} [r F_n(r)] + \left[ k^2 - \frac{2m}{\hbar^2} V(r) - n(n+1) \right] [r F_n(r)] = 0. $$

As shown in Chapter II, the solution of this equation which is finite at the origin will have the asymptotic form

$$ r F_n \sim A_n \sin(kr - \frac{1}{2} \pi + \eta_n), $$

$\eta_n$ being a phase constant. The amplitude $f(\theta)$ of the scattered wave was shown in Chapter II to be given by

$$ f(\theta) = \frac{1}{2k \pi} \sum_{n=0}^{\infty} (2n+1) [\exp(2i\eta_n) - 1] P_n(\cos \theta), \quad (4) $$

and the differential cross-section for elastic scattering into the solid angle $d\omega$ is

$$ I(\theta) \, d\omega = |f(\theta)|^2 \, d\omega. $$
The total elastic cross-section $Q$ is given by

$$Q = 2\pi \int_0^\pi I(\theta) \sin \theta \, d\theta;$$

we thus obtain

$$Q = \sum Q_n,$$

where

$$Q_n = 4\pi k^2 \eta^2 (2l+1) \sin^2 \eta.$$

Born's first approximation holds only when $\eta$ is small, so that $\sin \eta_n$ never passes through a maximum due to $\eta$ reaching the value $\frac{1}{2}\pi$. Thus to this approximation one expects no oscillations in $Q$ as a function of the energy. This is no longer the case if $\eta$ may become greater than $\frac{1}{2}\pi$.

The classical approximation to $I(\theta)$ is never valid for electron scattering problems, but Jeffrey's method, particularly as modified by Langer (see Chap. V, p. 99), may usually be employed to obtain a good approximation to a phase $\eta$ which is not too small.

It was shown in Chap. II, § 2, that, if

$$V(r) = \frac{k(l+1) \tilde{Z}^2}{2m},$$

for $r$ given by

$$kr \sim k(l+1),$$

the contribution of all phases $\eta_n$ with $n > l$ can be neglected. The convergence will thus be best for light atoms and slow electrons (see Chap. XVI, § 5).

Westin\* has carried out an analysis, in terms of phase shifts, of the experimental data on the total and differential cross-sections for helium, neon, and argon. The 'observed' phases which he derives are illustrated in Figs. 92, 93, and 94. It must be remembered that analysis in terms of real phases is strictly correct only for electron energies below the excitation threshold, but it should be reasonably satisfactory at higher energies if the coupling with inelastic processes is not strong. Furthermore, the accuracy of much of the experimental data used by Westin is poor so that too much reliance should not be placed on the 'observed phases' which he derives. They may be taken as a valuable guide which could be made much more reliable if accurate experimental data were available. The reliability of phase-shift analyses for nuclear collisions (see Chap. XI, § 6.3, and Chap. XX, § 7.2) is much greater because of the relatively high precision of the observed data.


---

**Fig. 92.** Phase shift for elastic scattering of electrons by helium atoms, derived from observed data by Westin.

---

**Fig. 93.** Phase shifts for elastic scattering of electrons by neon atoms, derived from observed data by Westin.

----- calculated from the Hartree field without allowance for exchange.
3. General application of the method of partial cross-sections

3.1. Condition for existence of a Ramanauer-Townsend effect

It has already been pointed out that a vanishing cross-section near the low-velocity limit can occur if the field is strong enough to introduce one or more additional wavelengths, i.e., one or more additional zeros of the wave function $\varphi$ within the field. For this to be possible the field must be strong enough to introduce respectively one or more discrete energy levels of zero angular momentum. The effect cannot occur with a repulsive field for the reasons discussed in Chap. II, § 5—the phase $\eta_0$ can only equal $\pi r$ for such a field if it eliminates one complete wavelength which would exist if the field were not present. This is not possible at low energies because the wavelength is then much greater than the range of the field. At higher energies it may occur, but then higher-order phases must be affected also and their contributions will prevent the total cross-section from becoming abnormally small.

Quantitative calculations, discussed in § 4, confirm this explanation of the Ramanauer-Townsend effect.

3.2. Explanation of other general features

We now give a general explanation of the following experimental facts:

1. The magnitude of the cross-section varies between wide limits, the maximum observed for the alkali metals being over 100 times that observed in neon.
2. The angular distributions of the scattered electrons show marked maxima and minima.
3. The cross-section—velocity curves have forms characteristic of the different columns of the periodic table.

In order to do this we make use of the following properties of the calculated phases:

(a) For any atomic field $\eta$ decreases monotonically with $l$.
(b) $\eta$ is small when, for $r$ such that $kr \sim \frac{1}{2}$,

$$\frac{2m}{\hbar^2} V(r) \ll \frac{l(l+1)}{r^2}.$$ 

It follows from (b) that the series of partial cross-sections will converge quite quickly for low-velocity impacts; the major contribution will arise from the partial cross-sections $Q_l$ of such an order that $\eta_l \ll \frac{1}{2} \pi$. The maximum value of the contribution from a partial cross-section of order $l$ is

$$Q_{l, \text{max}} = \frac{\Delta r}{\Delta} (2l + 1).$$

We may therefore say at once that, the lower the velocity and the larger the value of $l$ for which the phase $\eta_l$ attains the value $\frac{1}{2} \pi$, the bigger will be the cross-section. Referring to the condition (b), we see that the biggest collision cross-sections will be those of atoms whose fields extend out to the greatest distances, viz., the alkali metals. If we use the empirical rules due to Slater,† for the effective nuclear charges

† Phys. Rev. 36 (1930), 57.
of alkali atoms, and define the diameter of an atom as the distance at which the radial charge density \( r^2|\frac{\partial \rho}{\partial r}|^2 \) of the outer shell is a maximum, the following values of the radii \( r_0 \) of various atoms are obtained:

<table>
<thead>
<tr>
<th>( r_0 ) in atomic ( \text{watts} )</th>
<th>13 ( \text{watts} )</th>
<th>0-5 ( \text{watt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li</td>
<td>2-3</td>
<td>2-3</td>
</tr>
<tr>
<td>Ni</td>
<td>4-1</td>
<td>4-1</td>
</tr>
<tr>
<td>K</td>
<td>5-1</td>
<td>6-1</td>
</tr>
<tr>
<td>Zn</td>
<td>3-1</td>
<td>3-1</td>
</tr>
<tr>
<td>Na</td>
<td>0-6</td>
<td>0-6</td>
</tr>
<tr>
<td>Mg</td>
<td>0-7</td>
<td>0-7</td>
</tr>
<tr>
<td>A</td>
<td>1-3</td>
<td>1-3</td>
</tr>
<tr>
<td>Kc</td>
<td>1-7</td>
<td>1-7</td>
</tr>
</tbody>
</table>

Note. \( l \) is measured in units of 1\( \alpha_0 \).

We give also the values of \( kr_0 \) corresponding to 13 and 0-5 eV electrons. Thus, using the criterion (5), we find that for potassium at least 7 harmonics are required, and the cross-section may be greater than \( 50r_0^2 \), whereas one harmonic only is required for neon, and the effective area will not be greater than \( 10r_0^2 \). For \( k = 0-2/\alpha_0 \) (0-54 eV), the area may be as great as \( 50r_0^2 \) for potassium, but still not much greater than \( 10r_0^2 \) for neon or helium. There is, then, clearly no difficulty in explaining the wide range in magnitude observed in the effective cross-sections. It is equally clear that the method indicates the possibility of maxima and minima in the angular distribution given by (4). Again the dominant effect arises from terms in the series such that \( \eta_l \approx \frac{2\pi}{l} \). The angular distribution, then, is roughly of the form

\[
I(\theta) = \text{const.} \{P_l(\cos \theta)\}^2
\]

which has \( l \) minima. This will be especially marked for the lower velocity impacts, where only a few terms of the series (4) are required and the weight factor \( 2l + 1 \) is particularly effective. Thus for electrons of 30 eV energy in argon the angular distribution is given closely by \( \{P_l(\cos \theta)\}^2 \). Actually the calculated phase values at this velocity are

\[
\eta_l = 2\pi + 0.335, \eta_1 = 4.311, \eta_2 = 1.983, \eta_3 = 0.374, \eta_4 = 0.159
\]

†The cross-sections obtained in this way will only be approximately correct when the field is sufficiently strong to produce large phase changes. For collisions of high-velocity electrons a large number of terms are required in the series, but each is small and the total cross-section small also.

It must be realized that these remarks are only illustrative, and the actual effects produced by the sum of a number of partial cross-sections may be very complicated, particularly for heavy atoms. The diffraction of waves by spherical obstacles is a much more complicated process than diffraction by a grating or other symmetrical arrangement.

It is not possible to explain the third feature listed above in such simple terms as the preceding. The quasi-periodic behaviour of the partial cross-sections must be due to the behaviour of sin \( \eta_l \). At low velocities, for the lightest atoms, only the zero-order phase is appreciable. An atomic field will behave qualitatively in a similar way to the potential well discussed in Chap. II, § 4. At a given velocity, for some atomic field, the phase \( \eta_l \) will attain a value near \( \frac{2\pi}{l} \), and for some heavier atom a value of \( \frac{2\pi}{l} \) will be attained, giving an equal maximum of the zero-order cross-section, and so on. For some atom with intermediate properties \( \eta_l \) will become appreciable, and so on. In this way some quasi-periodic behaviour of the cross-sections might be expected, but we still require to show from the theory that the periodicity follows that of the periodic table. This was first done by Allis and Morey using a simplified model for the effective scattering potential of the form

\[
\gamma = \begin{cases} \frac{-Ze^4}{\gamma r_0} \frac{1}{r} (r \leq r_0), \\ 0 (r \geq r_0), \end{cases}
\]

which makes possible an analytic solution of equation (3). In order to illustrate the periodic behaviour of the cross-sections two quantities, \( x \) and \( \beta \), were defined such that

\[
\beta = \frac{Ze^4}{2r_0^2 \alpha_0}, \quad x = kr_0.
\]

The first of these depends only on the atomic field, while the second is a function also of the incident electron velocity. Allis and Morey then showed that the cross-sections are quasi-periodic in \( \beta \) with period unity. This is illustrated in Fig. 95, where a number of partial cross-sections corresponding to different values of \( x \) are illustrated as functions of \( \beta \).

If, now, the approximate atomic radii given by Slater are used (given above in Table 1) and the constant \( Z \) above is adjusted to give as good agreement as possible of the form (6) for \( \gamma \) with that obtained from Slater's values, it is found that a period of 1 in \( \beta \) is approximately a
used gives only a rough approximation, particularly for very low velocities. For such cases large effects may arise from the atomic field beyond the radius $r_0$. To allow for this Morse extended the calculations to the interaction $V = -Ze^2 r^{-1} \exp(-2r/r_0)$, and found that very similar results are obtained, the same quantities $\beta$ and $kr_0$ being again important.

![Graph showing absorption coefficient vs. wavelength for different elements.](image)

**Table II**

<table>
<thead>
<tr>
<th>Element</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium</td>
<td>1.38</td>
</tr>
<tr>
<td>Sodium</td>
<td>2.54</td>
</tr>
<tr>
<td>Potassium</td>
<td>3.31</td>
</tr>
<tr>
<td>Helium</td>
<td>0.77</td>
</tr>
<tr>
<td>Neon</td>
<td>1.73</td>
</tr>
<tr>
<td>Argon</td>
<td>2.65</td>
</tr>
<tr>
<td>Krypton</td>
<td>3.09</td>
</tr>
</tbody>
</table>

The lighter elements are to some extent anomalous in this respect; this is also borne out by the observations (see Fig. 98 of this chapter).

4. Quantitative application of method of partial cross-sections

The first quantitative application of the theory was made by Holtsmark for the scattering of electrons by argon; but we shall first consider the results obtained by Allis and Morse using their simplified model.

Having chosen the values of the parameter $\beta$ and $r_0$, using Slater's rules (loc. cit.), it was usually found that a good approximation to the observed cross-section is obtained with this model. In Fig. 96 the experimental curves are compared with calculated ones. The values of $\beta$ and $r_0$ used in obtaining the latter are slightly different from those obtained from Slater's rules (loc. cit.), but the differences are not great. In Table III the parameters which give the best fit with experiment are compared with Slater's values, both measured in atomic units.

The agreement obtained is very striking and leaves little doubt as to the correctness of the theoretical explanation of the Ramsauer-Townsend effect afforded by quantum mechanics. However, the field

---

**Table III**

<table>
<thead>
<tr>
<th>Element</th>
<th>$\beta$</th>
<th>$r_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>0.77</td>
<td>0.05</td>
</tr>
<tr>
<td>Neon</td>
<td>1.73</td>
<td>0.78</td>
</tr>
<tr>
<td>Argon</td>
<td>2.65</td>
<td>1.4</td>
</tr>
<tr>
<td>Sodium</td>
<td>3.31</td>
<td>3.15</td>
</tr>
</tbody>
</table>

Holtsmark obtained very good agreement with the observed cross-section for argon, including the Ramsauer-Townsend effect, by using for $V$ the Hartree field modified by an empirical polarization correction and evaluating the phases $\eta_0$ by numerical integration of the differential equations. Without the polarization modification the agreement is not good, the Ramsauer-Townsend effect not being found (cf. Fig. 104).

A much stricter test of the theory may be applied by comparing calculated and observed angular distributions. These are much more sensitive to inaccuracy in the theory. In Fig. 97 the angular distribution curves observed by Bullard and Massey and by Ramsauer and Kollath

---

† Zeits. f. Physik, 55 (1929), 497.

‡ Rev. Mod. Phys. 4 (1932), 577.

† Loc. cit.
for argon (loc. cit.) are compared with those calculated by using Holtsmark's values of the phases \( \eta \). The agreement for 30 and 12 volts is found to be very good. It is of interest to note that at the lower voltages the agreement with curves calculated from the simplified model of Morse and Allis discussed in § 3 is not nearly so satisfactory. At very low velocities, however, the observations of Ramsauer and Kollath are no longer in agreement even with Holtsmark's calculations, as seen from the figure.

Holtsmark\( ^\dagger \) carried out calculations for krypton similar to those for argon, using the Hartree field modified by a polarization correction. He obtained good agreement with observation, both in comparison with observed total cross-sections and with the angular distributions measured by Arnot\( ^\dagger \) and by Ramsauer and Kollath\( ^\dagger \) except at very low electron energies (less than 3 eV).

We now consider the problem of calculating the scattering directly given the Hartree–Fock wave functions for the atomic electrons. As in Chapter XVII we proceed first from the static field approximation (§ 5), then allow for exchange (§ 6) and finally for atom distortion and polarization (§ 7).

\( ^\dagger \) Sieds, F. Physik, 66 (1930), 49.


\( ^\dagger \) Ann. der Physik, 52 (1924), 887.


\( ^\dagger \) Proc. Roy. Soc. A, in course of publication.

\( ^\dagger \) Weert, loc. cit.; Thompson loc. cit.

\( ^\dagger \) Hartree, Kronig, and Petersen, Physica, 1 (1934), 301.


For a number of other atoms calculations have been carried out in which the large phases were determined from Jeffreys’ approximation (Chap. V, § 5) and the small ones from Born’s approximation (Chap. V, § 2). To illustrate the kind of accuracy which may be attained in this way† Table IV gives the values of the phase shifts calculated for

Table IV
Calculated phase shifts for 64-volt electrons scattered by krypton atoms

<table>
<thead>
<tr>
<th>Phase q</th>
<th>From accurate solution</th>
<th>Jeffreys’ approximation</th>
<th>Langer modification</th>
<th>Born approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.594</td>
<td>0.597</td>
<td>0.597</td>
<td>0.597</td>
</tr>
<tr>
<td>1</td>
<td>1.432</td>
<td>1.440</td>
<td>1.440</td>
<td>1.440</td>
</tr>
<tr>
<td>2</td>
<td>2.400</td>
<td>2.400</td>
<td>2.400</td>
<td>2.400</td>
</tr>
<tr>
<td>3</td>
<td>3.388</td>
<td>3.388</td>
<td>3.388</td>
<td>3.388</td>
</tr>
<tr>
<td>5</td>
<td>5.500</td>
<td>5.500</td>
<td>5.500</td>
<td>5.500</td>
</tr>
</tbody>
</table>

64-volt electrons in krypton calculated by accurate numerical solution of the differential equation‡ and by the approximate methods.§ Values obtained by use of Langer’s modification of Jeffreys’s approximation are also included. It will be seen that for $\eta < 0.6$ Born’s approximation is sufficiently accurate. For larger values of $\eta$ Jeffreys’s approximation is superior and is improved considerably for small-order phases by the Langer modification.

For mercury vapour Henneberg|| and Massey and Mohr|| carried out calculations in this way using the Thomas–Fermi field.|| The calculated angular distributions are compared with Arnot’s observations in Fig. 99, good general agreement being revealed. Similar calculations have been carried out for potassium,||| zine,||| cadmium,||| and bromine,||| reasonable agreement being obtained in all cases provided the electron energy is not too low.

6. Electron exchange in elastic collisions

In the discussion of § 4 the elastic scattering has been considered in terms of a semi-empirical model. At higher energies good results are obtained if the effective interaction assumed is taken to be the actual

† An empirical discussion of the effectiveness of these methods has been given by Arnot, Proc. Camb. Phil. Soc. 32 (1936), 161; see also Kale and Bonham, J. Chem. Phys. 40 (1964), 1936.
‡ Holtmark, Z. Physik, 66 (1930), 49.
§ Zelte, J. Physik, 83 (1935), 865.
‖‖ Loc. cit.
||| Zelte, J. Physik, 53 (1933), 855.
|||| McCutchen, Phys. Rev. 46 (1934), 988.
|||||| Shaw and Snyder, Phys. Rev. 59 (1941), 660.

mean static potential $V_0$ of the atom or rather the Hartree–Fock potential. For low energies, however, this is not adequate. Thus Holtmark found it necessary to include an additional potential which had the asymptotic form $-\alpha e^{-r/\beta}$ expected from a polarization effect but which otherwise was continued inwards to join smoothly to the Hartree field in an adjustable fashion so as to reproduce the observed results for argon and krypton, including the Ramsauer–Townsend effect. Even this empirical potential did not reproduce satisfactorily the observed angular distribution for electrons with energies below 4 eV.

The obvious failure of the static field as a representation of the effective potential for slow collisions presented by electron scattering from helium atoms has also been pointed out.

As discussed in Chap. XVII, § 2, the most important modification of the static field approximation in the theory of electron scattering by hydrogen atoms arises from electron exchange. It is natural, therefore, to examine the contribution which exchange makes to the scattering for other atoms.

The method used for including electron exchange in the one-body approximation for helium has already been outlined in Chap. XV, § 1.31. It involves the solution of integro-differential equations of the same.