

## Units and Constants

*The choice of units* suitable for the description of atomic and subatomic phenomena represents a delicate methodological problem.

It has become customary to measure the energies of particles in electronvolts (ev) and their momenta in ev/c (the momentum  $p$  in ev/c is numerically equal to the energy  $pc$  in ev). With these units, the energy of a particle with charge equal to the charge of the electron, that has been accelerated from rest through a potential difference  $V$ , is numerically equal to  $V$ . The momentum of this particle is related to the radius of curvature,  $R$ , of its trajectory in a magnetic field by the equation:

$$p \text{ (ev/c)} = 300\mathcal{B} \text{ (gauss)} \cdot R \text{ (cm)},$$

where  $\mathcal{B}$  represents the component of the magnetic induction perpendicular to the trajectory.

Also, it is customary to measure velocities in terms of the velocity of light,  $c$ , and charges in terms of the electron charge,  $e$ . For example, one says that a particle has a velocity of 0.9 the velocity of light, rather than a velocity of  $2.69798 \cdot 10^{10}$  cm sec<sup>-1</sup>, and one will speak of a doubly charged nucleus rather than of a nucleus with a charge of  $9.604 \cdot 10^{-10}$  e.s.u.

The writer has pointed out (RB40.2) that the above units can be incorporated into a consistent system whose fundamental units are the centimeter, the velocity of light, the charge of the electron and the volt. This system, however, has a serious disadvantage in that it fails to bring out the dimensional homogeneity of physical equations. For example, it yields the following expression for the fundamental relation between total energy  $U$ , momentum  $p$ , and mass  $m$  of a particle [see Eq. (A.2.5)]:

$$U^2 = p^2 + m^2.$$

This feature, of course, is a direct consequence of the fact that, in the proposed system, two quantities with physical dimensions (the velocity of light and the charge of the electron) are taken as equal to one.

In this book we have chosen to sacrifice consistency for convenience. Thus we have used the C.G.S. system in most of our equations, while, when referring to numerical data, we have always given energies in terms

of  $ev$ , momenta in terms of  $ev/c$ , charges in terms of  $e$  and velocities in terms of  $c$ .

The numerical values of the most important natural constants encountered in this book are listed in Table 1 [see refs. (DMJ48) and (DMJ49.2)].

Table A.1.1. Natural constants

Velocity of light:

$$c = 2.99776 \cdot 10^{10} \text{ cm sec}^{-1}.$$

Number of atoms per gram atom:

$$N = 6.024 \cdot 10^{23}.$$

Charge of the electron:

$$e = 4.802 \cdot 10^{-10} \text{ e.s.u.}$$

Mass of the electron:

$$m_e = 9.105 \cdot 10^{-28} \text{ g.}$$

Rest energy of the electron:

$$m_e c^2 = 5.1079 \cdot 10^5 \text{ ev.}$$

Classical radius of the electron:

$$r_e = e^2/m_e c^2 = 2.8176 \cdot 10^{-13} \text{ cm.}$$

Compton wavelength of the electron:

$$\lambda_e = h/m_e c = 2.4265 \cdot 10^{-10} \text{ cm.}$$

Fine structure constant:

$$\alpha = e^2/hc = 1/137.027.$$

Mass of the proton:

$$M_p = 1836 m_e.$$

Rest energy of the proton:

$$M_p c^2 = 938 \text{ Mev.}$$

Rest energy of the neutron:

$$M_n c^2 = 939 \text{ Mev.}$$

Mass of the  $\pi^\pm$  meson:

$$m_\pi = 276 m_e.$$

Rest energy of the  $\pi^\pm$  meson:

$$m_\pi c^2 = 141 \text{ Mev.}$$

Mass of the neutral meson:

$$m_0 = 266 m_e.$$

Rest energy of the neutral meson:

$$m_0 c^2 = 136 \text{ Mev.}$$

Mass of the  $\mu^\pm$  meson:

$$m_\mu = 209 m_e.$$

Rest energy of the  $\mu^\pm$  meson:

$$m_\mu c^2 = 107 \text{ Mev.}$$

Mean life of the  $\pi^\pm$  meson:

$$\tau_\pi = 2.65 \cdot 10^{-8} \text{ sec.}$$

Mean life of the  $\mu^\pm$  meson:

$$\tau_\mu = 2.10 \cdot 10^{-6} \text{ sec.}$$

APPENDIX

II

Review of Some Relativistic Equations

(a) **Lorentz transformation.** The equations

$$\begin{aligned} X' &= \frac{X - \beta ct}{\sqrt{1 - \beta^2}}, & X &= \frac{X' + \beta ct'}{\sqrt{1 - \beta^2}}, \\ Y' &= Y, \\ Z' &= Z, \\ ct' &= \frac{ct - \beta X}{\sqrt{1 - \beta^2}}, & ct &= \frac{ct' + \beta X'}{\sqrt{1 - \beta^2}}, \end{aligned} \tag{1}$$

express the relations between the space and time coordinates  $X, Y, Z, t$  and  $X', Y', Z', t'$  of a point in two frames of reference that are moving with respect to each other. The same units of length are used in the two systems. The motion is a uniform translation along the coincident  $X$ -axes of the two frames of reference;  $c$  is the velocity of light, and  $\beta$  is the velocity of the second system with respect to the first divided by the velocity of light. As shown in the figure, the  $Y$ -axes of the two systems are parallel, and consequently the  $Z$ -axes also are parallel.

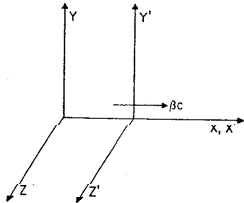


Fig. A.2.1. The Lorentz transformation.

From Eqs. (1) it follows that

$$\begin{aligned} X^2 + Y^2 + Z^2 - c^2 t^2 &= (X')^2 + (Y')^2 \\ &+ (Z')^2 - c^2 (t')^2 = \text{const.} \end{aligned} \tag{2}$$

Any four quantities, whose values in two different frames of reference are related by the same equations (1) connecting  $X, Y, Z, ct$  with  $X', Y', Z', ct'$ , are said to form a *four-vector*.

(b) **Energy and momentum.** The three Cartesian coordinates of the momentum of a particle, multiplied by  $c$ ,  $cp_x, cp_y, cp_z$ , and the total energy of the particle,  $U = E + mc^2$ , form a four-vector. Thus the following transformation equations hold:

$$\begin{aligned} cp'_{x'} &= \frac{cp_x - \beta U}{\sqrt{1 - \beta^2}}, & cp_x &= \frac{cp'_{x'} + \beta U'}{\sqrt{1 - \beta^2}}, \\ cp'_{y'} &= cp_y, \\ cp'_{z'} &= cp_z, \\ U' &= \frac{U - \beta cp_x}{\sqrt{1 - \beta^2}}, & U &= \frac{U' + \beta cp'_{x'}}{\sqrt{1 - \beta^2}}. \end{aligned} \tag{3}$$

From these equations one sees as before that

$$c^2 p_x^2 + c^2 p_y^2 + c^2 p_z^2 - U^2 = \text{const.} \tag{4}$$

Equation (4) indicates that the quantity  $U^2 - p^2 c^2$  is independent of the frame of reference. In the frame of reference in which the particle is at rest,  $p = 0$  and  $U = mc^2$ ; therefore in any frame of reference  $U^2 - c^2 p^2 = m^2 c^4$  and thus

$$U^2 = c^2 p^2 + m^2 c^4. \tag{5}$$

One can obtain the expressions for the momentum and the energy as functions of the velocity by considering a particle at rest in the second frame of reference and computing its momentum and energy in the first frame of reference. If we put in Eqs. (3),  $p' = 0$ , and if we call  $U$  the total energy and  $p$  the momentum in the first frame of reference we obtain

$$p = \frac{m\beta c}{\sqrt{1 - \beta^2}}; \quad U = \frac{mc^2}{\sqrt{1 - \beta^2}}; \tag{6}$$

where  $\beta c$  is the velocity of the particle in the first frame of reference.

(c) **Velocity of the center of mass.** Consider two particles with the same mass,  $m$ , one with momentum  $p$ , the other at rest. Let  $\beta_c$  be the velocity of the center of mass of the two particles. In the center-of-mass system, the total momentum of the two particles is zero, the total energy is  $2mc^2/\sqrt{1 - \beta_c^2}$ . In the laboratory system, the momentum of the two particles is equal to the momentum,  $p$ , of the particle in motion and it is given by [see Eqs. (3)]:

$$p = \frac{\beta_c}{c} \frac{1}{\sqrt{1 - \beta_c^2}} 2 \frac{mc^2}{\sqrt{1 - \beta_c^2}} = \frac{2m\beta_c}{1 - \beta_c^2}$$

Thus  $\beta_c$  satisfies the equation:

$$\frac{\beta_c}{1 - \beta_c^2} = \frac{p}{2mc} \tag{7}$$

One can obtain another expression for  $\beta_c$  as follows. Let  $U$  and  $U'$  be the total energies of both particles in the laboratory system and in the center-of-mass system respectively. These two quantities are related by the equation:

$$U = \frac{U'}{\sqrt{1 - \beta_c^2}}$$

On the other hand,  $p$  has the expression

$$p = \frac{\beta}{c} \frac{U'}{\sqrt{1 - \beta^2}}$$

One thus obtains:

$$\beta_c = \frac{cp}{U} \tag{8}$$

Note that this equation has more general validity than Eq. (7). It gives the velocity of the center of mass of any number of particles, with arbitrary masses, for which the vector sum of momenta has the magnitude  $p$  and the sum of the total energies is  $U$ .

(d) **Field of a charged particle in motion.** One can best obtain the relations between the quantities describing the electromagnetic field, as viewed from two frames of reference in relative motion to one another, by considering the vector potential  $A$  and the scalar potential  $V$ . The three components of  $A$  and  $V$  form a four-vector. Thus:

$$\begin{aligned} A'_{X'} &= \frac{A_X - \beta V}{\sqrt{1 - \beta^2}}, & A_X &= \frac{A'_{X'} + \beta V'}{\sqrt{1 - \beta^2}}, \\ A'_{Y'} &= A_Y, \\ A'_{Z'} &= A_Z, \\ V' &= \frac{V - \beta A_X}{\sqrt{1 - \beta^2}}, & V &= \frac{V' + \beta A'_{X'}}{\sqrt{1 - \beta^2}}. \end{aligned} \tag{9}$$

From  $A$  and  $V$ , one obtains the electric and magnetic field intensities by means of the equations:

$$\begin{aligned} \mathcal{E} &= -\text{grad } V - \frac{1}{c} \frac{\partial A}{\partial t}, \\ \mathcal{B} &= \text{curl } A. \end{aligned} \tag{10}$$

In the particular case where the field in the second system is purely electrostatic, i.e., set up by charges at rest, one can put  $A' = 0$ , and Eqs. (9) yield:

$$A_X = \frac{\beta V'}{\sqrt{1 - \beta^2}}, \quad A_Y = A_Z = 0, \quad V = \frac{V'}{\sqrt{1 - \beta^2}} \tag{11}$$

To derive the components of  $\mathcal{E}$  one uses Eqs. (10) and (11). For  $\mathcal{E}_X$  one obtains:

$$\begin{aligned} \mathcal{E}_X &= -\frac{\partial V}{\partial X} - \frac{1}{c} \frac{\partial A_X}{\partial t} \\ &= -\frac{1}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial X} - \frac{1}{c} \frac{\beta}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial t}. \end{aligned}$$

$V'$  is a function of  $X', Y', Z'$ ; these quantities are the functions of  $X, Y, Z, t$  given in Eq. (1); thus:

$$\begin{aligned} \mathcal{E}_X &= -\frac{1}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial X'} \frac{\partial X'}{\partial X} - \frac{1}{c} \frac{\beta}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial X'} \frac{\partial X'}{\partial t} \\ &= \frac{1}{1 - \beta^2} \mathcal{E}'_{X'} - \frac{\beta^2}{1 - \beta^2} \mathcal{E}'_{X'}. \end{aligned}$$

Therefore:  $\mathcal{E}_X(X, Y, Z, t) = \mathcal{E}'_{X'}(X', Y', Z', t')$  (12)

For  $\mathcal{E}_Y$  one obtains:

$$\begin{aligned} \mathcal{E}_Y &= -\frac{\partial V}{\partial Y} - \frac{1}{c} \frac{\partial A_Y}{\partial t} \\ &= -\frac{1}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial Y'} = -\frac{1}{\sqrt{1 - \beta^2}} \frac{\partial V'}{\partial Y'}. \end{aligned}$$

Thus:  $\mathcal{E}_Y = \frac{\mathcal{E}'_{Y'}}{\sqrt{1 - \beta^2}}$ , (13)

and, similarly:

$$\mathcal{E}_Z = \frac{\mathcal{E}'_{Z'}}{\sqrt{1 - \beta^2}} \tag{13a}$$

Equations (12) and (13) may be used to compute the electric field of a charged particle in motion. Suppose that this particle is at rest at the origin of the primed frame of reference, and therefore, moves with a velocity  $\beta c$  in the unprimed frame of reference. At a given point of the space, let us consider separately the components of  $\mathcal{E}$  in a direction parallel and perpendicular to the motion of the particle respectively. We may assume that the point under consideration is in the  $X, Y$  plane. In the primed frame of reference, the  $X$ - and  $Y$ -components of  $\mathcal{E}$  are given by

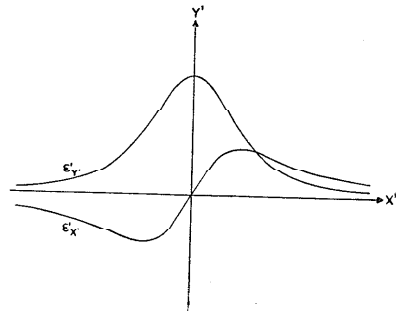


Fig. A.2.2. Components of the electric field of a point charge at rest.

$$\begin{aligned} \epsilon'_{X'} &= \frac{eX'}{(X'^2 + Y'^2)^{3/2}}, \\ \epsilon'_{Y'} &= \frac{eY'}{(X'^2 + Y'^2)^{3/2}}. \end{aligned}$$

These functions are represented, for a given value of  $Y$ , by the curves in Fig. 2.

In the unprimed frame of reference, the charged particle travels with the velocity  $\beta c$ . Consider the time  $t = 0$ , when the charged particle passes through the origin of the unprimed frame. At this time  $X' = X/\sqrt{1 - \beta^2}$ . This equation together with Eq. (12) shows that one obtains the curve representing  $\epsilon_x$  as a function of  $X$  from the curve

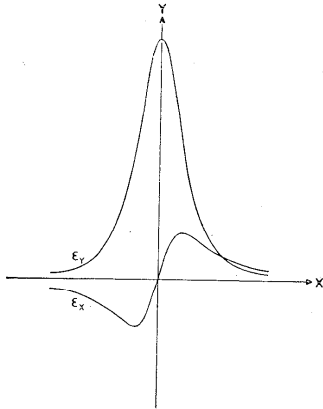


Fig. A.2.3. Components of the electric field of a point charge in motion.

representing  $\epsilon'_{X'}$  as a function of  $X'$  by contracting the scale of abscissae in the ratio of 1 to  $\sqrt{1 - \beta^2}$  and leaving the ordinates unchanged. Similarly Eq. (13) shows that one obtains the curve representing  $\epsilon_y$  as a function of  $X$  from the curve representing  $\epsilon'_{Y'}$  as a function of  $X'$  by contracting the scale of abscissae in the ratio of 1 to  $\sqrt{1 - \beta^2}$  and expanding the ordinate scale in the ratio of 1 to  $1/\sqrt{1 - \beta^2}$  (see Fig. 3).

For the convenience of the reader we give in Fig. 4 the graphs of various functions of  $\beta$  that appear in the equations of this Appendix.

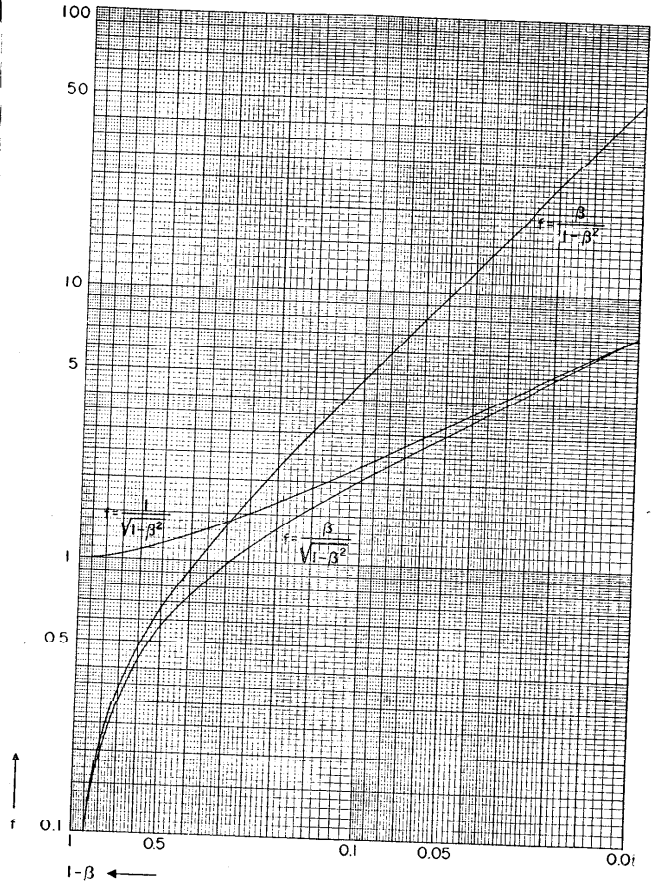


Fig. A.2.4. The quantities  $\beta/(1 - \beta^2)$ ,  $1/\sqrt{1 - \beta^2}$ , and  $\beta/\sqrt{1 - \beta^2}$  plotted as functions of  $1 - \beta$ .

## The Poisson Formula; Statistical Errors

Consider an experiment where one counts events occurring at random in time. Select a time interval  $T$ , and assume that the average number of events occurring during an interval of this duration is  $n$ . We want to calculate the probability  $p(m)$  that  $m$  events will occur in the selected interval.

First, assume the interval  $T$  to be subdivided into  $N$  equal intervals of duration  $T/N$ , such that the probability that an event will occur in any particular sub-interval is small. This probability is then  $n/N$ . Application of the multiplication theorem of probability theory gives a value of  $(n/N)^m$  for the probability that one event will occur in each of any  $m$  predetermined sub-intervals. Similarly, the probability that no event occurs in a given sub-interval is  $(1 - \frac{n}{N})$ , and the probability that no event occurs in any of  $(N - m)$  predetermined sub-intervals is  $(1 - \frac{n}{N})^{N-m}$ . The probability that events occur in  $m$  predetermined sub-intervals, but not in the remaining  $(N - m)$ , is the product of the two above probabilities and is:

$$\left(\frac{n}{N}\right)^m \left(1 - \frac{n}{N}\right)^{N-m}. \quad (1)$$

The number of ways in which one can select the first  $m$  intervals is:

$$\frac{N!}{m!(N-m)!} \quad (2)$$

Using the addition theorem of the probability theory, one obtains the following expression for the probability that  $m$  events occur in the original interval,  $T$ :

$$\frac{N!}{m!(N-m)!} \left(\frac{n}{N}\right)^m \left(1 - \frac{n}{N}\right)^{N-m}. \quad (3)$$

The desired distribution function  $p(m)$  is the limiting value of this expression as  $N$  tends to infinity, i.e.,

$$p(m) = \lim_{N \rightarrow \infty} \frac{N!}{m!(N-m)!} \left(\frac{n}{N}\right)^m \left(1 - \frac{n}{N}\right)^{N-m}. \quad (4)$$

Using Stirling's formula:

$$N! \rightarrow \sqrt{2\pi N} \left(\frac{N}{e}\right)^N,$$

one obtains from Eq. (4):

$$p(m) = \frac{e^{-n}}{m!} n^m. \quad (5)$$

Equation (5) is known as the *Poisson formula*. Several important properties of  $p(m)$  may be derived from the identity:

$$e^n = \sum_{l=0}^{\infty} \frac{n^l}{l!}. \quad (6)$$

If one divides both sides of Eq. (6) by  $e^n$ , and substitutes  $m$  for  $l$ , one obtains

$$\sum_{m=0}^{\infty} \frac{e^{-n}}{m!} n^m = \sum_{m=0}^{\infty} p(m) = 1, \quad (7)$$

as must be true from the definition of  $p(m)$ .

If one multiplies both sides of Eq. (6) by  $ne^{-n}$ , one obtains:

$$n = \sum_{l+1=1}^{\infty} (l+1) \frac{n^{l+1}}{(l+1)!} e^{-n} = \sum_{m=0}^{\infty} m \frac{n^m}{m!} e^{-n} = \sum_{m=0}^{\infty} mp(m). \quad (8)$$

This expresses the obvious result that the average value of  $m$  is  $n$ .

If one multiplies both sides of Eq. (6) by  $n^2e^{-n}$  one obtains, considering Eq. (8):

$$\sum_{m=0}^{\infty} m^2 p(m) = n^2 + n. \quad (9)$$

As a measure for the statistical accuracy of a particular counting experiment one may take the mean square value of the deviations of  $m$  from  $n$ , i.e., the quantity:

$$\sum_{m=0}^{\infty} (m-n)^2 p(m) = \sum_{m=0}^{\infty} m^2 p(m) - 2n \sum_{m=0}^{\infty} m p(m) + n^2 \sum_{m=0}^{\infty} p(m). \quad (10)$$

On combining Eqs. (7), (8), (9), and (10), one obtains:

$$\sum_{m=0}^{\infty} (m-n)^2 p(m) = n. \quad (11)$$

Thus the root mean square statistical error (or "standard statistical deviation") of an experiment in which  $n$  counts should be recorded is  $\sqrt{n}$ . The relative value of this error is  $\sqrt{n}/n = 1/\sqrt{n}$ .

## The Laplace and the Mellin Integrals

(a) *The Laplace integral.* Given a function,  $f(x)$ , of a real variable,  $x$ , the Laplace integral or Laplace transform of  $f(x)$  is defined by the equation:

$$\mathcal{L}_f(\lambda) = \int_0^{\infty} e^{-\lambda x} f(x) dx, \quad (1)$$

where  $\lambda$  is a complex parameter. We assume the integral to be convergent at the lower limit. If the integral converges at the upper limit for a certain value,  $\lambda_0$ , of  $\lambda$ , it also converges for all values of  $\lambda$  for which  $R(\lambda) > R(\lambda_0)$ , where  $R$  indicates the real part. On the other hand, if the integral diverges for a certain value,  $\lambda_0$ , of  $\lambda$ , it also diverges for all values of  $\lambda$  for which  $R(\lambda) < R(\lambda_0)$ . Hence, in general, the function  $\mathcal{L}_f(\lambda)$  is defined in the half-plane to the right of a straight line parallel to the imaginary axis.

The following formulae can easily be proved:

$$\mathcal{L}_f(\lambda) = \lambda \mathcal{L}_f(\lambda) - f(0), \quad (2)$$

where  $f' = df/dx$ ;

$$\mathcal{L}_F(\lambda) = -\frac{1}{\lambda} [\mathcal{L}_f(\lambda) - F(0)], \quad (3)$$

where  $F(x) = \int_x^{\infty} f(x') dx'$ ;

$$\left[ \frac{d^n}{d\lambda^n} \mathcal{L}_f(\lambda) \right]_{\lambda=0} = (-1)^n \int_0^{\infty} x^n f(x) dx. \quad (4)$$

An important property of the Laplace transformation is that, under not very restrictive conditions, the correspondence between  $f(x)$  and  $\mathcal{L}_f(\lambda)$  established by Eq. (1) is unique; i.e., there is only one function  $f(x)$  that has  $\mathcal{L}_f(\lambda)$  as its Laplace integral. If  $\mathcal{L}_f(\lambda)$  is known,  $f(x)$  can be determined by the following inversion formula:

$$f(x) = \frac{1}{2\pi i} \int_C e^{\lambda x} \mathcal{L}_f(\lambda) d\lambda, \quad (5)$$

where the integration path,  $C$ , is a straight line running parallel to the imaginary axis, in the half-plane of convergence of  $\mathcal{L}_f(\lambda)$ .

(b) *The Mellin integral.* Given a function,  $f(y)$ , of a real variable,  $y$ , the Mellin integral, or Mellin transform of  $f(y)$ , is defined by the equation:

$$\mathfrak{M}_f(s) = \int_0^{\infty} y^s f(y) dy, \quad (6)$$

where  $s$  is a complex parameter. If the integral diverges at the lower limit for a certain value,  $s_0$ , of  $s$ , it also diverges for all values of  $s$  for which  $R(s) < R(s_0)$ . If the integral diverges at the upper limit for a certain value  $s_0$  of  $s$ , it also diverges for all values of  $s$  for which  $R(s) > R(s_0)$ . Hence, if the Mellin integral converges anywhere, its field of convergence is a strip bounded by two straight lines parallel to the imaginary axis.

The following formulae can easily be proved:

$$\mathfrak{M}_F(s) = -s \mathfrak{M}_f(s-1), \quad (7)$$

where  $f' = df/dy$ ;

$$\mathfrak{M}_F(s) = \frac{1}{s+1} \mathfrak{M}_f(s+1), \quad (8)$$

where  $F(y) = \int_y^{\infty} f(y') dy'$ .

The Mellin transformation, like the Laplace transformation, is in general unique and can be inverted by the following formula:

$$f(y) = \frac{1}{2\pi i} \int_C y^{-(s+1)} \mathfrak{M}_f(s) ds, \quad (9)$$

where the integration path,  $C$ , runs parallel to the imaginary axis within the strip of convergence.

## The Gold Integral

The *Gold integral* is defined by the equation:

$$\varepsilon_n(x) = x^n \int_x^\infty \frac{e^{-s}}{s^{n+1}} ds, \quad (1)$$

or by the equivalent equation:

$$\varepsilon_n(x) = \int_1^\infty \frac{e^{-xs}}{s^{n+1}} ds. \quad (2)$$

Of particular interest in this volume are the Gold integrals corresponding to  $n = 0$  and to  $n = 1$ ,  $\varepsilon_0(x)$  and  $\varepsilon_1(x)$ . The function  $-\varepsilon_0(x)$  is also called the exponential integral,  $Ei(-x)$ :

$$-\varepsilon_0(x) = Ei(-x) = - \int_x^\infty \frac{e^{-s}}{s} ds = - \int_1^\infty \frac{e^{-xs}}{s} ds. \quad (3)$$

This function is extensively tabulated.\* The function  $\varepsilon_1(x)$  has the expression:

$$\varepsilon_1(x) = x \int_x^\infty \frac{e^{-s}}{s^2} ds, \quad (4)$$

or

$$\varepsilon_1(x) = \int_1^\infty \frac{e^{-xs}}{s^2} ds. \quad (5)$$

Integration by parts of Eq. (2) yields a relation between  $\varepsilon_n$  and  $\varepsilon_{n-1}$ ; in particular:

$$\varepsilon_1(x) = e^{-x} - x\varepsilon_0(x), \quad (6)$$

or

$$\varepsilon_1(x) = e^{-x} + xEi(-x). \quad (6a)$$

Differentiation of (5) with respect to  $x$  yields:

\* See, for example, Jahnke and Emde, Tables of Functions, Dover Publications, New York (1945).

$$\frac{d}{dx} \varepsilon_1(x) = -\varepsilon_0(x) = Ei(-x). \quad (7)$$

For  $x = 0$  Eq. (5) gives

$$\varepsilon_1(0) = \int_1^\infty \frac{ds}{s^2} = 1. \quad (8)$$

To investigate the behavior of  $Ei(-x)$  near  $x = 0$ , consider the identity:

$$-Ei(-x) = \int_x^{x_0} \frac{e^{-s}}{s} ds - Ei(-x_0).$$

For  $x$  and  $x_0$  very small compared with 1, one can take  $e^{-s} \approx 1$  and the above equation yields:

$$-Ei(-x) = \ln\left(\frac{x_0}{x}\right) - Ei(-x_0). \quad (9)$$

Equation (9) shows that  $-Ei(-x)$  diverges logarithmically as  $x$  approaches zero. For  $x \gg 1$ ,  $e^{-xs}$  decreases very rapidly with increasing  $s$ , so that the significant contributions to the integral in Eq. (5) come from values of the integrand in the neighborhood of  $s = 1$ . One thus obtains the following approximate expression for  $\varepsilon_1(x)$ :

for  $x \gg 1$ ,

$$\varepsilon_1(x) \approx \int_1^\infty e^{-xs} ds = \frac{e^{-x}}{x}. \quad (10)$$

Figure 1 gives a graphical representation of the function  $\varepsilon_1(x)$  for  $0 < x < 14$ . The approximate expression given by Eq. (10) is plotted for comparison.



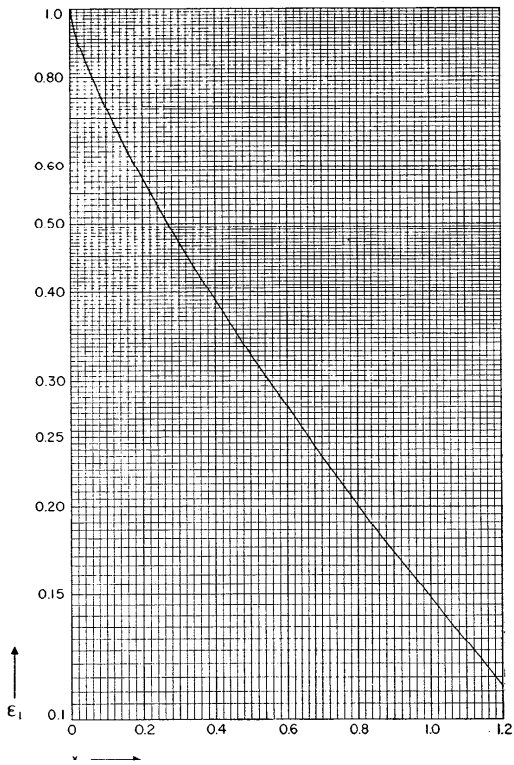


Fig. A.5.1. The solid lines represent the function  $\epsilon_1(x) = \int_1^\infty \frac{e^{-xs}}{s^2} ds$ ; the dashed line represents the function  $e^{-x}/x$ .

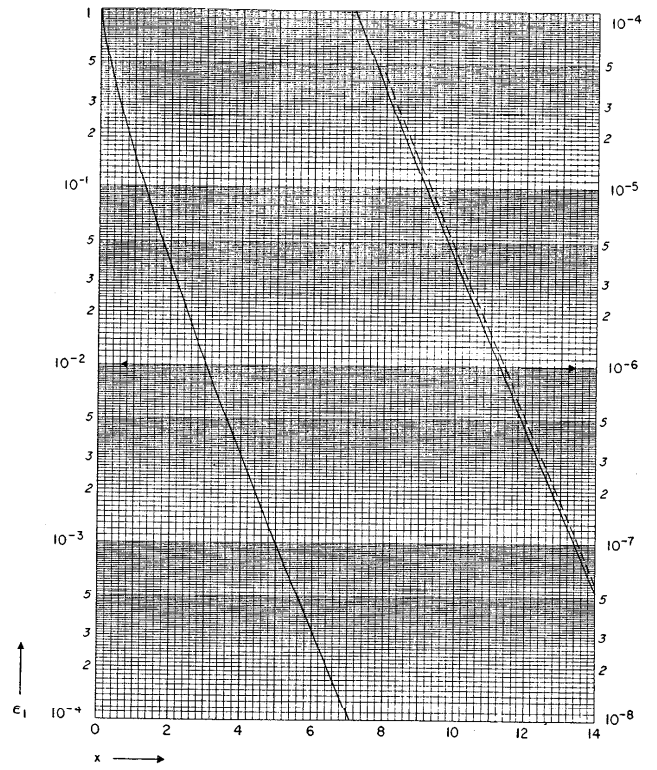


Fig. A.5.1. (Continued)

### The Standard Atmosphere

*In a static atmosphere*, the pressure at any point measures the amount of material above this point, i.e., the "atmospheric depth,"  $x$ . In a static and isothermal atmosphere,  $x$  is an exponential function of the altitude,  $z$ :

$$x = x_0 e^{-z/z_0}, \tag{1}$$

where  $z$  is measured from the level where the pressure is  $x_0$ .

The average relation between  $x$  and  $z$  in the actual atmosphere at latitudes around  $40^\circ$  or  $50^\circ$  is shown in Fig. 1. This figure is constructed from the data given in the Report No. 538 of the National Advisory Committee for Aeronautics.

Figure 2 gives the relation between  $x$  and the quantity:

$$z_0 = -x \frac{dz}{dx} \tag{2}$$

This quantity also represents the ratio of  $x$  to the density of air,  $\rho$ :

$$z_0 = \frac{x}{\rho} \tag{3}$$

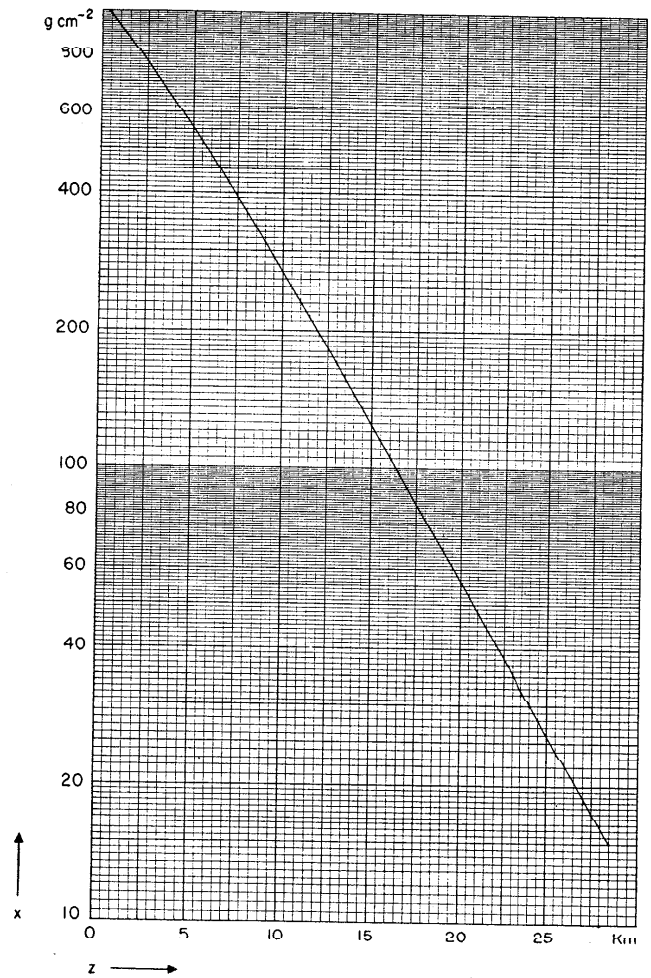


Fig. A.6.1. Atmospheric depth,  $x$ , as a function of altitude,  $z$ , from sea level in the standard atmosphere;  $z$  is measured in kilometers ( $10^5$  cm).

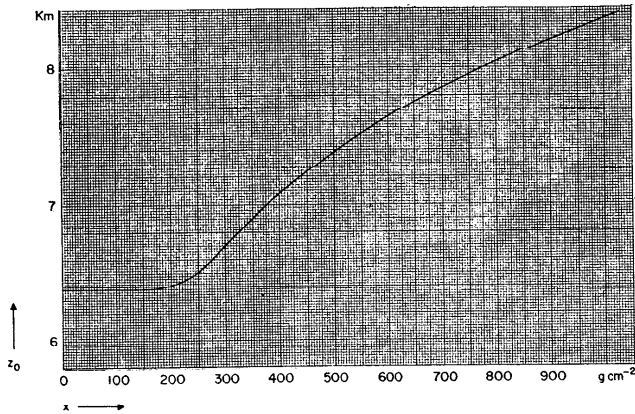


Fig. A.6.2. The quantity,  $z_0 = x/\rho = -x(dz/dx)$  as a function of  $x$  in the standard atmosphere;  $z_0$  is measured in kilometers.

## Bibliography

- (AAI48) A. I. Alikhanyan, A. Alikhanov, V. Morosov, G. Mushelzhvili and A. Hrinijan, *J. Exper. Theor. Phys.* *18*, 673 (1948).
- (AAI49) A. I. Alikhanyan, D. M. Samoilovich, I. I. Gurevich, K. P. Babayan and R. I. Geragimova, *J. Exper. Theor. Phys.* *19*, 664 (1949).
- (ACD32) C. D. Anderson, *Science* *76*, 238 (1932).
- (ACD33) C. D. Anderson, *Phys. Rev.* *43*, 491 (1933).
- (ACD34) C. D. Anderson and S. H. Neddermeyer, *Int. Conf. Phys. (London)* *1*, 171 (1934).
- (ACD36) C. D. Anderson and S. H. Neddermeyer, *Phys. Rev.* *50*, 263 (1936).
- (AE46) E. Amaldi, D. Bocciaelli, B. V. Cacciapuoti and G. C. Trabacchi, *Nuovo Cim.* *9*, 203 (1946).
- (AE51) E. Amaldi and G. Fidicaro, *Phys. Rev.* *81*, 339 (1951).
- (AGD48) G. D. Adams, *Phys. Rev.* *75*, 1707 (1948).
- (ALW50) L. W. Alvarez, A. Longacre, V. G. Ogren and R. E. Thomas, *Phys. Rev.* *77*, 752 (1950).
- (AM40) M. Ageo, G. Bernardini, N. B. Cacciapuoti, B. Feretti and G. C. Wick, *Phys. Rev.* *57*, 945 (1940).
- (AMM51) M. M. Addario and S. Tamburino, *Nuovo Cim.* *8*, 82 (1951).
- (AN38) N. Arley, *Proc. Roy. Soc. A* *168*, 519 (1938).
- (AN40) N. Arley and B. Eriksen, *Danske Videnskabernes Selskab* *17*, No. 11 (1940).
- (AP37) P. Auger, P. Ehrenfest, A. Freon and A. Fournier, *Comp. Rend.* *204*, 257 (1937).
- (AR51) R. Armenteros, K. H. Barker, C. C. Butler, A. Cachon and A. H. Chapman, *Nature* *167*, 501 (1951).
- (AWR46) W. R. Arnold and A. Roberts, *Phys. Rev.* *70*, 766 (1946).
- (BA47) A. Borsellino, *Helv. Phys. Acta* *20*, 136 (1947).
- (RAS40) A. S. Bishop, H. Bradner and F. M. Smith, *Phys. Rev.* *76*, 588 (1940).
- (BAS50) A. S. Bishop, J. Steinberger and L. J. Cook, *Phys. Rev.* *80*, 291 (1950).
- (BCC50) C. C. Butler, W. G. V. Rosser and K. H. Barker, *Proc. Phys. Soc.* *A63*, 145 (1950).
- (BdP50) P. Budini and N. Dallaporta, *Nuovo Cim.* *7*, 230 (1950).
- (BD49) D. Binder, *Phys. Rev.* *76*, 856 (1949).
- (BF33) F. Bloch, *Zeits. f. Physik* *81*, 363 (1933).
- (BgH44) H. Bagge, *Phys. Zeitsch.* *21*, 461 (1944).
- (BC41) C. Bernardini, B. N. Cacciapuoti, E. Pancini, O. Piccioni and C. C. Wick, *Phys. Rev.* *60*, 910 (1941).
- (BG45) G. Bernardini, M. Conversi, E. Pancini, E. Scrocco and G. C. Wick, *Phys. Rev.* *60*, 109 (1945).

- (BG48) G. Bernardini, G. Cortini and A. Manfredini, *Phys. Rev.* **74**, 845 (1948); **74**, 1878 (1948).
- (BG49) G. Bernardini, G. Cortini and A. Manfredini, *Phys. Rev.* **75**, 1792 (1949).
- (BG50) G. Bernardini, E. T. Booth, L. Lederman and J. H. Tinlot, *Phys. Rev.* **80**, 924 (1950).
- (BG51.1) G. Bernardini, E. T. Booth, L. Lederman and J. H. Tinlot, *Phys. Rev.* **82**, 105 (1951).
- (BG51.2) G. Bernardini, E. T. Booth and L. Lederman, *Phys. Rev.* **83**, 1075 (1951).
- (BGC45) G. C. Baldwin and H. W. Koch, *Phys. Rev.* **67**, 1 (1945).
- (BGC48) G. C. Baldwin and G. S. Klaiber, *Phys. Rev.* **73**, 1156 (1948).
- (BhA48) A. Bohr, *Danske Videnskabernes Selskab* **24**, No. 19 (1948).
- (BH50.1) H. Bradner, F. M. Smith, W. H. Barkas and A. H. Bishop, *Phys. Rev.* **77**, 462 (1950).
- (BH50.2) H. Bradner, D. J. O'Connell and B. Rankin, *Phys. Rev.* **79**, 720 (1950).
- (BHA30) H. A. Bethe, *Annalen d. Physik* **5**, 325 (1930).
- (BHA32) H. A. Bethe, *Zeits. f. Physik* **76**, 293 (1932).
- (BHA34) H. A. Bethe and W. Heitler, *Proc. Roy. Soc. A146*, 83 (1934).
- (BHA46) H. A. Bethe, *Phys. Rev.* **70**, 821 (1946).
- (BHH49) H. H. Barschall and G. S. Taschek, *Phys. Rev.* **75**, 1819 (1949).
- (BHH35) H. J. Bhabha, *Proc. Roy. Soc. A152*, 559 (1935).
- (BHH36) H. J. Bhabha, *Proc. Roy. Soc. A154*, 195 (1936).
- (BHH37) H. J. Bhabha and W. Heitler, *Proc. Roy. Soc. A159*, 432 (1937).
- (BHH38) H. J. Bhabha, *Proc. Roy. Soc. A164*, 257 (1938).
- (DIJ42) H. J. Bhabha and S. K. Chakrabarty, *Proc. Ind. Acad. Sci.* **15A**, 464 (1942).
- (BHJ43) H. J. Bhabha and S. K. Chakrabarty, *Proc. Roy. Soc. A181*, 267 (1943).
- (BHJ48) H. J. Bhabha and S. K. Chakrabarty, *Phys. Rev.* **74**, 1352 (1948).
- (BHL48) H. L. Bradt and B. Peters, *Phys. Rev.* **74**, 1828 (1948).
- (BHL50.1) H. L. Bradt and B. Peters, *Phys. Rev.* **77**, 54 (1950).
- (BHL50.2) H. L. Bradt and M. F. Kaplon, *Phys. Rev.* **78**, 680 (1950).
- (BHL50.3) H. L. Bradt, M. F. Kaplon and B. Peters, *Helv. Phys. Acta* **23**, 24 (1950).
- (BHL50.4) H. L. Bradt and B. Peters, *Phys. Rev.* **80**, 943 (1950).
- (BHS47) H. S. Bridge, B. Rossi and R. W. Williams, *Phys. Rev.* **72**, 257 (1947).
- (BHS48.1) H. S. Bridge and W. E. Hazen, *Phys. Rev.* **74**, 579 (1948).
- (BHS48.2) H. S. Bridge, W. E. Hazen, B. Rossi and R. W. Williams, *Phys. Rev.* **74**, 1083 (1948).
- (BHS49) H. S. Bridge and B. Rossi, *Phys. Rev.* **75**, 810 (1949).
- (BHS51.1) H. S. Bridge, F. Harris and B. Rossi, *New York Meeting of Amer. Phys. Soc.* (February 1951).
- (BHS51.2) H. S. Bridge and M. Annis, *Phys. Rev.* **82**, 445 (1951).
- (BHW51) H. W. Bochner and H. S. Bridge, *New York Meeting of Amer. Phys. Soc.* (February 1951).
- (BI47) I. Broser, H. Kallmann, *Zeits. f. Naturforschung*, **2a**, 642 (1947).
- (BI48) I. Broser, L. Herforth, H. Kallmann and U. Martius, *Zeits. f. Naturforschung*, **3a**, 6 (1948).

- (BIB50) B. I. Bernstein, *Phys. Rev.* **80**, 995 (1950).
- (BJ49) J. Burfenig, E. Gardner and C. M. G. Lattes, *Phys. Rev.* **75**, 382 (1949).
- (BJC51) J. C. Barton, F. P. George and A. C. Jason, *Proc. Phys. Soc. A64*, 175 (1951).
- (BJJ39) J. J. Braddick and G. S. Hensby, *Nature* **144**, 1012 (1939).
- (BJK50) J. K. Bowker, *Phys. Rev.* **78**, 87 (1950).
- (BKW50) W. Blocker, R. W. Kenney and W. K. H. Panofsky, *Phys. Rev.* **79**, 419 (1950).
- (BM32) M. Blau and H. Wambacher, *Sitzber. Akad. Wiss. Wien. Abt. 2a*, **141**, 617 (1932).
- (BM37) M. Blau and H. Wambacher, *Nature* **140**, 585 (1937).
- (BM48) M. Blau and J. A. DeFelice, *Phys. Rev.* **74**, 1198 (1948).
- (BM49) M. Blau, *Phys. Rev.* **75**, 279 (1949).
- (BnA49) A. Bonetti and C. C. Dilworth, *Phil. Mag.* **40**, 585 (1949).
- (BPM33) P. M. S. Blackett and G. P. S. Oechialini, *Proc. Roy. Soc. A139*, 699 (1933).
- (BPM34) P. M. S. Blackett, *Proc. Roy. Soc. A146*, 281 (1934).
- (BPM37) P. M. S. Blackett and J. G. Wilson, *Proc. Roy. Soc. A160*, 304 (1937).
- (BPM38) P. M. S. Blackett, *Proc. Roy. Soc. A165*, 11 (1938).
- (BPR51) P. R. Barker, *Phys. Rev.* **81**, 291 (1951).
- (BrD47) D. Broadbent and L. Janossy, *Proc. Roy. Soc. A190*, 497 (1947).
- (BrH51) H. Bramson and W. W. Havens, *Phys. Rev.* **83**, 861 (1951).
- (BrS49) S. Borowitz and W. Kohn, *Phys. Rev.* **76**, 818 (1949).
- (BR50) R. Bjorklund, W. E. Crandall, B. J. Moyer and H. F. York, *Phys. Rev.* **77**, 213 (1950).
- (BRB39) R. B. Brode, *Rev. Mod. Phys.* **11**, 222 (1939).
- (BRB49) R. B. Brode, *Rev. Mod. Phys.* **21**, 37 (1949).
- (BRH49.1) R. H. Brown, U. Camerini, P. H. Fowler, W. Heitler, D. T. King and C. F. Powell, *Phil. Mag.* **40**, 862 (1949).
- (BRH49.2) R. H. Brown, U. Camerini, P. Fowler, H. Muirhead, C. F. Powell and D. Ritson, *Nature* **163**, 47 (1949); *Nature* **163**, 82 (1949).
- (BRW50) K. W. Birge, *Phys. Rev.* **80**, 490 (1950).
- (BS44.1) S. Belenky, *J. Phys. U.S.S.R.* **8**, 305 (1944).
- (BS44.2) S. Belenky, *J. Phys. U.S.S.R.* **8**, 347 (1944).
- (BSC48) S. C. Brown, *Nucleonics* **2**, No. 6, 10 (1948); **3**, No. 2, 50 (1948); **3**, No. 4, 46 (1948).
- (BtA50) A. Bratenhal, S. Fernbach, R. H. Hildebrand, C. E. Leith and B. J. Moyer, *Phys. Rev.* **77**, 597 (1950).
- (BW29) W. Bothe and W. Kohlörster, *Zeits. f. Phys.* **56**, 751 (1929).
- (BWH48) W. H. Barkas, E. Gardner and C. M. G. Lattes, *Phys. Rev.* **74**, 1558 (1948).
- (BWH49) W. H. Barkas, *Phys. Rev.* **75**, 1109 (1949).
- (BWH51) W. H. Barkas, F. M. Smith and E. Gardner, *Phys. Rev.* **82**, 102 (1951).
- (BWW49) W. W. Brown, A. S. McKay and E. D. Palmatier, *Phys. Rev.* **76**, 506 (1949).

- (BWW50) W. W. Brown and A. S. McKay, *Phys. Rev.* **77**, 342 (1950).  
 (CAG50) A. G. Carlson, J. E. Hooper and D. T. King, *Phil. Mag.* **41**, 701 (1950).  
 (CAH34) A. H. Compton, W. O. Woilan and K. D. Bennett, *Rev. Sc. Inst.* **5**, 415 (1934).  
 (CB50) B. Cork, L. Johnston and C. Richman, *Phys. Rev.* **79**, 71 (1950).  
 (CC51) C. Chedester, P. Isaacs, A. Sachs and J. Steinberger, *Phys. Rev.* **82**, 958 (1951).  
 (CCY48) C. Y. Chao, *Phys. Rev.* **74**, 492 (1948).  
 (CCY49) C. Y. Chao, *Phys. Rev.* **75**, 581 (1949).  
 (CDR38) D. R. Corson and R. B. Brode, *Phys. Rev.* **53**, 773 (1938).  
 (CDR48) D. R. Corson and R. R. Wilson, *Rev. Sc. Inst.* **19**, 207 (1948).  
 (CFL41) F. L. Code, *Phys. Rev.* **59**, 229 (1941).  
 (CFS51) F. S. Crawford, K. M. Crowe and M. L. Stevenson, *Phys. Rev.* **82**, 97 (1951).  
 (CG48) G. Cocconi, V. Cocconi-Tongiorgi and K. Greisen, *Phys. Rev.* **74**, 1867 (1948).  
 (CG49.1) G. Cocconi, *Phys. Rev.* **75**, 1074 (1949).  
 (CG49.2) G. Cocconi, *Phys. Rev.* **76**, 318 (1949).  
 (CG49.3) G. Cocconi, *Phys. Rev.* **70**, 984 (1949).  
 (CG50) G. Cocconi, V. Cocconi-Tongiorgi and N. Widgoff, *Phys. Rev.* **79**, 768 (1950).  
 (CGB48) G. B. Collins, *Phys. Rev.* **74**, 1543 (1948).  
 (CGF50.1) G. F. Chew and J. L. Steinberger, *Phys. Rev.* **78**, 497 (1950).  
 (CGF50.2) G. F. Chew, *Phys. Rev.* **79**, 219 (1950).  
 (CGF50.3) G. F. Chew and M. L. Goldberger, *Phys. Rev.* **77**, 470 (1950).  
 (CH48) H. Carmichael, *Phys. Rev.* **74**, 1667 (1948).  
 (CI33.1) I. Curie and F. Joliot, *Compt. Rend.* **196**, 1581 (1933).  
 (CI33.2) I. Curie and F. Joliot, *Compt. Rend.* **196**, 1885 (1933).  
 (CJF37) J. F. Carlson and J. R. Oppenheimer, *Phys. Rev.* **51**, 220 (1937).  
 (CKM50) K. M. Crowe and E. Hayward, *Phys. Rev.* **80**, 40 (1950).  
 (CLJ49) L. J. Cook, E. E. McMillan, J. M. Peterson and D. C. Sewell, *Phys. Rev.* **75**, 7 (1949).  
 (CmM51) M. Camac, D. R. Corson, R. M. Littauer, A. M. Shapiro, A. Silverman, R. R. Wilson and W. M. Woodward, *Phys. Rev.* **82**, 745 (1951).  
 (CM44) M. Conversi and O. Piccioni, *Nuovo Cim.* **2**, 40 (1944), **2**, 71 (1944).  
 (CM45) M. Conversi, E. Pancini and O. Piccioni, *Phys. Rev.* **68**, 232 (1945).  
 (CM47) M. Conversi, E. Pancini and O. Piccioni, *Phys. Rev.* **71**, 209 (1947).  
 (CM50) M. Conversi and H. K. Ticho, *Nuovo Cim.* **7**, 677 (1950).  
 (CMG36) M. G. E. Cosyns, *Nature* **138**, 284 (1936).  
 (CO50) O. Chamberlain, F. R. Mozley, J. Steinberger and C. Wiegand, *Phys. Rev.* **79**, 394 (1950).  
 (CO51) O. Chamberlain, F. Segre and C. Wiegand, *Phys. Rev.* **81**, 284 (1951).  
 (CR44) R. Chaminade, A. Freon and R. Maze, *Compt. Rend.* **218**, 402 (1944).  
 (CRF41.1) R. F. Christy and S. Kusaka, *Phys. Rev.* **59**, 405 (1941).  
 (CRF41.2) R. F. Christy and S. Kusaka, *Phys. Rev.* **59**, 414 (1941).

- (CRL49) R. L. Cool, E. C. Fowler, J. C. Street, W. B. Fowler and R. D. Sard, *Phys. Rev.* **75**, 1275 (1949).  
 (CRS50) R. S. Christian, *U.C.R.L. 1011* (1950).  
 (CT48) T. Coor, Princeton University, Thesis (1948).  
 (CU48) U. Camerini, H. Muirhead, C. F. Powell and D. M. Ritson, *Nature* **162**, 433 (1948).  
 (CU49) U. Camerini, T. Coor, J. H. Davies, P. H. Fowler, W. O. Lock, H. Muirhead and N. Tobin, *Phil. Mag.* **40**, 1073 (1949).  
 (CU50) U. Camerini, P. H. Fowler, W. O. Lock, H. Muirhead, *Phil. Mag.* **41**, 413 (1950).  
 (CV49) V. Cocconi-Tongiorgi, *Phys. Rev.* **75**, 1532 (1949).  
 (CWB50) W. B. Cheston and L. J. B. Goldfarb, *Phys. Rev.* **78**, 683 (1950).  
 (CWF50) W. F. Cartwright, C. Richman, M. N. Whitehead and A. A. Wilcox, *Phys. Rev.* **78**, 823 (1950).  
 (CWF51) W. F. Cartwright, *Phys. Rev.* **82**, 460 (1951).  
 (CWY49) W. Y. Chang, *Rev. Mod. Phys.* **21**, 166 (1949).  
 (DBC50) B. C. Diven and G. M. Almy, *Phys. Rev.* **80**, 407 (1950).  
 (DCC48) C. C. Dilworth, G. P. S. Occhialini and R. M. Payne, *Nature* **162**, 102 (1948).  
 (DCC50.1) C. C. Dilworth, G. P. S. Occhialini and Vermaesen, *Univ. Bruxelles, Bull. 13A du Centre Phys. Nucl.* (1950).  
 (DCC50.2) C. C. Dilworth, S. J. Goldsack, Y. Goldschmidt-Clermont and F. Levy, *Phil. Mag.* **41**, 1032 (1950).  
 (DGN40) N. N. Das Gupta and P. C. Bhattacharya, *Proc. Nat. Inst. Sci. India* **6**, 713 (1940).  
 (DGN46) N. N. Das Gupta and S. K. Ghosh, *Rev. Mod. Phys.* **18**, 225 (1946).  
 (DJ44) J. Daudin, *Annales d. Phys.* **19**, 110 (1944).  
 (DJJ50.1) J. DeJuren and N. Knable, *Phys. Rev.* **77**, 606 (1950).  
 (DJJ50.2) J. DeJuren, *Phys. Rev.* **80**, 27 (1950).  
 (DJJ51) J. DeJuren and B. J. Moyer, *Phys. Rev.* **81**, 919 (1951).  
 (DLA37) L. A. Delsasso, W. A. Fowler and C. C. Lauritsen, *Phys. Rev.* **51**, 391 (1937).  
 (DMJ46) J. W. M. DuMond and E. R. Cohen, *Rev. Mod. Phys.* **20**, 82 (1948).  
 (DMJ49.1) J. W. M. DuMond, D. A. Lind and B. B. Watson, *Phys. Rev.* **75**, 1226 (1949).  
 (DMJ49.2) J. W. M. DuMond and E. R. Cohen, *Rev. Mod. Phys.* **21**, 651 (1949).  
 (DN50) N. Dallaporta, M. Merlin and G. Puppi, *Nuovo Cim.* **7**, 99 (1950).  
 (DWE36) W. E. Danforth and W. E. Ramsey, *Phys. Rev.* **49**, 854 (1936).  
 (EA37) A. Ehmert, *Zeits. f. Phys.* **106**, 751 (1937).  
 (EL48) L. Eyges, *Phys. Rev.* **74**, 1534 (1948).  
 (EL49) L. Eyges, *Phys. Rev.* **76**, 264 (1949).  
 (EP38) P. Ehrenfest, *Compt. Rend.* **207**, 573 (1938).  
 (ERD48) R. D. Evans and R. O. Evans, *Rev. Mod. Phys.* **20**, 305 (1948).  
 (FB42) B. Ferretti, *Ricerca Scient.* **13**, 532 (1942).  
 (FB49) B. Ferretti, *Nuovo Cim.* **6**, 379 (1949).  
 (FDT50) D. T. Feld, I. L. Lebow and L. S. Osborne, *Phys. Rev.* **77**, 131 (1950).

- (FC48) C. Franzinetti and R. M. Payne, *Nature* 161, 735 (1948).  
(FDH36) D. H. Follet and J. D. Crawshaw, *Proc. Roy. Soc. A155*, 546 (1936).  
(FDK38) D. K. Froman, J. C. Stearns, *Rev. Mod. Phys.* 10, 133 (1938).  
(FE39) E. Fermi, *Phys. Rev.* 56, 1242 (1939); 57, 485 (1940).  
(FF47.1) E. Fermi, E. Teller, and V. Weisskopf, *Phys. Rev.* 71, 314 (1947).  
(FF47.2) E. Fermi and E. Teller, *Phys. Rev.* 72, 399 (1947).  
(FE50) E. Fermi, *Progr. Theor. Phys.* 5, 570 (1950).  
(FE51) E. Fermi, *Phys. Rev.* 81, 683 (1951).  
(FF49) E. F. Fahy and M. Schein, *Phys. Rev.* 75, 207 (1949).  
(FFL49) F. L. Friedman, Massachusetts Institute of Technology, Thesis (1949).  
(FH49) H. Feschbach and V. F. Weisskopf, *Phys. Rev.* 76, 1550 (1949).  
(FHH50.1) H. H. Forster, *Phys. Rev.* 77, 733 (1950).  
(FHH50.2) H. H. Forster, *Phys. Rev.* 78, 247 (1950).  
(FHL51) H. L. Friedman and J. Rainwater, *Phys. Rev.* 81, 644 (1951).  
(FP48.1) P. Freier, E. J. Lofgren, E. P. Ney, F. Oppenheimer, H. L. Bradt and B. Peters, *Phys. Rev.* 74, 213 (1948).  
(FP48.2) P. Freier, E. J. Lofgren, E. P. Ney and F. Oppenheimer, *Phys. Rev.* 74, 1818 (1948).  
(FP50) P. Freier and E. P. Ney, *Phys. Rev.* 77, 337 (1950).  
(FPH50) P. H. Fowler, *Phil. Mag.* 41, 169 (1950).  
(FR50) R. Fox, C. Leith, L. Wouters and K. R. MacKenzie, *Phys. Rev.* 80, 23 (1950).  
(FRH47) R. H. Frost, University of California, Thesis (1947).  
(FS49) S. Fernbach, R. Serber and T. D. Taylor, *Phys. Rev.* 75, 1352 (1949).  
(FW50) W. Franzen, R. W. Peelle and R. Sherr, *Phys. Rev.* 79, 742 (1950).  
(FWB46) W. B. Fretter, *Phys. Rev.* 70, 625 (1946).  
(FWB48) W. B. Fretter, *Phys. Rev.* 73, 41 (1948).  
(FWB49.1) W. B. Fretter, *Phys. Rev.* 76, 511 (1949).  
(FWB49.2) W. B. Fretter, *Amer. J. Phys.* 17, 148 (1949).  
(FWB51.1) W. B. Fretter, New York Meeting of Amer. Phys. Soc., February 1951.  
(FWB51.2) W. B. Fretter, *Phys. Rev.* 83, 1053 (1951).  
(FWH37) W. H. Furry, *Phys. Rev.* 52, 569 (1937).  
(GBP50) B. P. Gregory, B. Rossi and J. H. Tinlot, *Phys. Rev.* 77, 299 (1950).  
(CBP51) B. P. Gregory and J. H. Tinlot, *Phys. Rev.* 81, 667 (1951); 81, 675 (1951).  
(GCY48) Y. Goldschmidt-Clermont, D. T. King, H. Muirhead and D. M. Ritson, *Proc. Phys. Soc. A61*, 138 (1948).  
(GCY50) Y. Goldschmidt-Clermont, *Nuovo Cim.* 7, 331 (1950).  
(GE48) E. Gardner and C. M. G. Lattes, *Science* 107, 270 (1948).  
(GE50) E. Gardner, W. H. Barkas, F. M. Smith and H. Bradner, *Science* 111, 191 (1950).  
(GEP47) E. P. George and A. C. Jason, *Nature* 160, 327 (1947).  
(GEP48) E. P. George and A. C. Jason, *Nature* 161, 248 (1948).  
(GEP49.1) E. P. George and P. T. Trent, *Nature* 164, 838 (1949).  
(GEP49.2) E. P. George and A. C. Jason, *Proc. Phys. Soc. A62*, 243 (1949).  
(GEP50.1) E. P. George and J. Evans, *Proc. Phys. Soc. A63*, 1248 (1950).

- (GEP50.2) E. P. George and A. C. Jason, *Proc. Phys. Soc. A63*, 1081 (1950).  
(GEP51) E. P. George and J. Evans, *Proc. Phys. Soc. A64*, 193 (1951).  
(CC46) G. Grotzinger and G. W. McClure, *Phys. Rev.* 74, 341 (1948).  
(GG50) G. Grotzinger, M. J. Berger and F. L. Ribe, *Phys. Rev.* 77, 584 (1950).  
(GG51) G. Grotzinger, M. J. Berger and G. W. McClure, *Phys. Rev.* 81, 969 (1951).  
(GH28) H. Geiger and W. Mueller, *Naturwiss.* 16, 617 (1928).  
(GK51) K. Gottstein, M. G. K. Menon, J. H. Mulvey, C. O'Ceallaigh, and O. Rochet, *Phil. Mag.* 42, 708 (1951).  
(GKI42) K. I. Greisen and N. Nereson, *Phys. Rev.* 62, 316 (1942).  
(GKI43) K. I. Greisen, *Phys. Rev.* 63, 323 (1943).  
(GKI48) K. I. Greisen, *Phys. Rev.* 73, 521 (1948).  
(GKI49) K. I. Greisen, *Phys. Rev.* 75, 1071 (1949).  
(GLH44) L. H. Gray, *Proc. Camb. Phil. Soc.* 40, 72 (1944).  
(GM48) M. L. Goldhaber and E. Teller, *Phys. Rev.* 74, 1046 (1948).  
(GML48) M. L. Goldhaber, *Phys. Rev.* 74, 1269 (1948).  
(GRW25) R. W. Gurney, *Proc. Roy. Soc. A107*, 332 (1925).  
(GSA40.1) S. A. Goudsmit and J. L. Saunderson, *Phys. Rev.* 57, 24 (1940).  
(GSA40.2) S. A. Goudsmit and J. L. Saunderson, *Phys. Rev.* 58, 36 (1940).  
(HaW50) W. Hartsough, M. Hill and W. M. Powell, *Phys. Rev.* 79, 219 (1950).  
(HAC47) A. C. Helmholtz, E. M. McMillan and D. C. Sewell, *Phys. Rev.* 72, 1003 (1947).  
(HAO49) A. O. Hanson, R. B. Duffield, J. D. Knight, B. C. Diven and H. Palevsky, *Phys. Rev.* 76, 578 (1949).  
(HCC49) C. C. Hanna and B. Pontecorvo, *Phys. Rev.* 75, 983 (1949).  
(HCS37) Hu Chien Shan, *Proc. Roy. Soc. A158*, 581 (1937).  
(IIEP48) E. P. Hincks and B. Pontecorvo, *Phys. Rev.* 73, 257 (1948).  
(HEP50) E. P. Hincks and B. Pontecorvo, *Phys. Rev.* 77, 102 (1950).  
(HFB51) F. B. Harrison, J. W. Keuffel and G. T. Reynolds, *Phys. Rev.* 83, 680 (1951).  
(HFL48) F. L. Hereford, *Phys. Rev.* 74, 574 (1948).  
(HG28) G. Hoffmann and F. Lindholm, *Carl. Beitr. Geoph.* 40, 12 (1928).  
(HG32) G. Hoffmann, *Phys. Zeits.* 33, 633 (1932).  
(HJ49) J. Hadley, E. Kelly, C. Leith, E. Segre, L. Wiegand and H. York, *Phys. Rev.* 75, 351 (1949).  
(HJB49.1) J. B. Harding, *Nature* 163, 440 (1949).  
(HJB49.2) J. B. Harding, *Phil. Mag.* 40, 530 (1949).  
(HJB49.3) J. B. Harding, S. Lattimore and D. H. Perkins, *Proc. Roy. Soc. A196*, 325 (1949).  
(HJB49.4) J. B. Harding, S. Lattimore, T. T. Li and D. H. Perkins, *Nature* 169, 319 (1949).  
(HJB50) J. B. Harding, *Phil. Mag.* 41, 405 (1950).  
(HO40) O. Halpern and H. Hall, *Phys. Rev.* 57, 459 (1940).  
(HO48) O. Halpern and H. Hall, *Phys. Rev.* 73, 477 (1948).  
(HW40) W. Horning and L. Baumhoff, *Phys. Rev.* 75, 370 (1940).

- (HR49) R. Hofstadter, Phys. Rev. 75, 796 (1949).  
 (HR50) R. Hofstadter, S. H. Liebson and J. O. Elliot, Phys. Rev. 78, 81 (1950).  
 (HRH50) R. H. Hildebrand and C. E. Leith, Phys. Rev. 80, 842 (1950).  
 (HsW36) W. Heisenberg, Zeits. f. Phys. 101, 533 (1936).  
 (HsW49) W. Heisenberg, Nature 164, 65 (1949).  
 (HtW34) W. Heitler and L. Nordheim, J. Phys. Radium 5, 449 (1934).  
 (HtW49) W. Heitler and L. Janossy, Proc. Phys. Soc. A62, 374 (1949).  
 (HTF50) Hoang-Tchang-Fong, University of Paris, Thesis (1950).  
 (HWE43.1) W. E. Hazen, Phys. Rev. 63, 107 (1943).  
 (HWE43.2) W. E. Hazen, Phys. Rev. 63, 213 (1943).  
 (HWE43.3) W. E. Hazen, Phys. Rev. 64, 7 (1943).  
 (HWE43.4) W. E. Hazen, Phys. Rev. 64, 257 (1943).  
 (HWE44) W. E. Hazen, Phys. Rev. 65, 67 (1944).  
 (HWE45) W. E. Hazen, Phys. Rev. 67, 269 (1945).  
 (JHE50) H. E. Johns, L. Katz, R. A. Douglas and R. N. H. Haslam, Phys. Rev. 80, 1062 (1950).  
 (JL40.1) L. Janossy, B. Rossi, Proc. Roy. Soc. A175, 88 (1940).  
 (JL40.2) L. Janossy and P. Ingleby, Nature 145, 511 (1940).  
 (JL41.1) L. Janossy, Nature 147, 56 (1941).  
 (JL41.2) L. Janossy, C. B. McCusker and G. D. Rochester, Nature 148, 660 (1941).  
 (JL42) L. Janossy, Proc. Roy. Soc. A179, 361 (1942).  
 (JL43.1) L. Janossy and G. D. Rochester, Proc. Roy. Soc. A182, 180 (1943).  
 (JL43.2) L. Janossy, Phys. Rev. 64, 345 (1943).  
 (JL44.1) L. Janossy and G. D. Rochester, Proc. Roy. Soc. A183, 181 (1944).  
 (JL44.2) L. Janossy and G. D. Rochester, Proc. Roy. Soc. A183, 186 (1944).  
 (JL50) L. Janossy, Proc. Phys. Soc. A63, 241 (1950).  
 (JSB50) S. B. Jones and R. S. White, Phys. Rev. 78, 12 (1950).  
 (JTH42) T. H. Johnson and R. P. Shutt, Phys. Rev. 61, 380 (1942).  
 (JWP50) W. P. Jesse, H. Forst and J. Sadauskis, Phys. Rev. 77, 782 (1950).  
 (KB33) B. Kunze, Zeits. f. Phys. 83, 1 (1933).  
 (KEL50) E. L. Kolly, C. Leith, E. Segre and C. Wiegand, Phys. Rev. 79, 96 (1950).  
 (KH50.1) H. Kallmann, Phys. Rev. 78, 621 (1950).  
 (KH50.2) H. Kallmann and M. Furst, Phys. Rev. 79, 857 (1950).  
 (KHW50) H. W. Koch, J. McElhinney and E. L. Gasteiger, Phys. Rev. 77, 329 (1950).  
 (KJM39) J. M. Kellogg, I. I. Rabi, N. F. Ramsey and J. R. Zacharias, Phys. Rev. 56, 728 (1939).  
 (KIH38) H. Kulenkampff, Verh. d. Deutsch. Phys. Ges. (3) 19, 92 (1938).  
 (KMF40) M. F. Kaplan, B. Peters and H. L. Bradt, Phys. Rev. 76, 1735 (1949).  
 (KO29) O. Klein and Y. Nishina, Zeits. f. Physik, 52, 853 (1929).  
 (KS10) S. Kinoshita, Proc. Roy. Soc. A83, 432 (1910).  
 (KWL50) W. L. Kraushaar, J. E. Thomas and V. P. Henri, Phys. Rev. 78, 486 (1950).  
 (LA50) A. Lovati, A. Mura, G. Salvini and G. Tagliaferri, Phys. Rev. 77, 284 (1950).

- (LCK50) K. J. LeCouteur, Proc. Phys. Soc. A63, 259 (1950).  
 (LCM47.1) C. M. G. Lattes, H. Muirhead, G. P. S. Occhialini and C. F. Powell, Nature 159, 694 (1947).  
 (LCM47.2) C. M. G. Lattes, G. P. S. Occhialini and C. F. Powell, Nature 160, 453 (1947).  
 (LCM47.3) C. M. G. Lattes, P. H. Fowler and P. Cuen, Proc. Phys. Soc. 59, 883 (1947).  
 (LCM47.4) C. M. G. Lattes, G. P. S. Occhialini and C. F. Powell, Nature 160, 486 (1947).  
 (LCM48) C. M. G. Lattes, G. P. S. Occhialini and C. F. Powell, Proc. Phys. Soc. 61, 173 (1948).  
 (LHW48) H. W. Lewis, J. R. Oppenheimer and S. A. Wouthuysen, Phys. Rev. 73, 127 (1948).  
 (LJJ49) J. J. Lord and M. Schein, Phys. Rev. 75, 1956 (1949).  
 (LJJ50) J. J. Lord, J. Fainberg and M. Schein, Phys. Rev. 80, 970 (1950).  
 (LJJ51) J. J. Lord, Phys. Rev. 81, 901 (1951).  
 (LJL49) J. L. Lawson, Phys. Rev. 75, 433 (1949).  
 (LJS50) J. S. Levinger and H. Bethe, Phys. Rev. 78, 115 (1950).  
 (LLD38) L. D. Landau and G. Rumer, Proc. Roy. Soc. A166, 213 (1938).  
 (LLD44) L. D. Landau, J. Phys. U.S.S.R. 8, 201 (1944).  
 (LLM51) L. M. Lederman, E. T. Booth, H. Byfield and J. Kessler, Phys. Rev. 83, 685 (1951).  
 (LMS37) M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 245 (1937).  
 (LnA50) A. Lundby, Phys. Rev. 80, 477 (1950).  
 (LRB49) R. B. Leighton, C. D. Anderson and A. J. Seriff, Phys. Rev. 76, 1432 (1949).  
 (LRB51) R. B. Leighton, S. D. Wanlass and W. L. Alford, Phys. Rev. 83, 843 (1951).  
 (LRL44) L. Leprince Ringuet and M. Lheritier, Comp. Rend. 219, 618 (1944).  
 (LRL49.1) L. Leprince Ringuet, Rev. Mod. Phys. 21, 42 (1949).  
 (LRL49.2) L. Leprince Ringuet, F. Bousser, Hoang-Tchang-Fong, L. Jauneau and D. Morellet, Phys. Rev. 76, 1273 (1949).  
 (LRI49.3) L. Leprince Ringuet, Hoang-Tchang-Fong, F. Bousser, L. Jauneau and D. Morellet, Comp. Rend. 229, 163 (1949).  
 (MC32) C. Møller, Annalen d. Phys. 14, 531 (1932).  
 (MCG47) C. G. Montgomery and D. D. Montgomery, Phys. Rev. 72, 131 (1947).  
 (MCG49.1) C. G. Montgomery and A. R. Tobey, Phys. Rev. 76, 1478 (1949).  
 (MCG49.2) C. G. Montgomery and D. D. Montgomery, Phys. Rev. 76, 1482 (1949).  
 (MEA50) E. A. Martinelli and W. K. H. Panofsky, Phys. Rev. 77, 465 (1950).  
 (MEJ49) J. McElhinney, A. O. Hanson, R. A. Becker, R. B. Duffield and B. C. Diven, Phys. Rev. 75, 542 (1949).  
 (MEJ50) J. McElhinney and W. E. Ogle, Phys. Rev. 78, 63 (1950).  
 (MFJ51) F. J. Milford and L. L. Foldy, Phys. Rev. 81, 13 (1951).  
 (MG47) G. Moliere, Zeits. f. Naturforschung 2a, 133 (1947).  
 (MG48) G. Moliere, Zeits. f. Naturforschung, 3a, 78 (1948).  
 (MG51) G. Moliere, in print.

- (MGW49) G. W. McClure and G. Groetzinger, *Phys. Rev.* **75**, 340 (1949).  
 (MH50.1) H. Messel and D. M. Ritson, *Proc. Phys. Soc. A63*, 1359 (1950).  
 (MH50.2) H. Messel and D. M. Ritson, *Phil. Mag.* **41**, 1190 (1950).  
 (MH51.1) H. Messel, *Phys. Rev.* **83**, 21 (1951).  
 (MH51.2) H. Messel, *Phys. Rev.* **83**, 26 (1951).  
 (MHJ39) H. J. Massey and H. C. Corben, *Proc. Camb. Phil. Soc.* **35**, 463 (1939).  
 (ML27) L. Myssowsky and P. Tschijow, *Zeits. f. Phys.* **44**, 408 (1927).  
 (MME49.1) E. M. McMillan and J. M. Peterson, *Science* **109**, 438 (1949).  
 (MME49.2) E. M. McMillan, J. M. Peterson and R. S. White, *Science* **110**, 579 (1949).  
 (MMW47) W. G. McMillan and E. Teller, *Phys. Rev.* **72**, 1 (1947).  
 (MNF29) N. F. Mott, *Proc. Roy. Soc. A124*, 425 (1929).  
 (MNG50) N. G. K. Menon, H. Muirhead and O. RoCHAT, *Phil. Mag.* **41**, 583 (1950).  
 (MO50) O. Martinson, P. Isaacs, H. Brown and J. W. Ruderman, *Phys. Rev.* **79**, 178 (1950).  
 (MRA36) R. A. Millikan and H. V. Neher, *Phys. Rev.* **50**, 15 (1936).  
 (MRE47) R. E. Marshak and H. A. Bethe, *Phys. Rev.* **72**, 506 (1947).  
 (MRE49) R. E. Marshak and A. S. Wightman, *Phys. Rev.* **76**, 114 (1949).  
 (MRF50) R. F. Mozley, *Phys. Rev.* **80**, 493 (1950).  
 (MTC50) T. C. Merkle, E. L. Goldwasser and R. B. Brode, *Phys. Rev.* **79**, 926 (1950).  
 (NA40) A. Nordsieck, W. E. Lamb and G. E. Uhlenbeck, *Physica* **7**, 344 (1940).  
 (NDB46) D. B. Nicodemus, Stanford University, Thesis (1946).  
 (NHV40) H. V. Neher and H. G. Stever, *Phys. Rev.* **58**, 766 (1940).  
 (NNG43) N. G. Nereson and B. Rossi, *Phys. Rev.* **64**, 199 (1943).  
 (NS46) S. Nassar and W. E. Hazen, *Phys. Rev.* **69**, 298 (1946).  
 (NSH35) S. H. Neddermeyer and C. D. Anderson, *Phys. Rev.* **48**, 486 (1935).  
 (NSH37) S. H. Neddermeyer and C. D. Anderson, *Phys. Rev.* **51**, 884 (1937).  
 (NSH39) S. H. Neddermeyer and C. D. Anderson, *Rev. Mod. Phys.* **11**, 191 (1939).  
 (NWM40) W. M. Nielson, C. M. Ryerson, I. W. Nordheim and K. Z. Morgan, *Phys. Rev.* **57**, 158 (1940).  
 (NWM41) W. M. Nielson, C. M. Ryerson and K. Z. Morgan, *Phys. Rev.* **59**, 547 (1941).  
 (OGP47.1) G. P. S. Occhialini and C. F. Powell, *Nature* **159**, 93 (1947).  
 (OGP47.2) G. P. S. Occhialini and C. F. Powell, *Nature* **159**, 186 (1947).  
 (OGP49) G. P. S. Occhialini, *Nuovo Cim.* **6** (supplement), 413 (1949).  
 (OJR40) J. R. Oppenheimer, H. Snyder and R. Serber, *Phys. Rev.* **57**, 75 (1940).  
 (OJR47) J. R. Oppenheimer, *New York Meeting of Amer. Phys. Soc.*, 1947.  
 (OLS50) L. S. Osborne, Massachusetts Institute of Technology, Thesis (1950).  
 (OWE50.1) W. E. Ogle, R. E. England, *Phys. Rev.* **78**, 63 (1950).  
 (OWE50.2) W. E. Ogle, L. J. Brown, A. N. Carson, *Phys. Rev.* **78**, 63 (1950).  
 (PC49) C. Peyrou, B. d'Espagnat and L. Leprince Ringuet, *Comp. Rend.* **228**, 1777 (1949).  
 (PC50) C. Peyrou and A. Lagarrigue, *J. Phys. Radium* **11**, 666 (1950).  
 (PCE51) C. E. Porter and H. Primakoff, *Phys. Rev.* **83**, 849 (1951).  
 (PCF49) C. F. Powell, *Colston Papers, Butterworth Publ.*, p. 83 (London, 1949).

- (PDH47.1) D. H. Perkins, *Nature* **159**, 120 (1947).  
 (PDH47.2) D. H. Perkins, *Nature* **160**, 707 (1947).  
 (PDH49) D. H. Perkins, *Nature* **163**, 682 (1949).  
 (PDH50) D. H. Perkins, *Phil. Mag.* **41**, 138 (1950).  
 (PE49) E. Pickup and L. Voyvodic, *Phys. Rev.* **76**, 1534 (1949).  
 (PE50) E. Pickup and L. Voyvodic, *Phys. Rev.* **80**, 89 (1950).  
 (PH50) H. Palevsky and A. O. Hanson, *Phys. Rev.* **79**, 242 (1950).  
 (PMA40) M. A. Pomerantz, *Phys. Rev.* **57**, 3 (1940).  
 (PN50) N. Page, *Proc. Phys. Soc. A63*, 250 (1950).  
 (PO48.1) O. Piccioni, *Phys. Rev.* **73**, 411 (1948).  
 (PO48.2) O. Piccioni, *Phys. Rev.* **74**, 1754 (1948).  
 (PO50) O. Piccioni, *Phys. Rev.* **77**, 1 (1950).  
 (PRF50) R. F. Post and M. S. Shiran, *Phys. Rev.* **78**, 80 (1950).  
 (PV50) V. Peterson, *U.C.R.L.* **713** (1950).  
 (PW41) W. Pauli, *Rev. Mod. Phys.* **13**, 203 (1941).  
 (PWK50.1) W. K. H. Panofsky, L. Aamodt and H. F. York, *Phys. Rev.* **78**, 825 (1950).  
 (PWK50.2) W. K. H. Panofsky and F. L. Fillmore, *Phys. Rev.* **79**, 57 (1950).  
 (PWK50.3) W. K. H. Panofsky, L. Aamodt, J. Hadley and R. Phillips, *Phys. Rev.* **80**, 94 (1950).  
 (PWK51) W. K. H. Panofsky, R. L. Aamodt and J. Hadley, *Phys. Rev.* **81**, 565 (1951).  
 (PWM41) W. M. Powell, *Phys. Rev.* **60**, 413 (1941).  
 (PWM46) W. M. Powell, *Phys. Rev.* **69**, 385 (1946).  
 (PWM51) W. M. Powell, W. Hartsough and M. Hill, *Phys. Rev.* **81**, 213 (1951).  
 (RB30) B. Rossi, *Nature* **125**, 636 (1930).  
 (RB31) B. Rossi, *Nature* **128**, 300 (1931).  
 (RB32.1) B. Rossi, *Naturwiss.* **20**, 65 (1932).  
 (RB32.2) B. Rossi, *Phys. Zeits.* **33**, 304 (1932).  
 (RB32.3) B. Rossi, *Nature* **130**, 699 (1932).  
 (RB33.1) B. Rossi, *Zeits. f. Phys.* **82**, 151 (1933).  
 (RB33.2) B. Rossi, *Nature* **132**, 173 (1933).  
 (RB33.3) B. Rossi, *Rend. Lincei* **17**, 1073 (1933).  
 (RB34) B. Rossi, *Int. Conf. Phys.* **1**, 233 (London, 1934).  
 (RD40.1) D. Rossi, N. Hilberry and J. B. Hoag, *Phys. Rev.* **57**, 461 (1940).  
 (RB40.2) B. Rossi, *Phys. Rev.* **57**, 660 (1940).  
 (RB40.3) B. Rossi and V. H. Regener, *Phys. Rev.* **58**, 837 (1940).  
 (RB41.1) B. Rossi and K. Greisen, *Rev. Mod. Phys.* **13**, 240 (1941).  
 (RB41.2) B. Rossi and D. B. Hall, *Phys. Rev.* **59**, 223 (1941).  
 (RB42.1) B. Rossi, K. Groicon, J. C. Stearns, D. K. Froman and P. Koontz, *Phys. Rev.* **61**, 675 (1942).  
 (RB42.2) B. Rossi and S. J. Klapman, *Phys. Rev.* **61**, 414 (1942).  
 (RD42.3) B. Rossi and N. Nereson, *Phys. Rev.* **62**, 417 (1942).  
 (RB48) B. Rossi, *Rev. Mod. Phys.* **20**, 537 (1948).  
 (RB49) B. Rossi, *Colston Papers, Butterworth Publ.*, p. 45 (London, 1949).



- (RC50) C. Richman and H. A. Wilcox, Phys. Rev. 78, 496 (1950).  
 (REH49) E. H. Rogers and H. H. Staub, Phys. Rev. 76, 980 (1949).  
 (RF41) F. Racetti, Phys. Rev. 59, 706 (1941); 60, 198 (1941).  
 (RG36) G. Racah, Nuovo Cim. 4, 66 (1936).  
 (RGD46) G. D. Rochester, Proc. Roy. Soc. A187, 464 (1946).  
 (RGD47.1) G. D. Rochester and C. C. Butler, Nature 160, 855 (1947).  
 (RGD47.2) G. D. Rochester, C. C. Butler and S. K. Runcorn, Nature 159, 227 (1947).  
 (RGD48) G. D. Rochester and C. C. Butler, Proc. Phys. Soc. 61, 535 (1948).  
 (RGT50) G. T. Reynolds, F. B. Harrison and G. Salvini, Phys. Rev. 78, 488 (1950).  
 (RJA48) J. A. Richards and L. W. Nordheim, Phys. Rev. 74, 1106 (1948).  
 (RJG49) J. G. Retallack and R. B. Brode, Phys. Rev. 75, 1716 (1949).  
 (RJM51) J. M. Robson, Phys. Rev. 83, 349 (1951).  
 (RJR48) J. R. Richardson, Phys. Rev. 74, 1720 (1948).  
 (RM11) M. Reiganum, Phys. Zeits. 12, 1076 (1911).  
 (RRH50) R. H. Rediker, Massachusetts Institute of Technology, Thesis (1950).  
 (RVH40) V. H. Regener, Ricerca Scient. 11, 66 (1940).  
 (SA51) A. Silverman and M. Stearns, Phys. Rev. 83, 853 (1951).  
 (SAH50) A. H. Snell, F. Pleasonton and R. V. McCord, Phys. Rev. 78, 310 (1950).  
 (SAJ50) A. J. Seriff, R. B. Leighton, C. Hsiao, E. W. Cowan and C. D. Anderson, Phys. Rev. 78, 290 (1950).  
 (SbM51) M. Schönberg, Nuovo Cim. 8, 159 (1951).  
 (SbR38) R. Serber, Phys. Rev. 54, 317 (1938).  
 (SbR47.1) R. Serber, Phys. Rev. 72, 1008 (1947).  
 (SbR47.2) R. Serber, Phys. Rev. 72, 1114 (1947).  
 (SB50) B. Smaller, J. May and M. Freedman, Phys. Rev. 79, 940 (1950).  
 (ScK39) K. Schmieder, Annalen d. Phys. 35, 445 (1939).  
 (SFM50.1) F. M. Smith, E. Gardner and H. Bradner, Phys. Rev. 77, 562 (1950).  
 (SFM50.2) F. M. Smith, W. H. Barkas, A. S. Bishop, H. Bradner and E. Gardner, Phys. Rev. 78, 86 (1950).  
 (SFM51) F. M. Smith, Phys. Rev. 81, 897 (1951).  
 (SgR51) R. Sagane, W. L. Gardner and H. W. Hubbard, Phys. Rev. 82, 557 (1951).  
 (SG39) G. Stetter and H. Wambacher, Phys. Zeits. 40, 702 (1939).  
 (SG43) G. Stetter, Zeits. f. Phys. 120, 639 (1943).  
 (SG44) G. Stetter and H. Wambacher, Sitzber. Akad. Wiss. Wien. Abt. 2a, 152, 1 (1944).  
 (SGR40) R. L. Sen Gupta, Nature 146, 65 (1940).  
 (SH31) H. Schindler, Zeits. f. Phys. 72, 625 (1931).  
 (SHG42) H. G. Stever, Phys. Rev. 61, 38 (1942).  
 (SHS38) H. S. Snyder, Phys. Rev. 53, 960 (1938).  
 (SHS49.1) H. S. Snyder and W. T. Scott, Phys. Rev. 76, 220 (1949).  
 (SHS49.2) H. S. Snyder, Phys. Rev. 76, 1563 (1949).  
 (SJ50.1) J. Steinberger and A. S. Bishop, Phys. Rev. 73, 493 (1950), 76, 494 (1950).  
 (SJ50.2) J. Steinberger, W. K. H. Panofsky and J. S. Steller, Phys. Rev. 78, 802 (1950).

- (SJC37) J. C. Street and E. C. Stevenson, Phys. Rev. 52, 1003 (1937).  
 (SJC39) J. C. Street, J. Frankl. Inst. 227, 765 (1939).  
 (SJH47) J. H. Smith, Phys. Rev. 71, 32 (1947).  
 (SJS50) J. S. Steller and W. K. H. Panofsky, Los Angeles Meeting of Amer. Phys. Soc. (December 1950).  
 (SK50) K. Bitte, Phys. Rev. 76, 714 (1950).  
 (SKR48) K. R. Symon, Harvard University, Thesis (1948).  
 (SL42) L. Seren, Phys. Rev. 62, 204 (1942).  
 (SLI46) L. I. Schiff, Phys. Rev. 70, 87 (1946).  
 (SLI49.1) L. I. Schiff, Phys. Rev. 76, 303 (1949).  
 (SLI49.2) L. I. Schiff and D. L. Weisman, Phys. Rev. 76, 1266 (1949).  
 (SLI49.3) L. I. Schiff, Phys. Rev. 76, 89 (1949).  
 (SLL46) L. L. Skolil, Phys. Rev. 70, 619 (1946).  
 (SM39) M. Schein and P. S. Gill, Rev. Mod. Phys. 11, 267 (1939).  
 (SMG50) M. G. Schorr and F. L. Torney, Phys. Rev. 80, 474 (1950).  
 (SMS45) M. S. Sinha, Phys. Rev. 68, 153 (1945).  
 (SO45) O. Sala, G. Wataghin, Phys. Rev. 67, 55 (1945).  
 (SpA40) A. H. Spees and C. T. Zahn, Phys. Rev. 58, 861 (1940).  
 (SR45) R. Sherr, Phys. Rev. 68, 240 (1945).  
 (SRB47) R. B. Sutton, T. Hall, E. E. Anderson, H. S. Bridge, J. W. DeWire, L. S. Lavatelli, E. A. Long, T. Snyder and R. W. Williams, Phys. Rev. 72, 1147 (1947).  
 (SRD48.1) R. D. Sard and E. J. Althaus, Phys. Rev. 74, 1364 (1948).  
 (SRD48.2) R. D. Sard, W. B. Ittner, A. M. Conforto and M. F. Crouch, Phys. Rev. 74, 97 (1948).  
 (SRD49) R. D. Sard, A. M. Conforto and M. F. Crouch, Phys. Rev. 76, 1134 (1949).  
 (SRD51) R. D. Sard, M. F. Crouch, D. R. Jones, A. M. Conforto and B. F. Stearns, Nuovo Cim. 8, 326 (1951).  
 (SRP42.1) R. P. Shutt, Phys. Rev. 67, 6 (1942).  
 (SRP42.2) R. P. Shutt, S. De Benedetti and T. H. Johnson, Phys. Rev. 62, 552 (1942).  
 (SRP46) R. P. Shutt, Phys. Rev. 69, 261 (1946).  
 (SS46) S. Sakata and T. Inoue, Progr. Theor. Phys. 1, 143 (1946).  
 (SSO49) S. O. C. Sørensen, Phil. Mag. 40, 947 (1949).  
 (StM49) M. Stearns, Phys. Rev. 76, 836 (1949).  
 (ST47) T. Sigurgeirsson and A. Yamakawa, Phys. Rev. 71, 319 (1947).  
 (SwJ40) J. Schwinger, Phys. Rev. 58, 1004 (1940).  
 (SW47) W. Seator, Phys. Rev. 72, 207 (1947).  
 (SWF38) W. F. G. Swann, J. Frankl. Inst. 226, 598 (1938).  
 (SWT42) W. T. Scott and G. E. Uhlenbeck, Phys. Rev. 62, 497 (1942).  
 (SWT49) W. T. Scott, Phys. Rev. 76, 212 (1949).  
 (SWT50.1) W. T. Scott and H. S. Snyder, Phys. Rev. 78, 223 (1950).  
 (SWT50.2) W. T. Scott, Phys. Rev. 80, 611 (1950).  
 (TA37) A. Trost, Zeits. f. Phys. 105, 399 (1937).  
 (THK48.1) H. K. Ticho and M. Schein, Phys. Rev. 73, 81 (1948).

- (THK48.2) H. K. Ticho, Phys. Rev. 74, 1337 (1948).  
 (TIE39) I. E. Tamm and S. Belenky, J. Phys. U.S.S.R. 1, 177 (1939).  
 (TJ34) J. Tinbaud, Phys. Rev. 45, 781 (1934).  
 (TJH48) J. H. Tinlot, Phys. Rev. 73, 1476 (1948); 74, 1197 (1948).  
 (TJH49.1) J. H. Tinlot and B. P. Gregory, Phys. Rev. 75, 519 (1949).  
 (TJH49.2) J. H. Tinlot and B. P. Gregory, Phys. Rev. 75, 520 (1949).  
 (TRW48) R. W. Thompson, Phys. Rev. 74, 490 (1948).  
 (TRW51) R. W. Thompson, H. O. Cohn and R. S. Flum, Phys. Rev. 83, 175 (1951).  
 (TS40) S. Tomonaga and G. Araki, Phys. Rev. 58, 90 (1940).  
 (US47) S. Ulam and J. von Neumann, Bull. Amer. Math. Soc. 53, 1120 (1947).  
 (VGE47) G. E. Valley, Phys. Rev. 72, 772 (1947).  
 (VGE48) G. E. Valley and B. Rossi, Phys. Rev. 73, 177 (1948).  
 (VGE49) G. E. Valley, Rev. Mod. Phys. 21, 35 (1949).  
 (VHG49) H. G. Voorhies and J. C. Street, Phys. Rev. 76, 1100 (1949).  
 (VJA39) J. A. Vargus, Phys. Rev. 56, 480 (1939).  
 (VRL49) L. Van Rossum, Comp. Rend. 228, 676 (1949).  
 (VSN46) S. N. Vernov and O. N. Vavilov, Phys. Rev. 70, 769 (1946).  
 (VV48) V. Votruba, Phys. Rev. 73, 1468 (1948).  
 (WC34) C. v. Weizsäcker, Zeits. f. Phys. 88, 612 (1934).  
 (WCT12) C. T. R. Wilson, Proc. Roy. Soc. A87, 277 (1912).  
 (WCT33) C. T. R. Wilson, Proc. Roy. Soc. A142, 88 (1933).  
 (WCT35) C. T. R. Wilson, Proc. Roy. Soc. A148, 523 (1935).  
 (WEJ30) E. J. Williams and F. R. Terroux, Proc. Roy. Soc. A126, 289 (1930).  
 (WEJ34) E. J. Williams, Phys. Rev. 45, 729 (1934).  
 (WEJ37) E. J. Williams, Proc. Camb. Phil. Soc. 33, 179 (1937).  
 (WEJ39) E. J. Williams, Proc. Roy. Soc. A169, 531 (1939).  
 (WEJ40.1) E. J. Williams and G. E. Roberts, Nature 145, 102 (1940).  
 (WEJ40.2) E. J. Williams and G. P. Evans, Nature 145, 818 (1940).  
 (WEO41) E. O. Wollan, Phys. Rev. 60, 532 (1941).  
 (WH47) H. Waffler and O. Hirzel, Helv. Phys. Acta 20, 373 (1947).  
 (WH48) H. Waffler and O. Hirzel, Helv. Phys. Acta 21, 200 (1948).  
 (WG40) G. Wataghin, M. D. de Souza Santos and P. A. Pompeia, Phys. Rev. 57, 61 (1940).  
 (WGC41) G. C. Wick, Ricerca Scient. 12, 858 (1941).  
 (WGC43) G. C. Wick, Nuovo Cim. 1, 302 (1943).  
 (WJA39) J. A. Wheeler and W. E. Lamb, Phys. Rev. 55, 858 (1939).  
 (WJA41) J. A. Wheeler and R. Ladenburg, Phys. Rev. 60, 754 (1941).  
 (WJA47) J. A. Wheeler, Phys. Rev. 71, 320 (1947).  
 (WJA40) J. A. Wheeler, Rev. Mod. Phys. 21, 133 (1949).  
 (WJG38) J. G. Wilson, Proc. Roy. Soc. A166, 482 (1938).  
 (WJG39) J. G. Wilson, Proc. Roy. Soc. A172, 517 (1939).  
 (WJG40) J. G. Wilson, Proc. Roy. Soc. A174, 73 (1940).  
 (WKC48) Kan Chang Wang and S. B. Jones, Phys. Rev. 74, 1547 (1948).  
 (WKM47) K. M. Watson, Phys. Rev. 72, 1060 (1947).

- (WmH40) H. Wambacher, Sitzber. Akad. Wiss. Wien. Abt. 2a, 149, 157 (1940).  
 (WRL49.1) R. L. Walker, Phys. Rev. 76, 527 (1949).  
 (WRL49.2) R. L. Walker, Phys. Rev. 76, 1440 (1949).  
 (WRR41) R. R. Wilson, Phys. Rev. 60, 749 (1941).  
 (WVF37) V. F. Weisskopf, Phys. Rev. 52, 295 (1937).  
 (WWD30.1) W. D. Walker, Phys. Rev. 77, 686 (1950).  
 (WWD50.2) W. D. Walker, S. P. Walker and K. I. Greisen, Phys. Rev. 80, 546 (1950).  
 (YgH49) H. Yagoda, N. Kaplan, C. H. Conner, Phys. Rev. 76, 171 (1949).  
 (YrH49) H. York, Phys. Rev. 75, 1467 (1949).  
 (YRT37) R. T. Young and J. C. Street, Phys. Rev. 52, 552 (1937).  
 (YuH35) H. Yukawa, Proc. Phys. Math. Soc. Japan 17, 48 (1935).  
 (ZAP35) A. P. Zhdanov, J. Phys. Radium 6, 233 (1935).

## Index

- Absorption:  
  coefficient of photons, 300-304, 325, 326  
  nuclear (see Nuclear absorption)  
  of  $\pi$ -mesons:  
    in condensed matter, 496-500  
    in deuterium, 207  
    in hydrogen, 202-207  
  of  $N$ -rays:  
    in the atmosphere, 490-496  
  of the star producing radiation, 445, 497  
  thickness, 490, 492, 497, 500  
Air showers, 8, 428, 429, 435, 436, 442  
Alpha particles:  
  from cosmic-ray stars, 468, 469  
  in the primary cosmic radiation, 6, 520  
  producing stars, 389, 470, 523, 524  
Altitude dependence:  
  of bursts, 441, 444  
  of cosmic-ray electrons, 7, 8  
  of cosmic-ray mesons, 7, 8  
  of cosmic-ray stars, 440-442  
  of nuclear interactions by cosmic rays,  
    438-445  
  of penetrating showers, 442-444  
Angular distribution:  
  of mesons produced in nucleon-nucleon  
    collisions, 451-454  
  of star tracks, 456, 462-464  
Annihilation process, 148  
Anomalous absorption of mesons in air, 155  
Anti-coincidences, 159, 184, 189, 190, 316,  
  222, 227, 507  
Atmosphere, 544-546  
Atomic radius, 58  
Auger showers (see Air showers)  
  
Beta decay, 150  
Betatron, 301  
Binding energy, 358  
Bohr magneton, 147  
Bose statistics, 146  
Bremsstrahlung (see Radiation process)  
Bursts:  
  produced by air showers, 435-438  
  produced by  $\mu$ -mesons, 340, 341  
  
Bursts (*Cont'd*):  
  produced by nuclear interactions of cosmic  
    rays, 434-438  
  
Capture (see Nuclear absorption)  
Cascade showers (see Electronic showers,  
  Showers, Shower theory)  
Cerenkov radiation, 29, 128  
Charge exchange, 342, 351, 466-468  
Clearing field, 117  
Cloud chamber, 115-127  
  application to the study of nuclear inter-  
    actions by cosmic rays, 408-425, 464-  
    468  
  counter-controlled operation, 118  
  expansion ratio, 115, 116  
  free-falling, 189  
  in magnetic field, 124-127  
  post-expansion tracks, 119, 120  
  pre-expansion tracks, 120-124  
  recycling time, 117  
  sensitive time, 118  
  with radial expansion, 117  
Cloud limit, 115  
Coincidence:  
  experiments, 109-111  
  circuit, 109, 110  
  delayed, 159, 160, 164, 479-482  
Collision, 3, 12, 13, 14, 343, 349, 348, 353, 359  
  elastic, 63, 348, 373, 374 (see also Scatter-  
    ing, elastic)  
  inelastic, 350, 362, 364, 367, 374 (see also  
    Scattering, inelastic)  
  loss, 22-27  
    fluctuations, 29-35  
    in close collisions, 24  
    in distant collisions, 22-24  
    numerical values, 25, 38, 39  
    of electrons, 27, 52  
    of particles heavier than electrons, 24-27  
  mean free path (see Mean free path)  
  of a particle with an electron, 13, 14  
  of a photon with an electron, 77  
  probability, 14  
    of electrons, 15, 16, 220, 221

- Collision (*Cont'd*):  
 probability (*Cont'd*):  
 of particles of spin 0, 16  
 of particles of spin  $\frac{1}{2}$ , 16  
 of particles of spin 1, 16  
 time, 18
- Complex nuclei (*see also* Multiply charged nuclei, Heavy nuclear fragments)  
 in primary cosmic rays, 6, 520-525  
 nuclear interactions, 520-525
- Compton effect, 4, 5, 13, 77-79, 84  
 Compton formula, 78
- Cosmic rays, 5-9, 299, 300  
 components in the atmosphere, 7, 8  
 particles, 151-154, 338, 445-447  
 experimental data on momentum loss, 310-312, 152, 153  
 experimental data on scattering, 312-316  
 primary, 6, 520
- Coulomb barrier, 371, 389  
 Coulomb scattering, 347, 353, 354 (*see also* Scattering)
- Counting loss, 109
- Cross-section:  
 for neutral meson production:  
 by photons in hydrogen, 385  
 by protons in hydrogen, 378  
 for neutron-neutron scattering, 355, 356  
 for neutron-proton scattering, 347-351  
 for nuclear interactions of cosmic rays, 508-510  
 for photo-disintegrations 304-308  
 for  $\pi$ -meson production:  
 by photons in hydrogen, 383, 384  
 by protons in hydrogen, 378  
 for proton-proton scattering, 351-355  
 geometric, 356  
 of nuclei:  
 for interactions of cosmic-ray particles, 508-510  
 for neutrons, 343-347
- Decay (*see* Disintegration)  
 Delta rays, 137, 138, 141, 142  
 Density effect, 27-29
- Detection:  
 of charged particles, 91-145  
 of neutral mesons, 200  
 of neutrons, 104, 184, 185, 343, 482-486  
 of protons, 343  
 Diffraction scattering, 364-368
- Disintegration (*see also* Nuclear disintegrations, Nuclear interactions, Stars)  
 curve of  $\mu$ -mesons, 160, 161, 168  
 curve of  $\pi$ -mesons, 195, 196  
 of mesons in flight, 157, 191, 192  
 of  $\mu$ -mesons, 154-162, 167-169, 188-197
- Disintegration (*Cont'd*):  
 of neutral mesons, 197-202  
 of neutrons, 150  
 of  $\pi$ -mesons, 102, 163, 187, 189
- Drift velocity, 91
- Efficiency of Geiger-Mueller counters, 111-114
- Electromagnetic field, 10, 11  
 of a moving charge, 532-534
- Electromagnetic Interactions, 5, 10, 13, 200  
 of mesons, 338, 341  
 of neutrons, 343  
 of photons, 300
- Electronic showers (*see also* Showers, Shower theory)  
 discovery, 316-318  
 experiments with cloud chambers, 317, 318, 334-338  
 experiments with Geiger-Mueller counters, 316, 317, 319-328  
 produced by mesons, 320, 327, 341  
 produced by photons, 323-328, 330-334  
 transition curves, 319-325, 327, 328
- Electrons, 1, 2, 11, 147 (*see also* Positons, Negatons)  
 charge, 147  
 classical radius, 15  
 collection, 100  
 collision loss, 27  
 collision probability, 15, 16, 220, 221  
 experimental data on momentum loss, 309-311  
 in cosmic rays, 7, 8  
 magnetic moment, 147  
 mass, 147  
 primary specific ionization, 113, 114, 119  
 probable specific ionization, 122-124  
 radiation loss, 50, 52  
 radiation probability, 48-51  
 spin, 147
- Elementary particles, 1, 2, 140
- Energy:  
 available for meson production in the collision between two nucleons, 374, 375, 450, 453  
 critical, 223, 295  
 loss (*see* Collision loss, Radiation loss)  
 per ion pair, 46, 47  
 -range relations, 37, 40, 41, 134, 135
- Evaporation of nuclei, 370, 371, 449, 473
- Excitation:  
 of atoms, 11  
 of nuclei, 370, 371
- Extensive showers, 429 (*see also* Air showers)
- Fermi energy in nuclei, 358  
 Fermi momentum in nuclei, 357

- Fermi statistics, 140
- Fluctuations (*see also* Statistical fluctuations)  
 in collision loss, 29-35  
 in range, 36, 37  
 in the development of showers, 217, 288-294  
 in the ionization current, 95  
 in the radiation loss, 00, 244
- Gamma rays (*see also* Photons)  
 absorption coefficients, 300-304, 325, 326  
 spectrum from synchrotron, 301, 302
- Gas multiplication, 102-104
- Geiger-Mueller tubes, 105-114  
 dead time, 109  
 efficiency, 111-114  
 quenching of the discharge, 106, 107  
 recovery time, 109  
 spontaneous time lag, 108
- Geometric cross-section of nuclei, 356
- Gold integral, 540-543
- Good geometry absorption measurements, 364
- Gross transformation, 440
- Hammer tracks, 392
- Hard radiation, 320
- Heavy nuclear fragments, 393, 394, 468
- Heavy particles:  
 collision loss, 24-27  
 radiation probability, 60-63  
 range, 35-45
- Impact parameter, 18, 21, 22, 55-59  
 limits in collision processes, 21, 22  
 limits in radiation processes, 58
- Interactions of high-energy particles, 3-5 (*see also* Electromagnetic interactions, Nuclear interactions)
- Ion limit, 115
- Ionization, 11 (*see also* Specific ionization, Primary specific ionization, Probable specific ionization, Total specific ionization)  
 bursts (*see* Bursts)  
 chamber, 91-102  
 current, 92  
 loss, 22-27 (*see also* Collision loss)  
 potential, 23  
 pulse, 96, 98-101, 434, 435
- Knock-on electrons, 338, 340
- Knock-on process (*see* Collision process)
- Laplace integrals, 600
- Light quanta (*see* Photons)
- Lorentz transformation, 530
- Magnetic lens, 161, 165
- Magnetic moment, 15, 146  
 of the deuteron, 150  
 of the electron, 147  
 of the neutron, 149, 150, 343  
 of the proton, 149, 150
- Magneton:  
 Bohr, 147  
 nuclear, 147
- Mass, 146  
 measurements in photographic emulsions, 139, 140, 178, 179  
 measurements with cloud chamber, 176-178  
 of electrons, 147-149  
 of  $\mu$ -mesons, 175-179, 188  
 of neutral mesons, 206  
 of neutrons, 149  
 of  $\pi$ -mesons, 176-179, 188  
 of protons, 149
- Materialization (*see* Pair production)
- Maximum energy:  
 available for meson production, 450  
 transferable, 14
- Mean free path, 346
- For inelastic nuclear collisions, 360-364  
 for nuclear interactions of  $\pi$ -mesons, 387, 512, 513  
 geometric, 356  
 in nuclear matter, 360  
 of high-energy nuclei, 521, 522  
 of high-energy nucleons in air, 496  
 of neutrons, 506-508  
 of N-rays, 500-508  
 of secondary particles from nuclear interactions, 510-513
- Mean life:  
 of  $\mu$ -mesons, 156, 160-162  
 of negative  $\mu$ -mesons, 166-170  
 of neutral mesons, 201, 202, 524  
 of neutrons, 150  
 of  $\pi$ -mesons, 192-197
- Mellin integral, 539
- Mesons, 1, 11 (*see also*  $\mu$ -mesons, Neutral mesons,  $\pi$ -mesons,  $\tau$ -mesons)  
 anomalous absorption in the atmosphere, 155, 156  
 artificial production, 172-175  
 collision loss, 311, 312  
 decay in flight, 157, 191, 192  
 discovery, 151-155  
 experimental data on momentum loss, 310-312  
 experimental data on scattering, 312-316  
 in cosmic rays, 7, 8  
 multiple production, 449, 514-517, 524  
 plural production, 449

- Mesons (*Cont'd*):  
 primary specific ionization, 113, 119  
 probable specific ionization, 122-124  
 production in nucleon-nucleon collisions, 449-454  
 relation to nuclear forces, 1, 212, 213  
 theoretical prediction, 152, 212, 213
- Mixed showers, 391
- Momentum:  
 loss, 27, 152, 308-312  
 maximum detectable, 126  
 measurement by means of cloud chamber, 124-127  
 -range relation, 40-41
- Monte Carlo method:  
 applied to nuclear disintegrations, 371  
 applied to the shower problem, 203, 294
- Mu-mesons, 2, 163  
 decay, 155-163  
 discovery, 151-155  
 disintegration products, 189-191  
 mass, 175-179, 188  
 mean life, 156, 160-162, 166-170  
 nuclear absorption, 164-170, 172, 181-186  
 nuclear interactions, 517-519  
 spin, 186, 341
- Multiple meson production, 401, 449, 514-517, 524
- Multiple scattering, 77
- Multiplication by collision in gases, 102-104
- Multiply charged nuclei:  
 identification in photographic emulsions, 140-142  
 in primary cosmic rays, 520
- Natural constants, 258, 259
- Negations, 1, 147  
 collision probability, 15  
 collision loss, 27
- Neutral mesons:  
 decay, 197-202  
 discovery, 197-202  
 mass, 200  
 mean life, 201, 202, 524  
 production  
 by cosmic rays, 390, 477-479  
 by nucleons, 373-382  
 by photons, 382-387  
 spin, 201
- Neutrinos, 150, 151
- Neutrons, 1, 2, 149  
 decay, 150  
 detection methods, 104, 184, 185, 343, 482-486  
 energy distribution in the atmosphere, 486, 489  
 from nuclear interactions of cosmic rays, 482-486, 513
- Neutrons (*Cont'd*):  
 magnetic moment, 150, 343  
 mass, 149  
 mean life, 150  
 nuclear interactions, 342-346, 368-372, 418, 419, 423, 466, 468  
 production by nuclear absorption of  $\mu^-$ -mesons, 184, 185  
 scattering by neutrons, 355, 356  
 scattering by nuclei, 343-347, 359-368  
 scattering by protons, 347-351  
 sources, 342, 343  
 spin, 149
- Nuclear absorption  
 of  $\pi^-$  mesons, 171, 172, 175, 179-181  
 of  $\mu^-$  mesons, 163-170, 175, 181-186  
 Nuclear binding energy, 358
- Nuclear disintegrations (*see also* Nuclear interactions, Stars)  
 by neutrons, 368-372  
 by photons, 304-308  
 by protons, 372, 373  
 theoretical interpretation, 371-373
- Nuclear evaporation, 370, 371, 449, 473
- Nuclear geometric cross-section, 366
- Nuclear interactions, 5 (*see also* Nuclear disintegrations, Nuclear scattering, Stars)  
 of complex nuclei, 520-525  
 of cosmic rays, 388-525  
 of high energy particles, 447-451  
 of  $\pi$ -mesons from artificial sources, 387  
 of  $\pi$ -mesons from cosmic rays, 497-500, 512, 513  
 of secondary particles from nuclear events, 510-513
- Nuclear matter, 360
- Nuclear model, 356-358
- Nuclear radius, 61, 356, 365, 366
- Nuclear scattering:  
 of cosmic-ray particles, 465  
 of neutrons, 343-347, 359-368  
 of  $\pi$ -mesons, 387  
 theoretical interpretation, 359-365
- Nuclei (*see* Complex nuclei, Heavy nuclear fragments, Multiply charged nuclei)
- Nucleons, 149, 150  
 in cosmic rays, 490-496
- N-rays, 490  
 absorption:  
 in condensed matter, 496-500  
 in the atmosphere, 490-496  
 collision mean free path, 500-508
- Pair production:  
 by charged particles, 86-90  
 by photons, 3, 2, 13, 79-80  
 differential probability, 79-83  
 experimental data, 300-304

- Pair production (*Cont'd*):  
 by photons (*Cont'd*):  
 in the field of electrons, 85, 86  
 r.m.s. angle of emission of secondary electrons, 82, 83, 85  
 total probability, 80, 81, 84
- Penetrating particles, 151, 152, 390
- Penetrating showers, 391, 426-434  
 altitude dependence, 442-444  
 extensive, 429  
 local, 428  
 transition curves, 428-433
- Photo-disintegrations, 304-308
- Photo-electric effect, 13
- Photo-fission, 306
- Photographic emulsion, 127-142  
 application to the study of nuclear interactions by cosmic rays, 454-464, 468-475  
 composition, 130  
 electron-sensitive, 131  
 fading of the image, 131  
 grain density, 129, 133, 134, 139, 140, 407  
 processing, 131  
 scattering measurements, 136, 137, 140  
 stripped, 131
- Photons, 1, 2, 11 (*see also* Gamma rays)  
 absorption measurements, 300-304, 325, 326  
 from a synchrotron source, 302  
 from nuclear interactions of cosmic rays, 476-479  
 in cosmic rays, 325  
 probability for Compton scattering, 77-79  
 probability for pair production, 79-83  
 shower production, 323-325, 327, 328, 330-334
- Pi-meson, 1, 2  
 capture:  
 by deuterons, 207  
 by protons, 202-207  
 decay, 162, 187, 188  
 discovery, 162, 163  
 disintegration products, 186-188  
 from nuclear interactions of cosmic rays, 389, 390, 470-474, 479-482  
 in cosmic rays, 446, 447, 492-494  
 in penetrating showers, 479-482  
 mass, 175, 178, 179, 188  
 mean life, 192-197  
 nuclear absorption, 171, 172, 175, 179-181  
 nuclear interactions, 387, 497-500, 512, 513  
 production:  
 by nucleons, 373-382  
 by photons, 382-387  
 spin, 186
- Plural meson production, 449
- Plural scattering, 77
- Poisson formula, 556, 557
- Poor geometry absorption measurements, 367, 368
- Positons:  
 annihilation, 148  
 collision probability, 15, 16  
 discovery, 148  
 energy loss, 27  
 mass, 149
- Primary specific ionization, 45, 46  
 measurement by means of Geiger-Mueller tubes, 112-114  
 measurements in cloud chambers, 119, 120
- Probable specific ionization, 45, 46  
 measurement in cloud chambers, 120-124
- Proportional counters, 102-105
- Protons, 1, 2, 149  
 de Broglie wave length, 364  
 energy spectrum in the atmosphere, 486-490  
 from nuclear interactions of cosmic rays, 468-474  
 in cosmic rays, 486-490  
 laboratory sources, 342  
 magnetic moment, 63, 150  
 mass, 149  
 radiation probability, 63  
 scattering by protons, 351-354  
 spin, 149
- Pulse shape:  
 in ionization chambers with electron collection, 96-101  
 in proportional counters, 103, 104
- Quenching in Geiger-Mueller tubes, 106, 107
- Radiation length:  
 definitions, 50, 54, 220  
 numerical values, 54, 295
- Radiation loss:  
 experimental data, 309-311  
 fluctuations, 60  
 of electrons, 48-54  
 of  $\mu$ -mesons, 89
- Radiation probability:  
 of electrons, 48-54  
 of particles heavier than electrons, 60-63  
 of particles of spin 0, 61  
 of particles of spin  $\frac{1}{2}$ , 61  
 of particles of spin 1, 61, 62  
 of protons, 63  
 semi-quantitative computation, 55-59
- Radiation process, 12, 48-63  
 in the field of electrons, 53, 54  
 r.m.s. angle of secondary photons, 53, 54
- Range, 25, 26  
 -energy relations, 37, 40, 41, 134, 135  
 fluctuations, 36, 37

Range (*Cont'd*):  
 in photographic emulsions, 134, 135, 139, 140  
 -momentum relations, 40-43  
 Resolving time, 110  
 Root mean square:  
 angle of electron emission in pair production, 83, 85  
 angle of photon emission in radiation processes, 53, 54  
 angle of scattering, 66-69  
 error, 537  
 Rutherford formula, 17, 64  
 derivation, 17-20  
 Sandwich plates, 180, 455, 509  
 Scattering, 12, 63-77 (*see also* Nuclear scattering)  
 distribution function, 69-77  
 elastic, 364, 367, 368  
 inelastic, 364, 367, 368  
 in photographic emulsions, 136, 147, 140  
 in the field of electrons, 68, 69  
 length, 73  
 mean absolute value of the angle, 69  
 mean square angle, 66-69  
 multiple, 77  
 of cosmic-ray particles, 312-316  
 of neutrons on protons, 346-351  
 of protons on protons, 351-355  
 plural, 77  
 probability, 63-66  
 Screening:  
 influence on pair production by charged particles, 87  
 influence of pair production by photons, 79, 80  
 influence on radiation phenomena, 48, 49, 58, 61  
 influence on scattering, 65  
 Scintillation counters, 142-145  
 Shielding (*see* Screening)  
 Showers (*see* Air showers, Electronic showers, Extensive showers, Mixed showers, Penetrating showers, Shower theory)  
 Shower theory:  
 approximation A, 223, 235-251  
 approximation B, 224, 251-265  
 center of gravity of shower curve, 219, 244, 245, 257, 260  
 critical energy, 223, 295  
 differential electron spectrum, 217, 245-251, 255, 256, 260  
 differential photon spectrum, 217, 245-251, 255, 256, 260  
 diffusion equations, 228-232, 235, 236  
 energy dissipation, 217, 222, 263  
 fluctuation problem, 217, 288-294

Shower theory (*Cont'd*):

integral electron spectrum, 217, 245-251, 257-259, 261, 262, 263, 265  
 integral photon spectrum, 217  
 Laplace integrals, 234, 235, 240-244, 257, 260  
 longitudinal spread, 219, 244, 245, 257, 260  
 maximum values of shower functions, 243, 257, 260, 284-288  
 Mellin integrals, 239-244  
 method of successive collisions, 227, 228  
 moments of the shower functions, 218, 235, 283, 284  
 optimum thickness, 218, 247, 248, 257-260, 284-288  
 qualitative theory, 225-227  
 stationary distributions, 233, 234, 237, 253-255  
 track lengths, 218, 244, 245, 257, 260, 265-282  
 Sensitive volume, 91  
 Soft radiation, 319, 320  
 Specific ionization, 45-48 (*see also* Primary specific ionization, Probable specific ionization, Total specific ionization)  
 Spin:  
 definition, 146  
 of the electron, 147  
 of the  $\mu$ -meson, 186, 381  
 of the neutral meson, 201  
 of the neutrino, 150  
 of the neutron, 149  
 of the  $\pi$ -meson, 186  
 of the proton, 149  
 Standard statistical deviation, 537  
 Stars (*see also* Nuclear disintegrations, Nuclear evaporation, Nuclear interactions)  
 angular distribution of prongs, 462-464  
 distribution according to number of prongs, 372, 373, 455-461  
 energy distribution of secondary particles, 468-473  
 nature of secondary particles, 468-473  
 production:  
 by cosmic rays, 388, 391-407, 409-409, 468-475  
 by nuclear absorption of  $\mu$ -mesons, 183  
 by nuclear absorption of  $\pi$ -mesons, 171, 172, 179-181, 175  
 by nucleons from artificial sources, 368-373  
 by photons, 305  
 theoretical interpretation, 368-372, 448-449  
 Statistical errors, 536, 537  
 Statistical fluctuations, 111, 536, 537 (*see also* Fluctuations)

Outgassing in range, 30, 37, 100 (*see also* Fluctuations)  
 Stripping, 342, 343  
 Symmetry principle, 449  
 Synchrotron, 301  
 Tau-meson, 208  
 Threshold detectors, 343  
 Threshold energy:  
 for meson production by nucleons in complex nuclei, 379, 380  
 for meson production by nucleons in hydrogen, 374, 375  
 for meson production by photons in hydrogen, 382  
 for photo-disintegrations, 306, 307  
 Temperature of a nucleus, 370  
 Total specific ionization, 45, 46

Tracks in photographic emulsion, 127-142  
 dense, 388  
 medium, 388  
 thin, 388  
 Transition curve:  
 for electronic showers, 319-325, 327, 328  
 for penetrating showers, 428-433  
 Transition effect:  
 in ionization-chamber measurements, 328-330  
 in the absorption of the  $N$ -component, 497, 498  
 Units, 527, 528  
 Variations, 208  
 $V$ -particles, 208-212, 412