The cumulants are defined by[8]

$$\ln\langle e^{\Sigma a_i X_i}\rangle = \sum_{\nu_1}\sum_{\nu_2}\cdots\prod_i \frac{1}{\nu_i!} \times a_i^{\nu_i} M_{\nu_1,\nu_2\ldots}(X_1,X_2\cdots), \quad (31)$$

where ν_1, $\nu_2\cdots$ range from zero to infinity, but the term with all $\nu_i=0$ is excluded. Consequently

$$M_{\nu_1,\nu_2\ldots}(X_1,X_2\cdots) = (-1)^{\Sigma\nu_i}\frac{\partial^{\nu_1}}{\partial F_1^{\nu_1}}\frac{\partial^{\nu_2}}{\partial F_2^{\nu_2}}\cdots\psi(F_1,F_2\cdots). \quad (32)$$

The relationship of the moments to the cumulants is derivable by derivative operators analogous to those in Eqs. (5)–(12); the relations for low order are given explicitly by Kubo.[8]

Measuring the Wavelength of Light with a Ruler

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A simple lecture demonstration is described, in which light from a gas-discharge laser is diffracted at grazing incidence by the rulings of a steel scale. The wavelength of light is obtained by measuring the pattern spacings and the distance from the ruler to the screen.

IT is well known that a reflecting grating with widely spaced grooves gives good diffraction spectra if the light is incident nearly parallel to the surface of the grating. Thus, several orders of reflection spectra may be seen by sighting along a good ruled scale at a small, distant light. It is even possible to estimate the spacing of the diffraction orders by viewing them against a background with scale markings, and so to measure the wavelength.[1]

However, with a small helium–neon gas-discharge laser, a good Fraunhofer diffraction pattern from a ruler can be projected for viewing by a large class. The arrangement is shown in Fig. 1. A good machinists' steel scale is positioned approximately horizontally in front of the laser, so that the light beam passes nearly parallel to, and just above one of the finer sets of rulings. The laser is then tilted downward so that part of the beam grazes the last few inches of the rulings. Several sharp diffraction orders are then seen on the screen. If some of the beam passes over the end or side of the ruler, the direct beam can also be seen. The spots from the direct beam and the zero-order diffracted (specularly reflected) beam define the angle of incidence, and so no direct measurement of angles is needed. All information required to measure the light wavelength comes from the spacings of the spots on the screen, and the distance from screen to ruler. These can be measured with another ruler, or even with the same one if the spot positions are marked.

Figure 2 shows the angles and distances involved. If i and θ are on opposite sides of the normal, and are both taken as positive quantities, the grating equation may be written

$$n\lambda = d(\sin i - \sin\theta),$$

where n is an integer, λ is the wavelength, and d is the spacing between rulings. It is more convenient to use the complements of these angles, i.e., $\alpha=90°-i$, $\beta=90°-\theta$ so that the equation becomes

$$n\lambda = d(\cos\alpha - \cos\beta).$$

In the experiment, the distance to the screen x_0, and the distance between spots on the screen are measured. Since $\alpha=\beta_0$ (the zeroth order is specularly reflected), the intersection of the plane of the grating with the screen lies half way between the spots of the direct beam and

[1] This was pointed out to me some years ago by Professor R. R. Richmond, University of Toronto, who remarked to some students on an appropriate occasion "If you don't behave, I'll make you measure the wavelength of light with a ruler."

the zeroth order diffracted beam. Take this point as the origin for measuring distances along the screen. Then the intersection of the direct beam is at $-y_0$, of the zeroth-, first-, second-, $\cdots$ order diffracted beams are at y_0, y_1, y_2 $\cdots$. For any of these, $\tan\beta = y/x_0$; but β is small, so that $\tan\beta \approx \sin\beta$,

$$\cos\beta = [1-\sin^2\beta]^{\frac{1}{2}} \approx [1-(y/x_0)^2]^{\frac{1}{2}}$$
$$= 1 - \tfrac{1}{2}(y^2/x_0^2),$$
$$\cos\alpha = \cos\beta_0 = 1 - \tfrac{1}{2}(y_0^2/x_0^2),$$
$$\cos\beta_n = 1 - \tfrac{1}{2}(y_n^2/x_0^2),$$
$$n\lambda = d(\cos\alpha - \cos\beta_n)$$
$$= d\left(1 - \frac{1}{2}\frac{y_0^2}{x_0^2} - 1 + \frac{1}{2}\frac{y_n^2}{x_0^2}\right) = \tfrac{1}{2}d\left(\frac{y_n^2 - y_0^2}{x_0^2}\right).$$

Thus, $(y_n^2 - y_0^2)$ is linearly proportional to n.

Since the spot spacings increase as d decreases, a widely spaced pattern is obtained by diffraction from a fine scale, such as $\frac{1}{64}''$ or even $\frac{1}{100}''$. It is worth demonstrating that coarser scales give finer patterns. One good way to show this is to slide the ruler a short distance sideways. Then part of the beam is diffracted from the coarser markings used to set-off every second, fourth, or fifth division of the fine scale. Extra spots appear between those seen originally; they are higher orders from the widely spaced long marks on the scale.

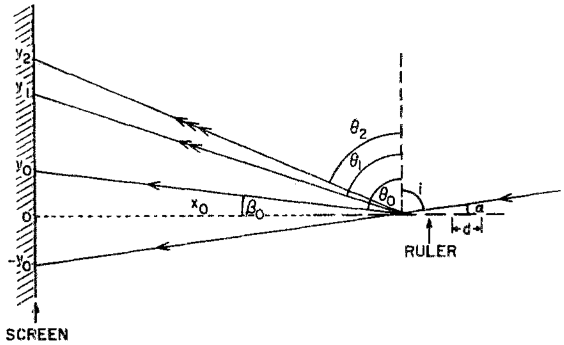

FIG. 2. Angles and distances for calculating wavelength from the diffraction pattern.

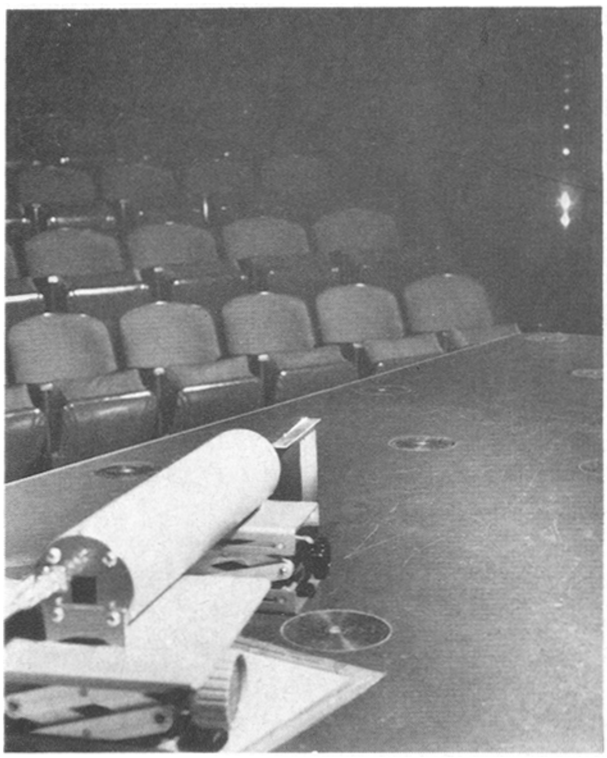

FIG. 1. Arrangement of the apparatus. The laser in the foreground shines on the rulings of a ruler, producing the diffraction pattern on the far wall.

It is also possible to demonstrate that the pattern is not clearly developed if the screen is too near the ruler, thus showing that the phenomenon is a case of Fraunhofer diffraction.

In a lecture–demonstration experiment, the distances x_0 and y_n were measured very hastily. Nevertheless, the differences of $y_{n+1}^2 - y_n^2$ all were constant to within two percent of their average value. With a little care, the wavelength of light could be measured to an accuracy about one percent, in a lecture experiment taking only a few minutes, and with all measurements clearly visible to the students. If desired, the theory and calculations can be given as an exercise.

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