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The solution of equation (4.97) is now expressed as a power series in terms of the magnetic induction on the symmetry axis ($B_0(z)$) and of its derivatives giving

$$A_{(r,z)} = \tfrac{1}{2}B_0(z)r - \frac{1}{2.2.4}B_0''(z)r^3 + \frac{1}{2.2.4.6}B_0''''(z)r^5 + \ldots \qquad (4.98)$$

The force acting on a particle of charge q and velocity V is, of course, $F = qV \times B$ and from this the three components of the equation of motion, which in this case must be solved simultaneously, are given by[44]

$$-\left(\frac{m}{q}\right)^2 \ddot{r} = \left(A_{(r,z)} - \frac{C}{r}\right)\left(\frac{\partial A_{(r,z)}}{\partial r} + \frac{C}{r^2}\right) \qquad (4.99a)$$

$$-\left(\frac{m}{q}\right)^2 \ddot{z} = \left(A_{(r,z)} - \frac{C}{r}\right)\frac{\partial A_{(r,z)}}{\partial z} \qquad (4.99b)$$

$$-\left(\frac{m}{q}\right)\dot{\phi} = \left(A_{(r,z)} - \frac{C}{r}\right)\frac{1}{r} \qquad (4.99c)$$

where C is a constant. The power series of equation (4.98) is substituted into equations (4.99) and then a numerical solution is obtained[43] for thirty-five different z-axis field profiles. Amongst these are the classic long-lens uniform field, the intermediate focus field[41] and the classic short-lens field.

Lindgren and Schneider[43] were able to show explicitly the ring focus characteristics of these fields. In addition they determined the source–image characteristics of the lenses obtained with these field profiles. This was the first time that this had been successfully achieved for any lens other than for the classical uniform-field lens and the so-called 'triangular-field' lens.[45]

A number of detailed design studies for the ring baffle slit have been made[2,37–40,46] and checked experimentally. Special twisted baffle systems have been designed by several groups for use with short and long spectrometer lenses[47,48] to enable electrons to be separated from positrons and vice versa. The only difference between the trajectories of these two particles in any lens spectrometer is that the electron trajectory is a right-hand screw spiral while the positron trajectory is a left-handed one.

This concludes the discussion of the theoretical description of charged-particle focusing and dispersion in various magnetic field configurations. It now remains to be seen how far these theoretical predictions can be achieved in practice. In the remaining two sections of this chapter the experimentally observed performance parameters for the various types of spectrometer are examined and, where possible, compared with predictions from the theory. Emphasis is placed on the experimental advantages and disadvantages of each type and on the techniques which have to be employed to achieve optimum performance.

4.2 Magnetic spectrometers for beta particles and electrons

4.2.1 *Introduction*

The magnetic beta spectrometer is one of the most important instruments in the whole history of the development of the study of nuclear structure. From the earliest days of the subject instruments which make use of the bending of the beta-particle trajectory in a magnetic field have been used to study the properties of the spectra of emitted beta particles. Several distinct types of instrument have been evolved, each with its own individual advantage, or, in some cases, disadvantage and the evolution process is still going on.

The requirement to study in coincidence the spectra of beta particles in a decay cascade or of beta particles followed by internal conversion electrons, or yet again of beta particles with gamma rays led to the development of rather specialised forms of these spectrometers from the early 1950s onwards. A parallel development was their use to study scattered electrons from the bombardment of targets by accelerated electron beams. This was started by Hofstadter[49] and several high-resolution instruments have since been designed and built for this work by other workers.

An associated field is the study of external conversion electrons, electron–positron pairs and Compton electrons from the interaction of high-energy gamma radiation with matter. Compton spectrometers and pair spectrometers have been evolved from most of the present-day types of standard beta spectrometers. Some types are more easily adapted than others but all have been used for pair or Compton studies.

A large number of the ideas used in beta spectrometers were carried over into heavy-ion spectrometers but here, the requirements being rather different, the development has been divergent rather than parallel.

Only the basic details of the designs will be discussed in this chapter, detailed references being given to the original publications on the instruments in each category.

4.2.2 *Flat-field semicircular-focusing beta spectrometers*

Historically these are the earliest types, being the direct descendants of those used in experiments by von Baeyer and Hahn[50] in 1910 in which direct deflection by a magnetic field was used to study beta particles from a radioactive wire. The semicircular-focusing devices were developed by Danysz[51] and by Rutherford[52] independently and this principle has been used ever since for this type of instrument.

The schematic representation of the focusing action in a uniform magnetic field is shown in figure 4.9. The focus is at $\phi = \pi$ (see, for example, the focus condition given by equation (4.40) for $n = 0$). There is a third-order aberration which causes a broad image of a point source. The width of this image is given by $\Delta X = 2R(1 - \cos\theta_0)$, where R is the usual radius of curvature of the central ray

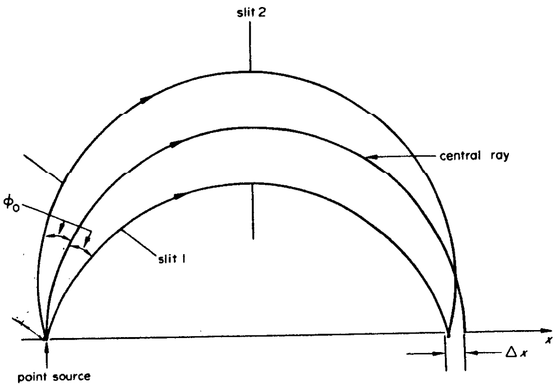

Figure 4.9 Focusing action in a uniform magnetic field

and θ_0 (usually equal to ψ_0) is the semi-opening angle at the source in the median plane. If the source is of width s, the image is then

$$\Delta x' = s + 2R(1 - \cos\theta_0) \approx s + R\theta_0^2$$

since the magnification for $\phi = \pi$ from equation (4.50) is unity. The base resolution is given by $R^0 = s/2R + \theta_0^2/2$ (see equation (4.90)) if the source height is neglected.

The line shape for the spectrum is distorted because of the aberration. A detailed treatment of this line shape has been given by Siegbahn[2,53] and the theoretical prediction for this shape is shown in figure 4.10(a), together with the experimentally observed line shape in figure 4.10(b).[54] Calculations which include the length of the source have been made by several authors[55–57] but these give essentially the same theoretical profile as that of figure 4.10(a).

Detailed design studies taking into account detector slit width w, source height h, and half-opening angle θ_0 were made by Geoffrion[57] and from these a very precise instrument was built.[58] The spectrometer had a mean radius of 305 mm, source and counter slit widths of 1 mm and heights of 5 mm. The resolution obtained was 0·25 per cent for a transmission of 0·07 per cent giving a figure of merit of 0·28. The theoretical values of these quantities for this instrument from the first-order and second-order theory given in section 4.1 would be 0·26 per cent, 0·07 per cent and 0·27 respectively, showing that it approaches the theoretical limit of accuracy. This instrument was equipped with an electrostatic acceleration system allowing the detection of electrons down to very low energies but necessitating transmission corrections for these energies.

In these spectrometers the detector may be a counter of the GM type, proportional type, organic or inorganic scintillator type, a solid-state detector or it

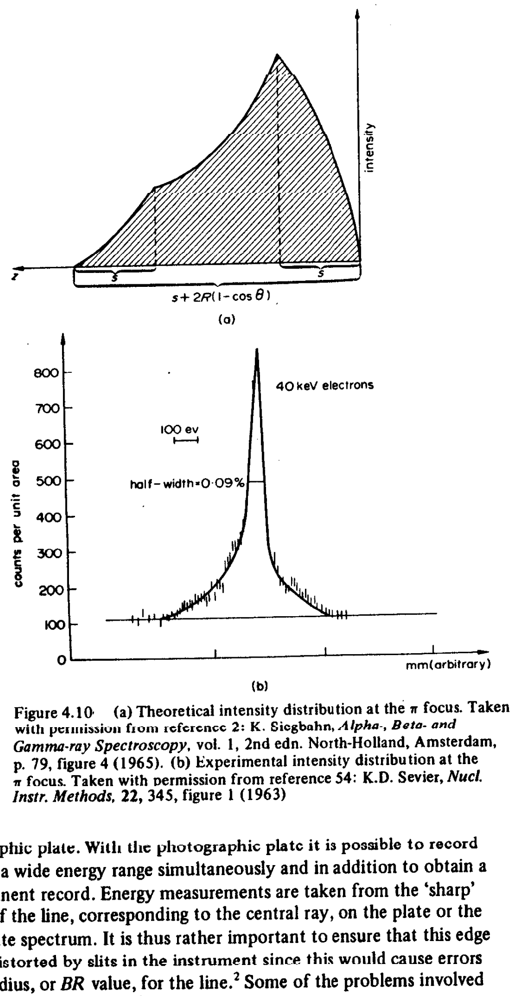

Figure 4.10 (a) Theoretical intensity distribution at the π focus. Taken with permission from reference 2: K. Siegbahn, *Alpha-, Beta- and Gamma-ray Spectroscopy*, vol. 1, 2nd edn. North-Holland, Amsterdam, p. 79, figure 4 (1965). (b) Experimental intensity distribution at the π focus. Taken with permission from reference 54: K.D. Sevier, *Nucl. Instr. Methods*, **22**, 345, figure 1 (1963)

may be a photographic plate. With the photographic plate it is possible to record beta particles over a wide energy range simultaneously and in addition to obtain a more or less permanent record. Energy measurements are taken from the 'sharp' high-energy edge of the line, corresponding to the central ray, on the plate or the BR versus count-rate spectrum. It is thus rather important to ensure that this edge is not artificially distorted by slits in the instrument since this would cause errors in the measured radius, or BR value, for the line.[2] Some of the problems involved in this work are discussed by Slätis.[59]

Two possible positions for the photographic plate are in general use. These are shown in figure 4.11. The arrangement in figure 4.11(a) increases the dispersion and reduces the background considerably but causes problems in accurate measurement of the radius of curvature, *R*. The arrangement shown in figure 4.11(b) is the more usual one and in it the source and plate can both be mounted on one holder, making the measurement of the radius *R* a relatively simple job on a standard laboratory comparator.[2]

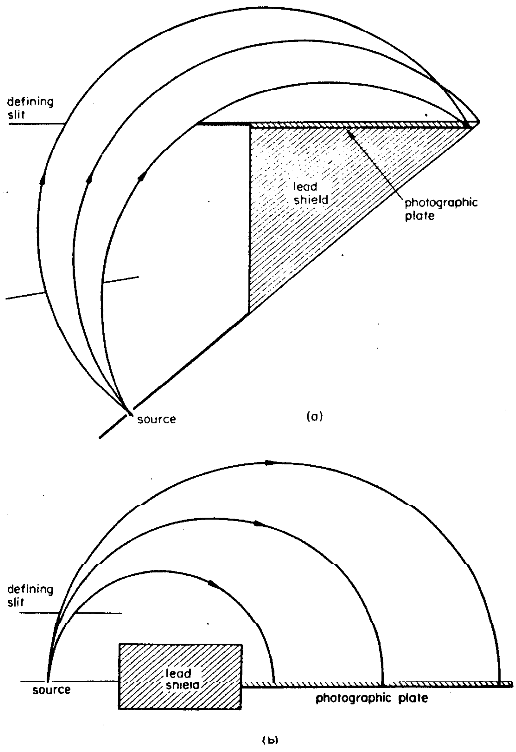

Figure 4.11 (a) The tilted plate-holder position. (b) The normal plate-holder position

The magnetic field is measured in a number of ways, rotating search coil, proton n.m.r. or Hall probe. The problems involved are similar for all spectrometers. Details of the systems in use can be found, for example, in references 60, 61.

The low-energy spectra of beta particles present special problems of measurement. Most counters have windows which must be penetrated to ensure detection. Photographic plates require fairly large numbers of grains to be developed for definite recording and the surface is often less sensitive. The acceleration method used by Geoffrion[58] is typical of what has been done to try to overcome these problems. However, problems are raised in turn by these pre- or post-acceleration methods, mainly to do with the absolute, or relative, efficiency. These problems have been studied by several authors.[62]

The spectra obtained are often very complex with several superimposed decays and many conversion lines, the spectrum of ^{194}Au being a good example of this. An automatic analysis programme has been set up by Schneider and Lindqvist[63] to analyse the complex spectra observed in many decays. The coordinates of a line on the plate are measured and used as input data for a computer programme which has stored in it all the K, L, M, etc., binding energies in the *Z* region being examined. Similar analysis systems, some directly on-line, have been developed for other types of spectrometer.

The presence of the lines in a spectrum is a great help in establishing an absolute energy scale. This problem is greatest with spectrometers with non-uniform fields and tabulations exist of accurately known conversion lines for calibration purposes.[2] With uniform-field spectrometers an absolute energy scale can usually be determined from the direct measurement of the magnetic field and of the radius of curvature in that field, hence the importance of these two measurements.

A very large number of uniform-field semicircular-focusing beta spectrometers have been built. Many have fields given by permanent magnets[2,64–66] while the rest have electromagnets. Great care has to be taken to ensure high uniformity of field and complicated cycling programmes are used to eliminate the effects of hysteresis.[2,67]

The absolute measurement of the line intensity in photographic plates causes some difficulties but various methods have been used to overcome this.[54,68] The big advantage of the use of plates is that very long exposures can be used, up to one month, with the permanent magnet spectrometers. Overall resolution of the order of 0·1 per cent FWHM can be achieved with these instruments but with the disadvantage that the one-dimensional focusing means that a very small solid angle is obtained. The figures of merit for these instruments are usually between 0·1 and 0·3.

The third-order θ_0 aberration can be removed by use of a field which is non-uniform. This was shown by Beiduk and Konopinski.[69] If the *C* parameter of table 4.2 is put equal to 1 then third-order focusing occurs at an angle of π. At least two instruments have been constructed with this field form as beta spectrometers.[70,71] The angle of acceptance from the source can be increased to be of the order of 33° without loss of resolution. The line shape is more nearly symmetric and resolution of 0·4 per cent for a transmission of 0·2 per cent can be obtained, giving a figure of merit of 0·5. The main use of the Belduk–Konopinski field profile has been for mass dispersion in the 'Calutron' isotope separator.

The semicircular-focusing principle has been used extensively for the examination of electron–positron pairs produced in a heavy-element converter by high-energy gamma rays. The broad range which is available in such an instrument is a very big advantage. One of the earliest instruments is that due to McDaniel[72] in which effectively two semicircular spectrometers are contained within one uniform-field region, one for electrons and the other for positrons. The schematic representation of this type of pair spectrometer is shown in figure 4.12.

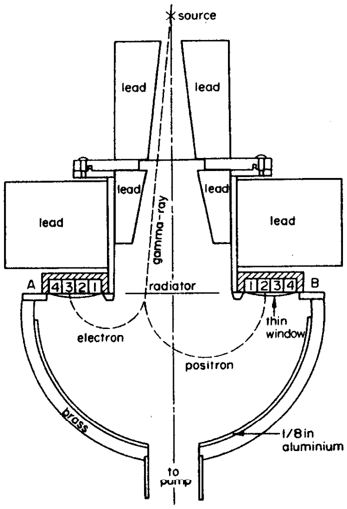

Figure 4.12 Schematic representation of the pair spectrometer of references 72–74. Taken with permission from reference 74: J. Terrell, *Phys. Rev.*, **80**, 1076, figure 1 (1950)

As originally used[72,73] only one detector was employed in each spectrometer side. In a modified design due to Terrell[74] four detectors were used on each side. In the design of Leslie and Main[75] triggered spark chambers were used to cover the complete focal plane on each side and determine the positions of the electrons and positrons. An overall energy resolution of 3 per cent FWHM for gamma rays from 10 to 20 MeV was achieved.[75]

A recently reported instrument by Golubnichiy *et al.*[76] again used triggered spark chambers for gamma rays in the 120 MeV region. To increase the working range of the instrument an array of coincident spark chambers was used round the sides of the uniform-field region and coincidences were required between a pair of these momentum-analysing detectors and an entrance detector. The schematic representation of this system is shown in figure 4.13. For normal operation coincidences are required between detectors k_2, k_3 and k_4. For an extended range further coincident pulses are required between k_2, k_5, k_7 and k_2, k_6, k_8 with again k_2 determining the position of formation of the pair. Counter k_1 is used in anti-coincidence with all these sets to exclude charged particles in the incident photon beam on the converter target. An overall energy resolution of 1·5 per cent FWHM was achieved at a gamma-ray energy of 120 MeV for a converter of thickness $0{\cdot}014X_0$, X_0 being the radiation length.

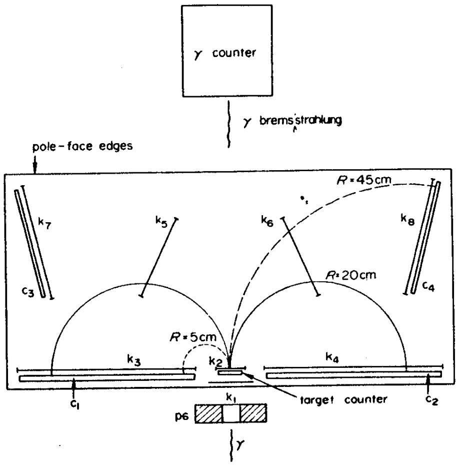

Figure 4.13 Schematic representation of the high-energy pair spectrometer of reference 76. Taken with permission from P.I. Golubnichiy, L.M. Kurdadze, D.M. Nikolenko, A.P. Onuchin, S.G. Popov and V.A. Sidorov, *Nucl. Instr. Methods*, **67**, 22, figure 1 (1969)

All these broad-range semicircular-focusing pair spectrometers suffer from poor transmission because of the one-dimensional focusing. They do, however, have very good overall energy resolution. The overall efficiency of the Golubnichiy instrument at 120 MeV is estimated to be 6 per cent for a resolution of 2·3 per cent from a counter of thickness X_0.

4.2.3 Double-focusing iron beta spectrometers with $n = \frac{1}{2}$

The principle of double focusing in an $n = \frac{1}{2}$ magnetic field was originally developed for the betatron by Kerst and Serber.[12] The first application of this principle to beta spectrometers was by Svartholm and Siegbahn[77] in 1946. Since then a number of different designs have been developed, some using iron-cored magnets and some using air-cored devices. Only iron-cored spectrometers will be discussed in this section. These devices have the advantage over the air-cored types in that they provide their own magnetic shield against the magnetic field of the earth or the stray fields from other local magnets.

The construction details of these iron-cored spectrometers can be divided into three main classes, as is shown in figure 4.14. The type shown in figure 4.14(a) is widely used and is similar to one of the very earliest of the double-focusing instruments to be built.[78] On a small magnet, fringe field effects may be a problem and in that case the configuration of figure 4.14(b) is better, since in this design the fringing fields are eliminated; this is the so-called 'inside-out' configuration.[79] A combination of these two designs, originally due to Bartlett[80] and Wild and Huber *et al.*[31,81] and shown in figure 4.14(c) has the advantage that it can be used as a very-high-aperture instrument. The other two basic types have restricted vertical apertures and although field shapes for high aperture are the normal ones in use, the wide-aperture profile is better suited to these designs. The two sets of coils in the basic type in figure 4.14(c) allow variation of the field profile to fit a range of values of the β field coefficient (equation (4.87)) and so the instrument can be made very versatile. A disadvantage of the types in figures 4.14(b) and 4.14(c) is that access to the median plane through the side is obstructed by the coils and this means that, for example, it is not as easy to use these instruments with photon beams and a converter as it is in the case of the figure 4.14(a) design.

From equations (4.40) and (4.41) it is found that, when $n = \frac{1}{2}$, double focusing occurs at $\phi = \pi\sqrt{2}$. The dispersion for this focus condition, given by equation (4.51), is 4. This should be compared with the value of 2 for the dispersion in the $n = 0$ homogeneous-field semicircular-focusing spectrometer. The resolution is half that for the semicircular case and the transmission, because of the double focusing, is much greater. Figures of merit of as high as 7 can be obtained with these instruments.[84,81]

Schematic drawings of the instruments of references 78, 79 and 80 are shown in figures 4.15, 4.16 and 4.17 respectively. Typical performance figures are given in table 4.3 for these instruments together with the theoretical resolution estimates from the second-order theory given by equation (4.90).

Copies of these original instruments have been made in, or supplied to, several laboratories, the Brookhaven 500 mm double-focusing instrument being an example.[82] These spectrometers are all corrected to at least third order but nevertheless aberrations still exist. Detailed calculations for the exact field form to reduce these were made originally by Daniel[4] and more recently by Groth.[5] Other effects, for example those due to hysteresis, have been studied to try to improve the performance.[83]

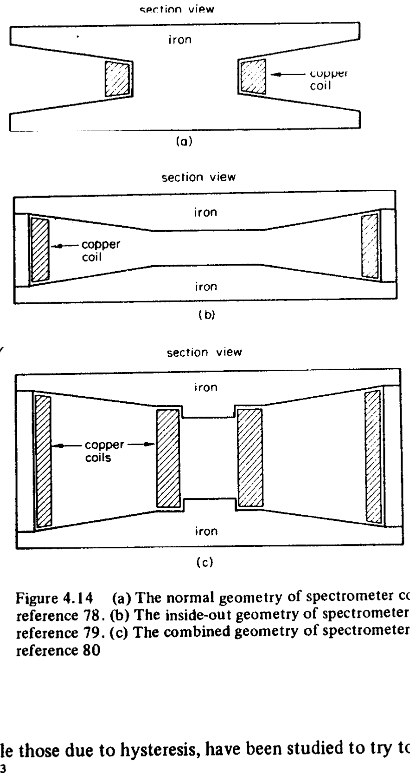

Figure 4.14 (a) The normal geometry of spectrometer coils of reference 78. (b) The inside-out geometry of spectrometer coils of reference 79. (c) The combined geometry of spectrometer coils of reference 80

Improvements have been made to the original designs as a result of these studies. The internal slits and baffles now used give much cleaner spectra. The overall luminosity can be much improved by the use of electrostatic correctors and deflectors at the source and along the orbit, as has been shown by Bergkvist.[84]

Spectrometers of this iron-cored $n = \frac{1}{2}$ type have made a major contribution to the acquisition of data for nuclear structure studies. More recently designed iron-free types give better performance[1,85] but nevertheless the 'flat' iron spectrometers form the backbone of the study of beta-particle and conversion electron spectra.

The construction and field geometry of these $n = \frac{1}{2}$ spectrometers make them unsuitable for pair spectrometry but their use with converter targets for external

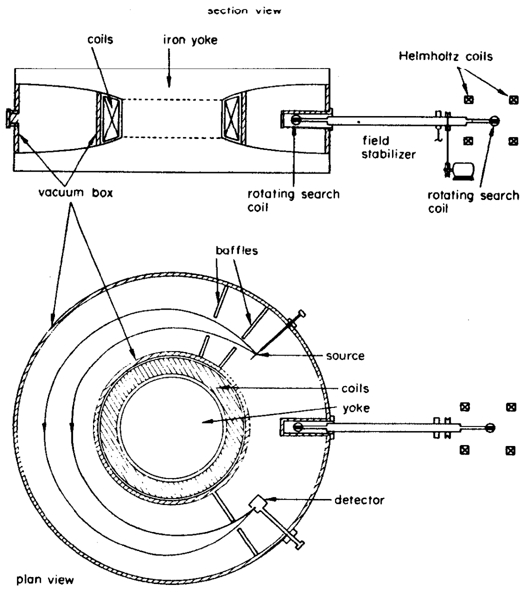

Figure 4.15 Plan and section views of the spectrometer of reference 78

conversion electrons and Compton electrons is of as great importance as is their use with beta particles and internal conversion sources. Modifications to the Hedgram *et al.* spectrometer[78] to allow its use as one element in a coincidence and angular correlation system have recently been reported.[86] The plan view of this modified system, which includes new orbit-defining baffles, is shown in figure 4.18.

The dispersion of these instruments can in principle be improved if the particles are allowed to pass the first focus at $\pi\sqrt{2}$ and continue to a second focus at $n \times \pi\sqrt{2}$, where n is an integer greater than unity. This means passing the source and the final detector at least once before stopping the beam. The scattering and loss of particles caused by this may outweigh any gain in dispersion. An instrument with a complete angle of $2\pi\sqrt{2}$ has been built.[87] The plan view of this is shown in figure 4.19.

Particles from the source pass the final detector, form a focus in the first detector which is a thin-window gas-filled transmission detector, diverge again, pass the source and come to a second focus at the final detector. Coincidences are recorded between the transmission detector at the first focus and the final stopping detector and in this way background is considerably reduced. A resolution of 3·5 per cent is obtained with an overall transmission of 0·7 per cent for a solid angle at the source of 6 per cent of 4π. The figure of merit for this instrument is 0·2.

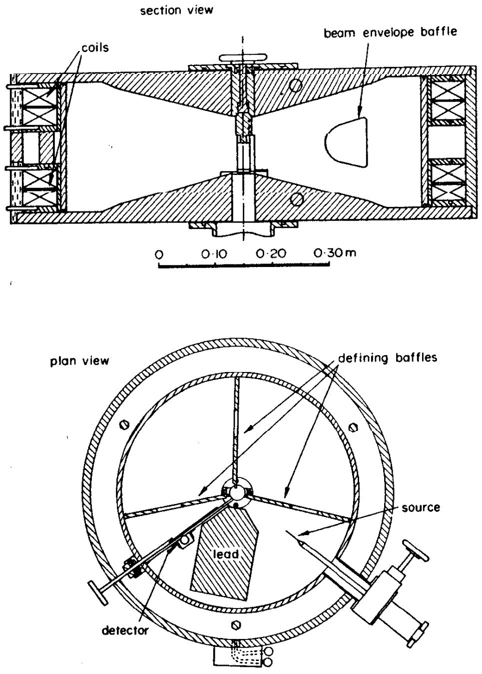

Figure 4.16 Plan and section views of the spectrometer of reference 79. Taken with permission from: E. Arbman and N. Svartholm, *Arkiv Fysik*, **10**, 1, figures 2 and 3 (1956)

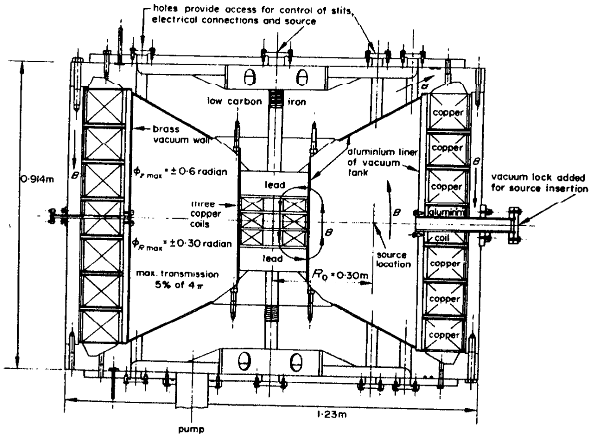

Figure 4.17 Section view of the spectrometer of reference 80. Taken with permission from A.A. Bartlett, R.A. Ristinen and R.P. Bird, *Nucl. Instr. Methods,* 17, 188, figure 4 (1962)

Table 4.3

Reference	Mean radius/ mm	Measured transmission/ %	Resolution/% Measured	Resolution/% Theory	Source slit/mm	Detector slit/mm	Figure of merit *T/R*
Hedgram *et al.*[78]	500	0·02	0·045	0·042	0·8	0·5	0·44
	500	0·12	0·14	0·15	2·0	2·0	0·86
	500	0·24	0·27	0·29	4·0	4·0	0·86
Arbman *et al.*[79]	185	4·0	1·3	1·45	2	2	3·4
	185	1·0	0·85	0·87	4	4	1·2
	185	0·5	0·50	0·46	2	2	1·0
Bartlett *et al.*[80]	300	0·5	0·067	0·066	0·5	–	7·0
	300	2·5	0·8	0·82	(0·105 rad)	–	3·2

Experimental data taken from references 78, 79 and 80 with theoretical resolutions and figures of merit calculated by the author.

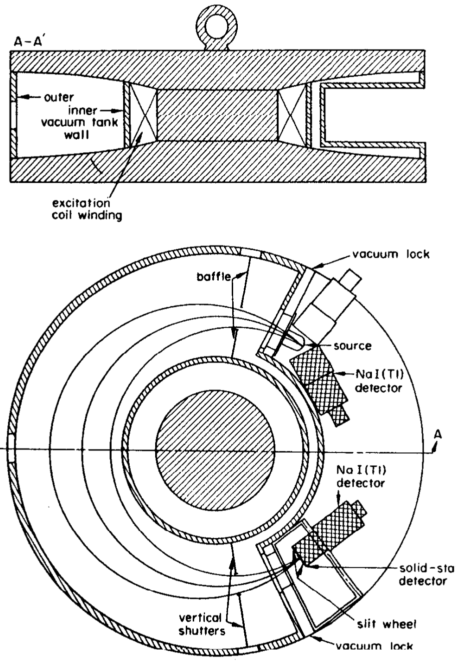

Figure 4.18 Plan view of the double-focusing spectrometer modified for coincidence and correlation work. Taken with permission from reference 86: S. Antmann, Y. Grunditz, A. Johansson, B. Nyman, H. Pettersson, B. Svahn and K. Siegbahn, *Nucl. Instr. Methods,* 82, 13, figure 2 (1970)

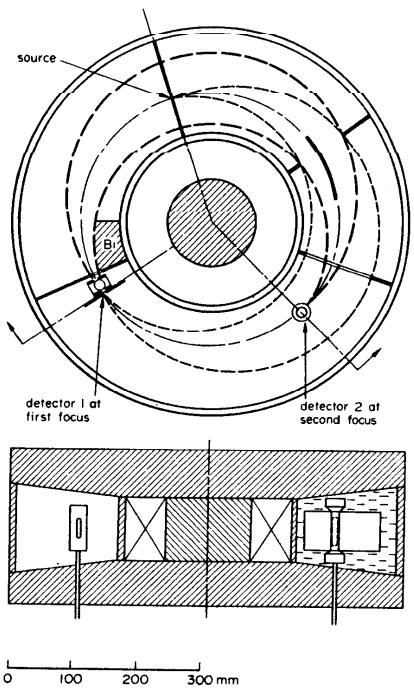

Figure 4.19 Plan view of the $2\pi\sqrt{2}$ double-focusing spectrometer of reference 87. Taken with permission from H.U. Gersch, E. Hentschel, P. Gippner and W. Rudolph, *Nucl. Instr. Methods*, 25, 314, figure 1 (1964)

Other suggested configurations to improve the dispersion by causing a focus at $n \times \pi\sqrt{2}$ have tried to displace the source and detector with respect to the central ray. This removes the scattering from the source and detector but reduces the solid angle and upsets the vertical focus condition so that double focusing is no longer achieved and the image is astigmatic.

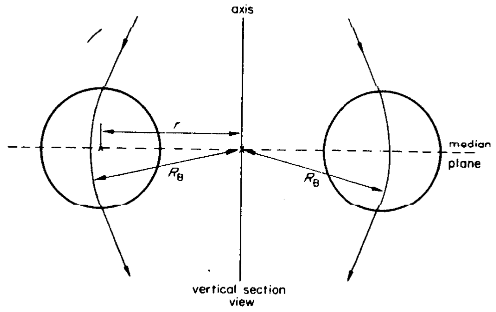

Figure 4.20 Plan view of the motion of a particle in the axial plane of a torus

4.2.4 Toroidal-field spectrometers

The field inside an air-cored torus has a very simple form. In the median plane of the torus, see figure 4.20, the field at any point inside the torus is proportional to R_B^{-1}, the distance from the torus axis. If the particle radius of curvature in the median plane is also R_B then particle motion similar to that in an $n = 1$ field results and from equation (4.40) no focusing takes place in the radial direction, although z-direction focusing will occur (equation (4.41)). If, however, the path of the particle is normal to the median plane, lies on a plane containing the axis of symmetry of the torus, and has a radius of curvature in the field, R_c, which is not equal to R_B, then a condition can be found in which the orbit is in the form of trochoidal loops, figure 4.21, and a radial focus can be formed. Focusing in the z direction is also present but usually *not* double focusing so the system is astigmatic. A similar type of trochoidal orbit will exist in the median plane if the particles are injected into the field on the median plane from some point in the median plane.

If the physical dimensions of the field are such that the path of the particle through the magnetic field of the solenoid is only a small part of one trochoidal orbit, as shown in figure 4.20, then a toroidal magnet can be designed to have a lens action for particles leaving the symmetry axis on one side of the magnet and coming to an astigmatic focus on the symmetry axis on the other side of the magnet. To describe the particle trajectory, and to define the shape of the entrance and exit boundaries to the field, a new parameter is introduced. This is generally known as the 'k' parameter, given by R_c/R_B, but in some texts it is known as the 'b' parameter.

The field shape can of course be obtained with an iron-free toroid or it can be achieved by the use of gaps between conical or wedge-shaped iron pole pieces. The

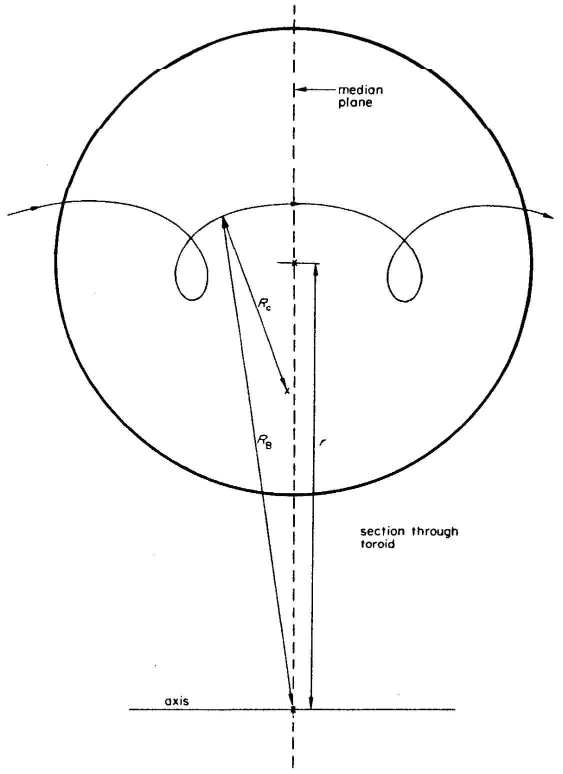

Figure 4.21 Trochoidal motion in a toroidal field

majority of the instruments are complete toroids with many gaps between the coils to allow passage of the beta particles. These multi-gap spectrometers may have as many as 100 gaps,[88] the minimum is usually 6 gaps.[89] Attempts to give artists' three-dimensional sketches of these spectrometers, particularly the early 6-gap instruments have led to the name of 'orange' spectrometers being applied to them. A plan and section view of an iron-cored six-gap orange spectrometer is shown in figure 4.22.[90]

Iron-free toroids are probably the best way of achieving the correct field form,

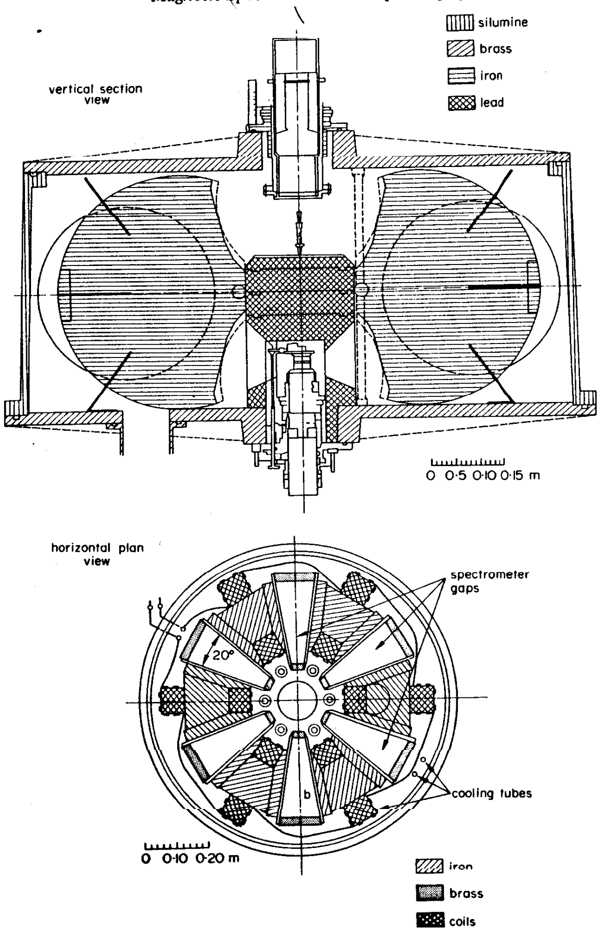

Figure 4.22 Plan and section view of the six-gap 'orange' spectrometer of reference 90. Taken with permission from K. Siegbahn, *Alpha-, Beta- and Gamma-ray Spectroscopy*, vol. 1, 2nd edn, North-Holland, Amsterdam, p. 79, figure 35 (1965)

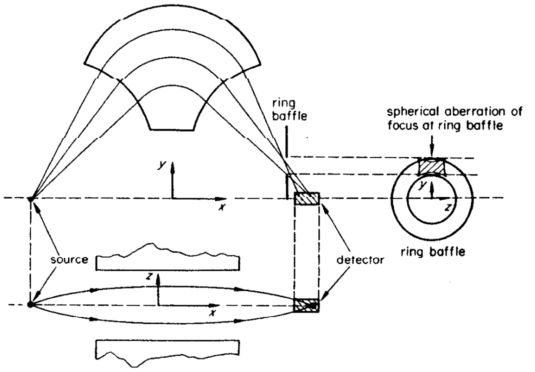

Figure 4.23 The trajectories in a multi-gap 'orange' spectrometer showing astigmatic focus and the spherical aberration at the ring baffle

although here the problems of fringe fields near the entrance and exit have to be carefully examined. The parameter k is used in the description of the shape of the entrance and exit boundaries. These boundary shapes are shown, for example, in reference 2. The most common k values in use lie between 0·5 and 0·6 but one instrument, due to Bartis,[91] has 38 gaps and a k (or b) value of 1·0. Bartis claims that some contribution to focusing comes from azimuthal effects in his instrument and that this reduces the defocusing which must otherwise be expected with $k = 1$. This suggestion has been refuted by Wagner *et al.*[92] who state that some other cause should be sought for the large transmission reported for this instrument.

A number of orange spectrometers are now in use, some directly on beam lines on accelerators. Typical examples are reported in references 88–91 and 93–96. The focusing is usually astigmatic and in the dispersive direction there is usually a segmented ring focus. The energy-defining baffle is placed at this point. Loss of transmission results from the spherical aberration of the image in the z direction at this radial (dispersive) focus. These effects are shown schematically in figure 4.23. Detailed calculations on these focusing effects have been reported by Siegbahn.[2]

The transmission available with these multi-gap instruments is very large, 10 per cent of 4π being a commonly achieved figure. The resolution is to some extent sacrificed in a number of these instruments to ensure very high transmission but figures of less than 1 per cent can be achieved. Figures quoted for the instrument of Bisgard[89] and for the Argonne double 100-gap instrument[88] are shown in table 4.4. These are typical of what can be achieved with instruments of this type.

Table 4.4 Performance figures for orange spectrometers

Reference	Transmission/%	Resolution/%	Figure of merit T/R
Bisgard[89]	1	0·4	2·5
	5	0·8	6·2
	10	1·4	7·1
Freedman *et al.*[88]	19	0·93	20
	2·8	0·12	13

When the luminosity is taken into account they do not appear as good as the figures of merit would suggest. For example, Bisgard[89] estimates that the luminosity of his six-gap instrument is at best 20 per cent of that obtainable with the $n = ½$ spectrometers. The reason for this is that the source must be small and be situated on the axis of the toroid. However, for direct use on a beam line with an accelerator, where the source is usually the dimension of the beam spot on the target, this problem is not of importance since these beam spots are usually of order 1–3 mm in diameter.

Several single-gap iron-cored instruments have been reported which have toroidal field profiles and are essentially one section from a multi-gap orange spectrometer.[97,98] These can be used singly as straightforward beta spectrometers or as part of a beta–gamma coincidence system.[97,98] Alternatively they can be used in pairs to measure beta–beta or beta–electron coincidences.[97] The instrument described by Armini *et al.*[98] is fully automated for collection of short-lived beta-particle spectra from sources produced by cyclotron bombardment of targets remote from the spectrometer and brought to it by a 'rabbit' system.

4.2.5 *Trochoidal spectrometers*

If the physical dimensions of the toroidal magnetic field are sufficiently large then one or more trochoids may be performed by the particles in traversing the field. A number of studies have been made of the focusing conditions for fields varying as $(r)^{-1}$ and $(r \sin \theta)^{-1}$. The most comprehensive of these are studies of the $(r)^{-1}$ field by Riche[99] and of the $(r \sin \theta)^{-1}$ field by Hofmann.[100] Very-high-order double focusing with correspondingly very high dispersion can in principle be achieved. A method for achieving the correct field shape for the $(r)^{-1}$ field is given by Mugnier and Lafoucriere.[101]

An instrument with the $(r \sin \theta)^{-1}$ configuration has been constructed by Balzer *et al.*[102] according to the design studies of Hofmann.[100] This instrument has very-high-order focusing. Very high dispersion can be achieved for a small source dimension. The performance figures are very good indeed and a resolution of 0·1 per cent at 1 per cent transmission, giving a figure of merit of 10, can easily be achieved. The transmission and resolution are almost linearly related so that the high figure of merit is maintained as the source slits are closed to improve resolution.

The luminosity is very high, being at least as good as the $n = \frac{1}{2}$ double-focusing instruments.[102]

A trochoidal pair spectrometer with zero dispersion, and thus no energy resolution, has been designed and built by Malmfors,[103] for use with pulsed beams. The electron and positron from a pair event perform trochoidal orbits in opposite directions round a radially symmetric field region to be detected at 180° from the target by two detectors. With the pulsed beams time-of-flight techniques can be used to determine the electron and positron energies and thus the original photon energy.[2,103]

A more recent instrument for the study of internal pairs has been built by Allan[104] in which the electron and positron are detected in a pair of solid-state detectors after traversing approximately 130° in the field. An energy resolution of 6 keV is obtained at 500 keV with a detection efficiency in the peak of 20 per cent (of 2π). The pair full peak efficiency is about 10^{-2} with a resolution of 15 keV for $E_\gamma = 3$ MeV.[104]

It seems probable that more of these multi-loop highly focusing instruments will be developed. The main problems lie in achieving a high order of accuracy in the field profile to obtain aberration-free double focusing after several trochoidal loops. This is illustrated in the instrument of Balzer *et al.*[102] where the theoretical calculations showed that the resolution should be almost independent of transmission and dependent only on the width of the source. In theory this would show a resolution of 0·013 per cent for a 1·04 per cent transmission for a source 0·05 mm wide, giving a figure of merit of 80. In the actual instrument as built, residual aberrations meant that the source had to be very much smaller in width even to approach these figures and that only for large source widths and resolutions of the order of 0·4 per cent were the theoretical figures realised.[102]

4.2.6 Sector-focusing spectrometers

The focusing properties of a sector magnet were developed in section 4.1. A number of instruments with various field profiles have been built using this sector-focusing principle, which is also used in the multi-gap 'orange' spectrometers. The advantage of these sector spectrometers is that both the source and detector can be outside the magnetic field and so they are of great importance in coincidence work and correlation studies. Some achieve all their focusing and dispersive properties by virtue of this field index alone, as indeed did the first of these spectrometers built by Lauritsen *et al.*[105] Other designs achieve this focusing by fringe field focusing from non-normal entry to a homogeneous magnetic field.[14,15,106–108] Yet another class achieve part of their focusing from their non-zero field index and part from the fringe field.[97,98]

In the case of a homogeneous-field sector magnet with normal entry and exit the source, image and 'bending edge' lie on a straight line (figure 4.24(a)). This is known as Barber's law of magneto-optics. For a non-uniform field magnet this law has to be generalised using the field coefficients of equation (4.87).

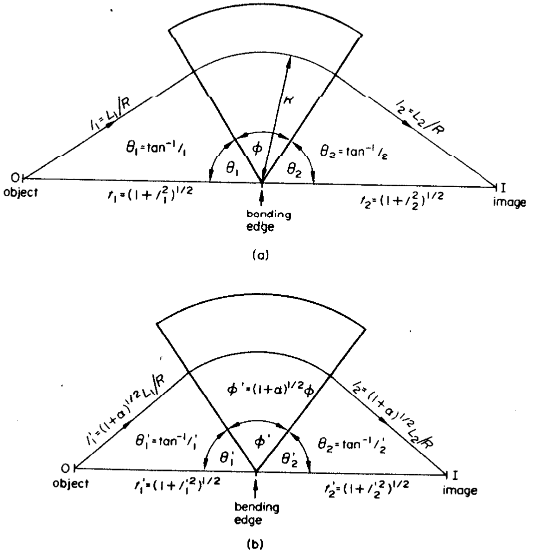

Figure 4.24 (a) Illustration of Barber's law for a uniform-field wedge magnet. (b) Illustration of the generalised form of Barber's law for a non-uniform-field wedge magnet

$$\phi' = (1 + \alpha)^{1/2}\phi \tag{4.100}$$

$$l'_{1,2} \equiv \frac{L'_{1,2}}{R} \equiv (1 + \alpha)^{1/2}\frac{L_{1,2}}{R} \tag{4.101}$$

Here ϕ is the actual bending angle and L_1 and L_2 are the actual free flight lengths from the source to the magnet and from the magnet to the image. R is the radius of curvature of the central ray. This generalised version of the rule is shown in figure 4.24(b) and can be applied to all sector magnets.

The second-order focusing with a non-uniform sectorial field is described by the second-order focusing coefficients given in reference 3. A slightly different version of the description of this focusing using the factorial expansion of equation (4.87) is given by Rosenblum[109] and by Sakai and Ikegami[110] and is quoted in reference 2.

Instruments with $n = \frac{1}{2}$ have been modified for use in electron scattering experiments and, further, for heavy-ion spectrometry. Probably the best known of the sector magnets for the magnetic analysis of electron scattering is that due to Chambers and Hofstadter.[49] More recent instruments in this field have, for example, been described by De Jager *et al.*[111] and by Alvarez *et al.*[112] All these sector magnet spectrometers have the angle of deflection and the field index n optimised to reduce as far as possible the second-order aberrations at the final focus. These instruments are therefore sometimes known as 'magic angle' spectrometers. The spectrometer of Alvarez *et al.*,[112] for example, has a focus after an angle of deflection of 220° with a field index $n = 0{\cdot}25$ while the spectrometer of De Jager *et al.*[111] has an angle of deflection of 163·3° with a field index $n = \tfrac{1}{2}$. A detailed study of the properties of the focal plane, both for electron and heavy-ion work, has been given recently by Freiesleben *et al.*[113] Earlier studies of the second-order focusing properties are given in references 109, 110 and 114. In these spectrometers for the study of electron, and heavy-ion, scattering the total bending angle for $n = \tfrac{1}{2}$ is usually reduced from $\pi\sqrt{2}$ to be of the order of π. The solid angles are small compared with the normal double-focusing instruments, being measured in millisteradians in most cases. The momentum range is also limited because of the high magnetic rigidity of the particles being studied. These are not serious disadvantages since usually only a limited momentum range is being studied at any one time and the solid angle accepted from the target has in any case to be small for angular resolution requirements. The time of flight of the particle from the target to the detector at the focal plane of the spectrometer can be an important property of the spectrometer (see section 4.1.8) when used for coincidence studies. This time of flight can also be used to distinguish between particles of the same magnetic rigidity but different mass and charge.[113]

Applications of sector-focusing spectrometers to coincidence and angular correlation studies are probably the most important uses of these instruments. One big advantage of magnetic spectrometers in this context is that they remove the particles of interest to a region where the background is much lower than it is close to the source or the target. This applies to measurements with heavy ions as well as with electrons. In several laboratories twin beta spectrometers have been built for electron–electron studies.[97,110] Practically all single instruments can be used for beta–gamma correlation studies.[97,115–117]

The performance figures vary considerably from one instrument to another. This is a result of the specialisation of the design to suit particular experimental conditions. In the latest single spectrometer built to the design of Ikegami *et al.*[115] a maximum transmission of 1·2 per cent of 4π is obtained under normal operating conditions. At 0·2 per cent transmission a resolution of 0·2 per cent is obtained, giving a figure of merit of 1. The optimum resolution achieved with this instrument is 0·08 per cent for a ^{137}Cs source which is of area 1 × 10 mm^2. In the instrument of Wolm and Wilkinson[116] which has a double wedge magnet, and an orbit which is in effect part of a trochoidal loop, the maximum transmission is 2·3 per cent and the best resolution 0·2 per cent. Figures of merit are somewhat ambiguous for this instrument since the resolution is a complicated function of source size and source opening angles. For a particular source size, an almost linear relationship exists between the values of the transmission and resolution which give optimum figures of merit, and the solid angle of opening at the source. From this relationship a figure of merit of 1·2 for a resolution of 1·5 per cent and a transmission of 1·8 per cent is obtained from a 2 mm diameter source with opening solid angle of 2·3 per cent.

Sectorial magnets have been used for the study of external conversion electrons and Compton electrons produced by high-energy gamma rays. A sector magnet preceded by a quadrupole triplet has been reported by Bezic *et al.*[22] for use in examining Compton electrons from gamma rays in the 10 to 30 MeV region. An overall resolution of 2 per cent is obtained with this instrument which has been used to determine the total photoabsorption cross sections in this region of gamma-ray energies. Other designs of Compton spectrometer using two sectorial magnets have been reported by Mahlein[118] and by Bosi.[119]

Other applications of magnetic sectorial spectrometers to experiments with accelerated electron beams are reviewed by Penner.[120] These include the energy loss spectrometer designed at M.I.T.[120] for use on the electron linear accelerator. Bendel *et al.*[121] report the use of a magnetic sector spectrometer for the examination of the spectra of electrons scattered at 180° to the incident beam direction. Here an auxiliary magnet is used to steer the incident beam to the target and the same auxiliary magnet then deflects the 180° scattered beam into the spectrometer. An important point to note in the context of inelastic electron scattering with $n = \tfrac{1}{2}$ spectrometers is that because of the limited momentum range which can be examined at one time care must be taken in the design of the spectrometer vacuum box and magnet yoke to ensure that the elastic electrons do not strike these parts of the system and then scatter into energy regions of interest in the inelastic spectrum. At high electron energies an important group of experiments involve reactions of the type (e, e′p) in which a proton is scattered out of the target nucleus. Magnetic spectrometers have been used to examine the spectrum of the recoil protons[122] and in some experiments the electrons are also detected in coincidence with these protons by use of a second, electron, spectrometer. These measurements are analogous to the two spectrometer coincidence measurements on (p, 2p) reactions of the type reported by Andrews *et al.*[123]

4.2.7 *Iron-free $\pi\sqrt{2}$ and $\tfrac{1}{2}\pi\sqrt{13}$ beta spectrometers*

Probably the best known and most widely discussed $\pi\sqrt{2}$ double-focusing iron-free spectrometers are the Uppsala[1,124] and Chalk River designs.[85] These instruments achieve a double-focusing condition to a very high order of accuracy. Only brief descriptions of the spectrometers will be given here since they have been reported and commented on in detail elsewhere.[2]

The Uppsala design, shown schematically in figure 4.25 employs two coaxial iron-free solenoids whose lengths are slightly greater than the diameter of the inner solenoid. The current flows in opposite directions in the two coils so that their internal magnetic fields are opposed. However, the external magnetic field of the

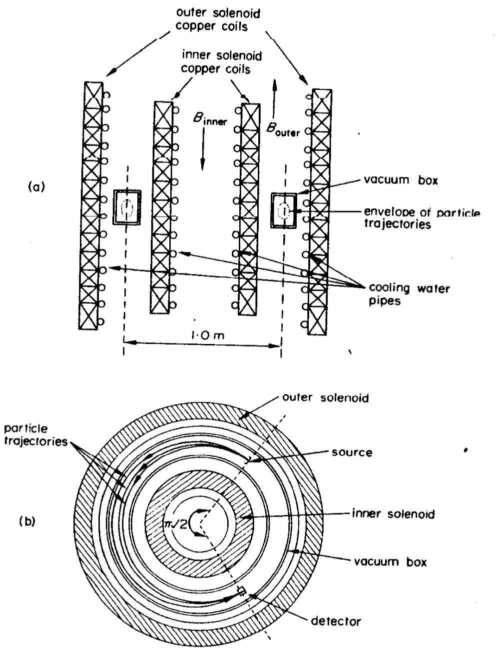

Figure 4.25 (a) Section view of the iron-free spectrometer designed by Siegbahn, reference 1. (b) Plan view of the spectrometer of reference 1

inner solenoid is in the same direction as the internal field in the larger, outer, solenoid so that an addition of field occurs. Figure 4.26 shows how with the superposition of these two fields a $1/r^{1/2}$ radial field dependence can be achieved. The earlier Uppsala design[125] had a central ray radius of 300 mm with inner and outer coil radii of 240 mm and 360 mm respectively. The height of the coils in that instrument was 487 mm. In the latter instrument, of which two were built simultaneously at Berkeley[124] and Uppsala,[1] the central ray has an orbit radius of 500 mm. The spectrometer has an outer radius of inner coil of 382·5 mm and an inner radius of outer coil of 637·5 mm. The height of the coils is 943 mm. Both the earlier 300 mm design and the later 500 mm design have $\beta = {}^3/_8$.

Since these instruments are iron free they are susceptible to interference from local magnetic fields and from the magnetic field of the earth. They are therefore sited in specially constructed buildings which are free of ferromagnetic materials in their construction and are at a considerable distance from any building containing heavy steel equipment.[1] An elaborate Helmholtz coil arrangement is used to balance the earth's field and any other small local variations in magnetic field. To ensure accuracy over long periods of time independent of external temperature conditions the buildings are accurately temperature controlled and heavily insulated.[1] The residual vertical magnetic field at the orbit is of order of 1×10^{-8} T and the horizontal magnetic field of the order of 5×10^{-8} T.

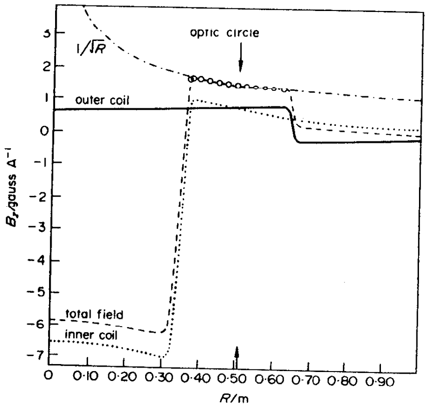

Figure 4.26 Superposition of the fields in the Uppsala iron-free spectrometer. The full line is the field due to the outer coil, the dotted line is that due to the inner coil and the dashed line is the combined field. Taken with permission from reference 1: K. Siegbahn, C. Nordling, S.E. Karlsson, S. Hagstrom, A. Fahlman and I. Andersson, *Nucl. Instr. Methods*, **27**, 173, figure 6 (1964)

The 500 mm instrument at Uppsala can be run either on fully automatic control or on manual control. Since no iron is involved only precision measurements of the coil currents are necessary and these measurements can be made to an accuracy of better than[1] 1 part in 10^5. A specially designed set of baffles[124] are described in detail in reference 1. The best resolution which is obtainable with this 500 mm spectrometer is of the order of 0·012 per cent. Figures of merit for the instrument are ambiguous because of the different baffle arrangements which can be used but a typical figure would be 2·9, from a resolution of 0·045 per cent at 0·13 per cent transmission. The performance is sufficiently good to allow observation of the natural widths of the lines in a conversion electron spectrum. For example, the

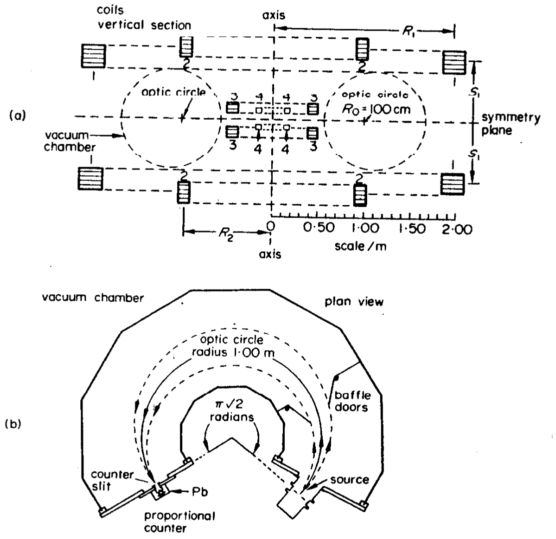

Figure 4.27 (a) A section view of the Chalk River iron-free double-focusing spectrometer. Taken with permission from reference 85: R.L. Graham, G.T. Ewan and J.S. Geiger, *Nucl. Instr. Methods*, 9, 245, figure 2 (1960). (b) A plan view of the Chalk River iron-free double-focusing spectrometer. Taken with permission from reference 85: R.L. Graham, G.T. Ewan and J.S. Geiger, *Nucl. Instr. Methods*, 9, 245, figure 2 (1960)

F line from the ThB spectrum gives a resolution of 0·039 per cent whereas the I line gives 0·012 per cent under identical conditions. A number of other broadened lines have been observed with this instrument.[1]

The Chalk River design[85] approaches the problem of obtaining the double-focusing field profile from a different point of view. Instead of the simple two-coil system a multi-coil system of large dimension is used to achieve the required field profile over a large volume. A schematic view of the Chalk River instrument is shown in figure 4.27 and the resultant field profile obtained from the superposition of the many coils is shown in figure 4.28. The radius of the orbit of the central ray is 1·0 m and the radius of the largest coils is 2 m. The mean separation of these largest coils is 1·337 m. In principle the field is set for $\beta = {}^3\!/_8$ and can be corrected to fourth order by the field coil arrangement provided. A detailed study of the arrangement of the Helmholtz coils for degaussing the active volume of the

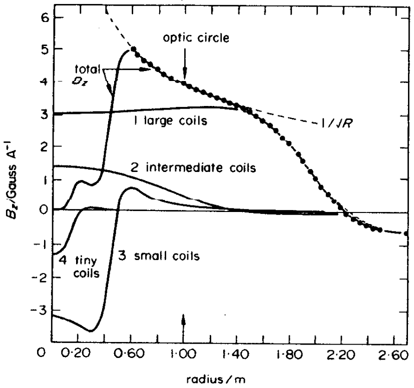

Figure 4.28 The field contributions from the coil sets in the Chalk River spectrometer. Taken with permission from reference 85: R.L. Graham, G.T. Ewan and J.S. Geiger, *Nucl. Instr. Methods*, 9, 245, figure 4 (1960)

spectrometer is given in references 85 and 126. The residual field achieved in the region of the spectrometer orbit is of the order of 10^{-8} T.

An elaborate baffle system is incorporated in the spectrometer to allow its use at high resolution or high transmission. These are discussed in detail in reference 85. The optimum resolution obtainable with this instrument is again of the order of 0·01 per cent and yet again the figures of merit depend rather critically on what baffle settings are being used. A typical resolution of 0·1 per cent for a transmission of 0·2 per cent, giving a figure of merit of 2, can be achieved with high luminosity. An overall luminosity of 0·1 can in principle be obtained but in practice the best experimentally obtained value is smaller than this by about a factor of 10, partly because the optimum source size is difficult to achieve.[85]

A comparison of the Chalk River spectrometer and the 500 mm Berkeley–Uppsala spectrometer is shown in table 4.5. The performances of the two designs are very similar and from a comparison of the absolute values of the experimental results obtained from the two types of spectrometer it is clear that the agreement is well within the errors of 0·02 per cent quoted for these results.[2]

A number of similar instruments have been built. Apart from the identical instruments at Berkeley and Uppsala a 300 mm instrument of similar design to the earlier Uppsala spectrometer has been built at Amsterdam[127] and a scaled-up version with a central ray orbit radius of 500 mm has been built at Belgrade.[128] A smaller version of the Chalk River spectrometer was built by Wolfson *et al.*[129] and another 500 mm version of this spectrometer has been built at Grenoble.[130]

Table 4.5

Spectrometer	Resolution FWHM/%	Transmission/%	Luminosity/cm²	Figure of merit
Chalk River 1 m[**]	0·1	0·3	6×10^{-4}	3·0
	0·013	0·07	2×10^{-5}	5·4
Berkeley–Uppsala 0·5 m[1]	0·1	0·3	$5{\cdot}4 \times 10^{-4}$	3·0
	0·012	0·05	$1{\cdot}8 \times 10^{-5}$	4·1

Note. Resolutions and transmissions are those quoted in references 1 and 85. The figures for luminosity were derived by the author from various other experimental measurements published for these instruments and are not from the original accounts of these spectrometers.

A different version of the multi-coil system to that used at Chalk River has been used in the spectrometer built by Moussa and Bellicard[131] and by Baird *et al.*[132] with performance very similar to the Chalk River spectrometer. Other coil arrangements can of course be used to produce the required field in an iron-free way but the designs discussed briefly here are the ones most commonly used.

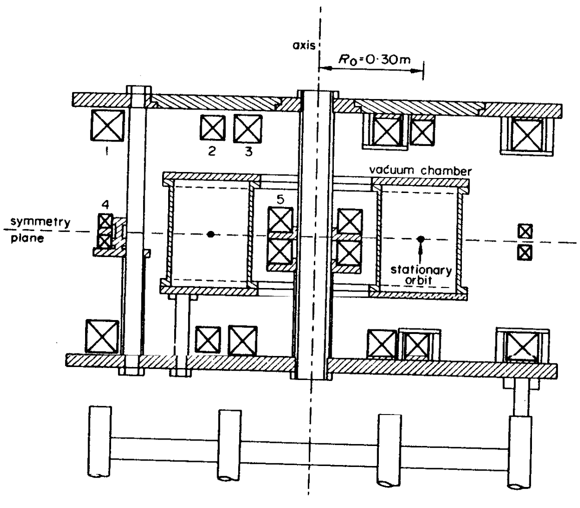

Figure 4.29 A section view of the ½π√13 iron-free spectrometer of reference 134. Taken with permission from: H. Daniel, P. Jahn, M. Kuntze and B. Martin, *Nucl. Instr. Methods*, **82**, 29, figure 5 (1970)

It has been pointed out by several authors that if higher dispersion is required then an angle larger than π√2 (254·5°) is required. Daniel *et al.*[4,133] have described the advantages of a system giving radial focusing at an angle of ½π√13 (324·5°) with a dispersion of 6·5 compared with 4 for the π√2 spectrometer. The system is not double focusing and a curved exit slit of the correct profile is required. Both the transmission and the resolution of such an instrument should be high.

Very recently Daniel *et al.*[134] have reported an iron-free spectrometer which achieves the required field profile using 5 pairs of coils, a section view of which is shown in figure 4.29. The orbit radius of the central ray is 300 mm. Baffles designed to give the maximum transmission with high resolution for the instrument have been fitted.[134] An optimum resolution of 0·013 per cent has been achieved with this spectrometer. Typical values of transmission and resolution are 0·6 per cent and 0·022 per cent respectively giving a figure of merit of 27 with a luminosity of 3×10^{-5}. Performance figures from reference 134 are shown in table 4.6 and should be compared with those shown in table 4.5 for the Chalk River and Berkeley–Uppsala spectrometers.

Table 4.6

Resolution FWHM/%	Transmission/%	Luminosity/cm²	Figure of merit
0·013	0·15	$0{\cdot}75 \times 10^{-5}$	12
0·022	0·60	3×10^{-5}	27
0·030	0·60	$1{\cdot}2 \times 10^{-4}$	20
0·048	1·00	2×10^{-4}	21

Data taken from reference 134.

The figures of merit are higher than those achieved with any other iron-free spectrometer and are comparable with those of the trochoidal spectrometer of Balzer *et al.*[102] This instrument shows what can be achieved if the double-focus condition is relaxed and the exit slit then made to the correct profile.

4.2.8 *Magnetic lens beta spectrometers*

The first attempts to use the axial focusing properties of a solenoid magnetic lens were made by Kapitza[135] and Tricker[136] in 1924. They used what is now usually referred to as a uniform-field long lens. The theory of the focusing properties of such a lens was developed by Busch[37] in 1926. In 1935 Klemperer[137] reported the use of a short lens where the field profile was almost gaussian. The potential of this short lens was developed by a number of workers[47,138,139] and resolutions of the order of 1 per cent were achieved. In parallel with this the long lens was developed by Witcher[38] and eventually gave better figures of merit than the short lens. The advantages of a lens with a field profile which is the inverse of that of the short lens and which gives an intermediate image and a final point focus were pointed out by

Siegbahn.[41,140] The field conditions for this lens to give the focus required were originally found by detailed photographic ray tracing inside the lens.[41]

These lens spectrometers can be iron-free instruments, in which case they require external Helmholtz coils to balance the magnetic field of the earth and other local fields. Alternatively they can have an external iron surround which acts as a return path for the solenoid flux and as a screen for other local fields. These iron sheaths help to reduce the leakage field at the external photomultiplier tubes of the plastic scintillator detectors which are used as electron detectors with many of these instruments.

A modified iron-sheathed lens, with the source outside the magnetic field, for angular correlation studies has been reported by Siegbahn[141] and by Kleinheinz *et al.*[142] who use up to three of these instruments, together with scintillation detector gamma-ray counters, for electron–electron and electron–gamma coincidence and correlation studies.

Details of the ring focus and baffle systems for these lenses have been studied by several authors[38,39,42,46,143,144] as was mentioned in section 4.1. Probably the most widely used baffle system at this ring focus is that designed by Hubert[46] and this type is used, for example, in the iron-sheathed lenses of Kleinheinz *et al.*[142] Detailed calculations of the focusing properties of 35 field profiles have been carried out by Lindgren *et al.*[43] and the characteristics of the ring focus examined in each case for radially extended sources as well as for point sources. In effect, to get good energy resolution one dimension of the source has to be sacrificed so that for high resolution only point sources can be used.[2] However, very high transmissions can be achieved, up to 5 per cent being reported for some lenses.[2]

The coil configurations and field profiles most commonly used for the four lens types discussed briefly above are shown in figure 4.30. All these field profiles have been studied by Lindgren[43] and the optimum condition for source size and ring focus obtained for each. The performance figures for these lens spectrometers are shown in table 4.7. They are typical experimental figures for these spectrometers and are not necessarily the best that has been achieved with each type.

High-resolution examples of the long-lens spectrometer have been built by Schmidt[145] and Jungerman *et al.*[146] Resolutions down to 0·018 per cent have been achieved with these instruments. The short-lens spectrometer of Mann and Payne[144] is typical of what can be obtained with the short-lens field profile and an ultimate resolution of 1·37 per cent has been obtained with a 1·6 mm diameter source for this instrument.

Table 4.7

Lens type	Resolution FWHM/%	Transmission/%	Luminosity	Figure of merit
Long-lens	0·4	2	4×10^{-4}	5
	0·05	0·36	$0{\cdot}6 \times 10^{-5}$	7·2
Short lens	1·37	1·6	$1{\cdot}8 \times 10^{-3}$	1·18
	5·7	1·5	$1{\cdot}8 \times 10^{-3}$	0·26
Intermediate image	3·0	7·0	$1{\cdot}6 \times 10^{-4}$	2·3
	1·6	4·5	1×10^{-4}	2·8
Iron-sheathed with external source[142]	0·324	0·210	2×10^{-5}	0·6
	2·45	1·33	$1{\cdot}33 \times 10^{-4}$	0·54

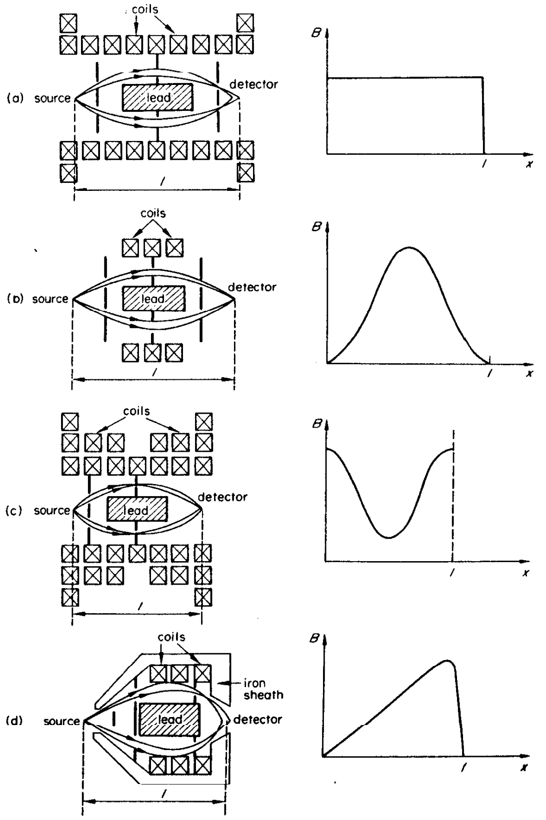

Figure 4.30 Coil arrangements and field profiles in lens spectrometers: (a) long magnetic lens; (b) short magnetic lens; (c) intermediate focus lens; (d) modified iron-sheathed lens

With an intermediate image lens the aim is for high transmission. As a result the energy resolution is not usually as good but 0·5 per cent resolution has been obtained by Alburger[147] in his intermediate image pair spectrometer when used as a normal beta spectrometer. The iron-sheathed spectrometers of Kleinheinz *et al.*,[142] built to a design of Siegbahn,[141] for coincidence and angular correlation studies show up badly where resolution figures are concerned because of the special line shape which arises from the use of the Hubert envelope baffle. This line shape makes FWHM resolution figures somewhat meaningless.[2,142] It is pointed out that because of this line shape the same information can be extracted from a spectrum with 2·5 per cent FWHM resolution as can be extracted from one with 0·3 per cent resolution.[142]

The lens spectrometer is widely used as an external conversion spectrometer, Compton spectrometer and pair spectrometer, the instrument designed by Alburger[147] being an excellent example of the latter use. The coincidence system of Kleinheinz *et al.*,[142] which is shown schematically in its three-spectrometer arrangement in figure 4.31, has been further discussed by Falk *et al.*[148] It is an extremely versatile system and much elegant work has been carried out with it.

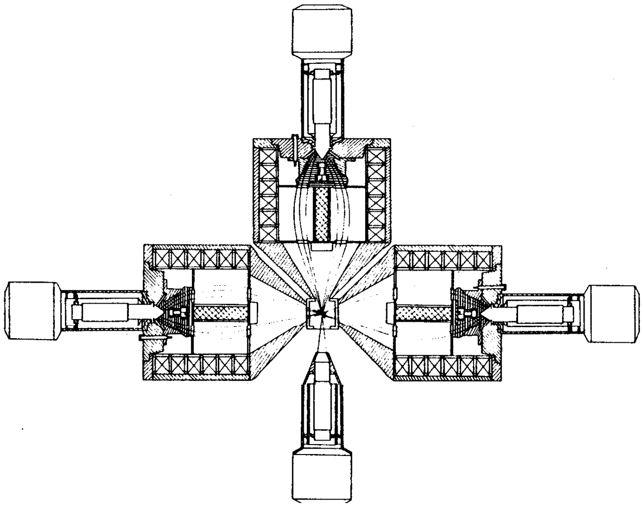

Figure 4.31 The plan view of the three-lens electron coincidence and correlation system of reference 142. Taken with permission from P. Kleinheinz, L. Samuelsson, R. Vukanovic and K. Siegbahn, *Nucl. Instr. Methods*, **32**, 1, figure 4 (1965)

Other designs for coincidence work have been reported but these usually have only one magnetic lens, or at most two lenses back to back.[149–152] Examples of the use of lens spectrometers for the study of pair production, in addition to that described by Alburger,[147] are given in references 153 and 154. Corrections for solid angles in the angular correlation studies with these spectrometers are discussed by Vos *et al.*[155] and a comparison of the experimentally obtained focusing conditions with theory is given by Besev *et al.*[156]

By its construction, the lens spectrometer lends itself to use with very-high-energy elementary particles of sub-proton mass produced in high-energy interactions. The experimental difficulty is primarily in achieving a sufficiently high field. With normal high-conductivity copper coils, the axial field of 1·94 T in a large spectrometer needed to focus particles with a momentum of 500 MeV c^{-1} would require a power input[2] of 10 MW and this would raise many problems. However, the idea of using a superconducting solenoid lens is attractive for this high-momentum work and that it would be a practical proposition with present-day technology is illustrated by the small superconducting solenoid lens described by Shera *et al.*[157]

4.2.9 Detection systems for beta spectrometers

The detection of the particles in a beta spectrometer can be either by electrical methods or by a photographic plate method. If a permanent record is required then obviously the photographic method is best. One problem which occurs in this method is the non-linear response of the normal photographic emulsion. This is important when absolute intensities of lines are required. A method for overcoming this difficulty has been discussed by Albridge *et al.*[68] An alternative photographic method which is ten times more sensitive than the normal spectrometer emulsion plate is the use of the special fine-grain nuclear emulsions developed by Ilford and by Kodak. In this technique the track of each electron is counted and so absolute numbers can easily be obtained.[54] This method has of course been developed to a high degree of precision, with automatic track scanning, in the heavy-ion spectrometers.

From the earliest days of beta spectrometers GM tubes have been used as recorders of the beta particles. These devices will operate quite satisfactorily in a magnetic field, provided the applied voltage is high enough. The main problem is the window thickness required in the tubes. The gas pressure is often about atmospheric, or higher if high-energy electrons are to be collected, and the window must withstand the full pressure differential between the gas filling and vacuum. Post-or pre-acceleration of the low-energy part of the electron spectrum helps to overcome this problem by ensuring that the particles have enough energy to get into the detector. By using a variable-pressure GM tube and an ultra-thin Zapon window electrons down to 4 keV can be detected.[86]

A very large number of spectrometers use scintillation counters for the detection of the electrons. This raises problems because of the effect of the magnetic field on the photomultiplier associated with the scintillator. One solution, used for example by Alburger,[147] is to have very long Perspex light-guides to take the light out of the high-field region to a region where only fringe fields of a few gauss are present. Iron screening is then used locally round the photomultiplier to reduce this field to a very low value. The disadvantage of this system is the loss of light intensity which usually results. An alternative method is to screen the photomultiplier tube heavily with several coaxial soft-iron cylindrical screens and an inner set of mu-metal screens. This is well illustrated in the design of Kleinheinz *et al.*[142] The disadvantage of this system for double-focusing spectrometers of the 'flat' type is that it produces a rather large local variation in the magnetic field of the spectrometer which may be difficult to compensate. For fast coincidence work this scintillator is usually an organic plastic material and various shapes and sizes of these scintillators have been used. In the long magnetic lens spectrometer the scintillator is often a cylinder with its long axis on the field axis. In the intermediate image type it can be a flat disc perpendicular to the field axis or a thin-walled cone with axis on the field axis and with apex pointing into the lens.

The sector magnet spectrometer and the 'orange' spectrometer are especially favourable for the use of a scintillation detector since here the photomultiplier can be outside the main field completely. It should be noted that for angular correlation work the source also should preferably be in a field-free region to prevent perturbation of the correlation due to precession in the field.

When the magnetic spectrometer is used to analyse high-energy electrons scattered by a target placed in an accelerated electron beam then Čerenkov detectors are often used to detect the electrons arriving at the focal plane of the spectrometer.[49] This is a particularly good detection system when fluxes of heavy particles are present, such as fast neutrons. The threshold and directional characteristics of the Čerenkov radiation can then be used to eliminate the recoil protons produced by the neutron flux which would be a serious source of background if a scintillation detector were to be used.

Solid-state detectors offer many advantages for use in magnetic spectrometers. They are relatively insensitive to external magnetic fields and can be used satisfactorily in fields of many tesla.[158] Surface barrier and lithium-drifted silicon detectors are now used in most spectrometers and detailed studies of their peak area response functions for electrons have been made to allow relative and absolute intensities of conversion lines to be determined.[159] Position-sensitive detectors have been used in some beta spectrometers but their position resolution is usually too poor with electrons to make them of much use. A more usual system is an array of narrow surface barrier detectors, each with effectively its own collimation system. Special surface barrier detectors which are hollow cylinders with the active 'front' contact as the outside surface of the cylinder and the 'back' contact as the inner surface of the cylinder have been reported.[160,161] These cylindrical detectors are of special interest for use in 'orange' spectrometers and magnetic lens spectrometers.

The use of spark chambers for position recording in beta-ray spectrometers is at present mainly confined to pair spectrometers,[75,76] although their use with a sector spectrometer for electron scattering studies has been reported.[162] The more recently developed multi-wire proportional counter would seem to offer better spatial resolution and faster response than the spark chamber, with the additional advantage of high-count-rate capability up to 10^5 per second.[163,164] The application of these multi-wire proportional counters to heavy-ion spectrometers has recently been reported[21] but although they have been tested with beta-particle sources their usefulness with beta spectrometers is still to be proved.

4.2.10 Absolute determination of beta-particle energies and the calibration of beta spectrometers

Absolute energy measurements can in principle be made with any spectrometer if the field and the orbit radius are known, giving the BR value for the particle entering the detector. This is not in fact very easily done in practice. The radius can usually be determined accurately from the geometry of the spectrometer but the value of the field B at the orbit may be very difficult to determine. Only in uniform-field semicircular-focusing spectrometers is the measurement of the field easy and an absolute calibration directly obtainable.[2] In most other spectrometers the field is measured at some point away from the radius of the particle, or by measurement of the field coil current, and so only relative BR values and energies can be determined. To make these absolute some reference calibration source is required. This source should have a fairly long half-life, it should have a large number of intense lines whose absolute energies and intensities are well known and it should be readily available in a form suitable for use in a spectrometer. It turns out that by far the best source which satisfies nearly all these requirements is Th(B + C + C″) and detailed tabulations of the lines in the spectrum of this source have been made and are available, for example, in reference 2. These line spectra from the Th(B + C + C″) source cover an energy range from 24 keV to 2·6 MeV.

Another commonly used standard is ^{137}Cs with its intense internal conversion K and L lines and the weaker M and N lines. A somewhat less commonly used one is ^{192}Ir. Energy measurements for most spectrometers are referred to the lines in these reference spectra and any new line is usually calibrated relative to a reference line on either side of it.

In any spectrometer the accuracy is directly related to the quality of the source. Even when all spectrometer aberrations are minimised, the final resolution, and for low energies the absolute energy scale as well, is critically dependent on the energy loss and self-absorption in the source.[2] For absolute measurements with his permanent-magnet uniform-field spectrometer Siegbahn[2] developed a method for evaporating the source onto the source disc in the spectrometer itself so as to obtain the highest accuracy. In general, the problems of obtaining good sources which do not distort the spectrum are identical with those met with in preparing sources for absolute intensity measurements. These problems are discussed in reference 2.