ELECTRIC CONDUCTION IN A MAGNETIC FIELD: THE HALL EFFECT

6.9 When a current flows in a conductor in the presence of a magnetic field, the force $(q/c)\mathbf{v} \times \mathbf{B}$ acts directly on the moving charge carriers. Yet we observe a force on the conductor as a whole. Let's see how this comes about. Figure 6.28*a* shows a section of a metal bar in which a steady current is flowing. Driven by a field $\mathbf{E}$, electrons are drifting to the left with average speed $\bar{v}$, which has the same meaning as the $\bar{u}$ in our discussion of conduction in Chapter 4. The conduction electrons are indicated, very schematically, by the white dots. The black dots are the positive ions which form the rigid framework of the solid metal bar. Since the electrons are negative, we have a current in the y direction. The current density $\mathbf{J}$ and the field $\mathbf{E}$ are related by the conductivity of the metal, σ, as usual: $\mathbf{J} = \sigma E$. There is no magnetic field in Fig. 6.28*a* except that of the current itself, which we shall ignore. Now an external field $\mathbf{B}$ in the x direction is switched on. The state of motion immediately thereafter is shown in Fig. 6.28*b*. The electrons are being deflected downward. But since they cannot escape at the bottom of the bar, they simply pile up there, until the surplus

of negative charge at the bottom of the bar and the corresponding excess of positive charge at the top create an electric field $\mathbf{E}_t$ in which

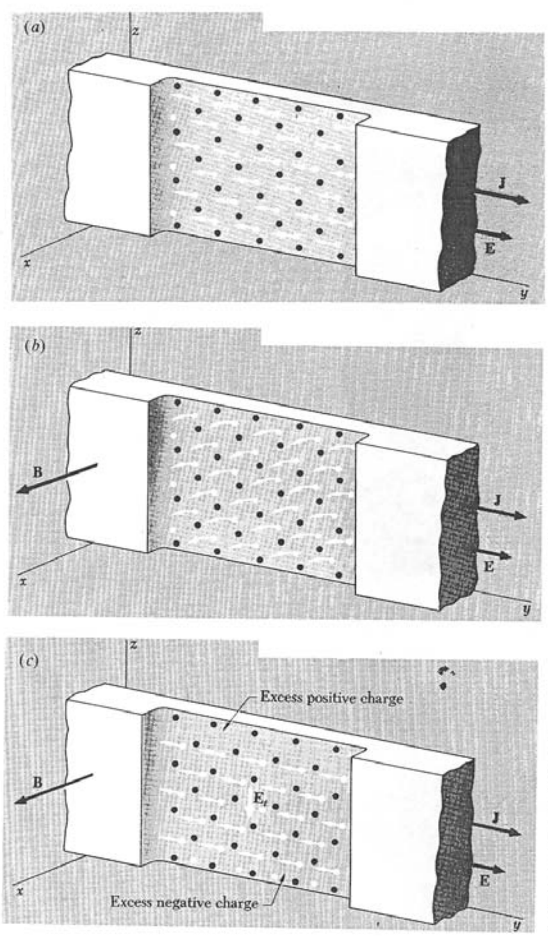


FIGURE 6.28
(*a*) A current flows in a metal bar. Only a short section of the bar is shown. Conduction electrons are indicated (not in true size and number!) by white dots, positive ions of the crystal lattice by black dots. The arrows indicate the average velocity $\bar{v}$ of the electrons. (*b*) A magnetic field is applied to the x direction, causing (at first) a downward deflection of the moving electrons. (*c*) The altered charge distribution makes a transverse electric field E_t. In this field the stationary positive ions experience a downward force.

the upward force, of magnitude eE_t, exactly balances the downward force $(e/c)\bar{v}B$. In the steady state (which is attained very quickly!) the average motion is horizontal again, and there exists in the interior of the metal this transverse electric field $\mathbf{E}_t$, as observed in coordinates fixed in the metal lattice (Fig. 6.28*c*). This field causes a downward force on the positive ions. That is how the force, $(-e/c)\bar{\mathbf{v}} \times \mathbf{B}$, on the electrons is passed on to the solid bar. The bar, of course, pushes on whatever is holding *it*.

The condition for zero average transverse force on the moving charge carriers is

$$\mathbf{E}_t + \frac{\bar{\mathbf{v}}}{c} \times \mathbf{B} = 0 \qquad (64)$$

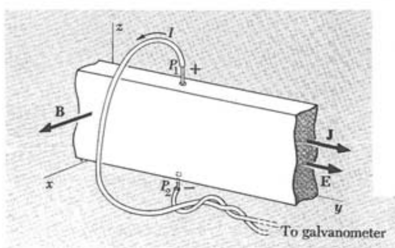
Suppose there are n mobile charge carriers per cm^3 and, to be more general, denote the charge of each by q. Then the current density $\mathbf{J}$ is $nq\bar{\mathbf{v}}$. If we now substitute $\mathbf{J}/nq$ for $\bar{\mathbf{v}}$ in Eq. 64, we can relate the transverse field $\mathbf{E}_t$ to the directly measurable quantities $\mathbf{J}$ and $\mathbf{B}$:

$$\mathbf{E}_t = \frac{-\mathbf{J} \times \mathbf{B}}{nqc} \qquad (65)$$

For electrons $q = -e$, so $\mathbf{E}_t$ has in that case the direction of $\mathbf{J} \times \mathbf{B}$, as it does in Fig. 6.28*c*.

The existence of the transverse field can easily be demonstrated. Wires are connected to points P_1 and P_2 on opposite edges of the bar (Fig. 6.29), the junction points being carefully located so that they are at the same potential when current is flowing in the bar and $\mathbf{B}$ is zero: The wires are connected to a voltmeter. After the field $\mathbf{B}$ is turned on P_1 and P_2 are no longer at the same potential. The potential difference is E_t times the width of the bar, and in the case illustrated P_1 is positive relative to P_2. A steady current will flow around the external circuit from P_1 to P_2, its magnitude determined by the resistance of the voltmeter. Notice that the potential difference would be reversed if the current $\mathbf{J}$ consisted of positive carriers moving to the right rather than electrons moving to the left. Here for the first time we have an experiment that promises to tell us the *sign* of the charge carriers in a conductor.

FIGURE 6.29
The Hall effect. When a magnetic field is applied perpendicular to a conductor carrying current, a potential difference is observed between points on opposite sides of the bar—points which, in the absence of the field, would be at the same potential. This is consistent with the existence of the field E_t inside the bar. By measuring the "Hall voltage" one can determine the number of charge carriers per cubic centimeter, and their sign.



The effect was discovered in 1879 by E. H. Hall who was studying under Rowland at Johns Hopkins. In those days no one understood the mechanism of conduction in metals. The electron itself was unknown. It was hard to make much sense of the results. Generally the sign of the "Hall voltage" was consistent with conduction by negative carriers, but there were exceptions even to that. A complete understanding of the Hall effect in metallic conductors came only with the quantum theory of metals, about 50 years after Hall's discovery.

The Hall effect has proved to be especially useful in the study of semiconductors. There it fulfills its promise to reveal directly both the concentration and the sign of the charge carriers. The n-type and p-type semiconductors described in Chapter 4 give Hall voltages of opposite sign, as we should expect. As the Hall voltage is proportional to B, an appropriate semiconductor in the arrangement of Fig. 6.29 can serve, once calibrated, as a simple and compact device for measuring an unknown magnetic field. An example is described in Problem 6.35.