EXACT RELATIONS AMONG SI AND CGS UNITS

In 1983 the General Conference on Weights and Measures officially redefined the meter as the distance that light travels in vacuum in 1/299,792,458 of a second. The second is defined in terms of a certain atomic frequency in a way that does not concern us here. The nine-digit integer was chosen to make the assigned value of c agree with the most accurate measured value to well within the uncertainty in the latter. Henceforth the velocity of light is, by *definition*, 299,792,458 meters/sec. An experiment in which the passage of a light pulse from point A to point B is timed is to be regarded as a measurement of the distance from A to B, not a measurement of the speed of light.

While this step has no immediate practical consequences, it does bring a welcome simplification of the exact relations connecting various electromagnetic units. As we learn in Chapter 9, Maxwell's equations for the vacuum fields, formulated in SI units, have a solution in the form of a traveling wave with velocity $c = (\mu_0\epsilon_0)^{-1/2}$. The SI constant μ_0 has always been defined exactly as $4\pi \times 10^{-7}$, whereas the value of ϵ_0 has depended on the experimentally determined value of the speed of light, any refinement of which called for adjustment of the value of ϵ_0. But now ϵ_0 acquires a permanent and perfectly precise value of its own, through the requirement that

$$(\mu_0\epsilon_0)^{-1/2} = 299{,}792{,}458 \text{ meters/sec} \qquad (1)$$

In our CGS system no such question arises. Wherever c is involved, it appears in plain view, and all other quantities are defined exactly, beginning with the electrostatic unit of charge, the esu, whose definition by Coulomb's law involves no arbitrary factor.

With the adoption of Eq. 1 in consequence of the redefinition of the meter, the relations among the units in the systems we have been using can be stated with unlimited precision. These relations are given in Table E.1 for the principal quantities we deal with. In the table the symbol "3" stands for the precise decimal 2.99792458; the symbol "9" stands for the 17-digit square of that number, 8.9875517873681764.

The exact numbers are uninteresting and for our work quite unnecessary. That "3" happens to be so close to 3 is sheer luck, an accidental consequence of the length of the meter and the second. When 0.1 percent accuracy is good enough we need only remember that "300 volts is a statvolt" and "3×10^9 esu is a coulomb." Much less precisely, but still within 12 percent, a capacitance of 1 cm is equivalent to 1 picofarad.

An important SI constant is $(\mu_0/\epsilon_0)^{1/2}$, which is a resistance in ohms. Its precise value is stated below the table. One tends to remember it, and even refer to it, as "377 ohms." It is the ratio of the electric field strength E, in volts/meter, in a plane wave in vacuum, to the strength in amperes/meter of the accompanying magnetic field H. For this reason the constant $(\mu_0/\epsilon_0)^{1/2}$ is sometimes denoted by Z_0 and called, rather cryptically, the *impedance of the vacuum*. In a plane wave in vacuum in which E_{rms} is the rms electric field in volts/meter, the mean density of power transmitted, in watts/m^2, is E_{rms}^2/Z_0.

The logical relation of the SI electrical units to one another takes on now a slightly different aspect. Before the redefinition of the meter it was customary to designate one of the electrical units as *primary*, in this sense: Its precise value could, at least in principle, be established by a procedure involving the SI mechanical and metrical units only. Thus the ampere, to which this role has usually been assigned, was defined in terms of the force in newtons between parallel currents, using the relation Eq. 7′ of Chapter 6. This was possible because the constant μ_0 in that relation has the precise value $4\pi \times 10^{-7}$. Then with the ampere as the primary electrical unit, the coulomb was defined precisely as 1 ampere-second. The coulomb itself, owing to the presence of ϵ_0 in Coulomb's law, was not eligible to serve as the primary unit. Now with ϵ_0 as well as μ_0 assigned an exact numerical value, the system can be built up with any unit as the starting point. All quantities are in this sense on an equal footing, and the choice of a primary unit loses its significance. Never a very interesting question anyway, it can now be relegated to history.

TABLE E.1

	In SI units		In CGS units
Energy	1 joule	=	10^7 erg
Force	1 newton	=	10^5 dyne
Electric charge	1 coulomb	=	"3" $\times$ 10^9 esu
Electric current	1 ampere	=	"3" $\times$ 10^9 esu/sec
Electric potential	"3" $\times$ 10^2 volts	=	1 statvolt (1 erg/esu)
Electric field E	"3" $\times$ 10^4 volts/m	=	1 statvolt/cm (1 dyne/esu)
Magnetic field B	1 tesla	=	10^4 gauss (10^4 dynes/esu)
Magnetic field H	1 ampere/m	=	$4\pi \times 10^{-3}$ oersted
Capacitance	1 farad	=	"9" $\times$ 10^{11} cm
Inductance	1 henry	=	("9" $\times$ $10^{11})^{-1}$ sec^2/cm
Resistance	1 ohm	=	("9" $\times$ $10^{11})^{-1}$ sec/cm

$\mu_0 = 4\pi \times 10^{-7}$ ohm-sec/m $\epsilon_0 = (4\pi \times$ "9" $\times 10^9)^{-1}$ sec/ohm-m

$(\mu_0/\epsilon_0)^{1/2} = 40\pi \times$ "3" ohms $= 376.73 \cdots$ ohms

"3" $= 2.9979245800000 \cdots$ "9" = "3" $\times$ "3"