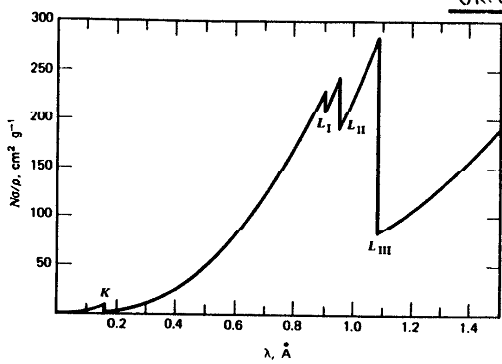


**Fig. 25-1.** Mass absorption coefficient $N\sigma/\rho$ for platinum as a function of photon wavelength.

understood classically; electromagnetic radiation impinging on the electron accelerates it, and the radiation emitted by the accelerated charge is the scattered radiation. The classically calculated *Thomson* cross section is

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \tag{25-29}$$

In quantum mechanics, the scattering amplitude (matrix element) must be proportional to $e^2$, since two photons are involved. Since the perturbation in the Hamiltonian is

$$\frac{e}{mc}\mathbf{p}\cdot\mathbf{A}(\mathbf{r},t) + \frac{e^2}{2mc^2}\mathbf{A}^2(\mathbf{r},t) \tag{25-30}$$

when both terms in the expansion of (22-11) are kept, we see than an $e^2$ contribution to the scattering amplitude can come from two sources.

(i) the first source is a first-order contribution from the term $e^2\mathbf{A}^2(\mathbf{r},t)/2mc^2$.

(ii) the second source is a second-order perturbation term from the coupling $e\mathbf{p}\cdot\mathbf{A}(\mathbf{r},t)/mc$. Since we have not developed the second order perturbation formalism, we will restrict ourselves to stating the results.

(a) At threshold, with the gauge that we have been using, $\nabla\cdot\mathbf{A}(\mathbf{r},t) = 0$, the whole amplitude comes from the term involving $e^2\mathbf{A}^2(\mathbf{r},t)/2mc^2$.

(b) The matrix element in second order has the form

$$-\sum_n \frac{\langle f|e\mathbf{p}\cdot\mathbf{A}/mc|n\rangle\langle n|e\mathbf{p}\cdot\mathbf{A}/mc|i\rangle}{E_n - E_i} \tag{25-31}$$



where the "sum" over intermediate states "$n$" also implies integration over all the momenta, when "$n$" includes continuum states.[6] It is not enough to include intermediate one-electron states corresponding to the sequence

$$\gamma_i + e_i \to e' \to \gamma_f + e_f$$

and the intermediate states containing an electron and two photons, corresponding to the process

$$e_i + \gamma_i \to \gamma_i + \gamma_f + e' \to \gamma_f + e_f$$

It turns out that it is necessary to include the possibility of the "virtual" creation of an electron-positron pair by the incident photon, followed by the annihilation of the positron by the incident electron, with the emission of the final photon, as in

$$e_i + \gamma_i \to e_i + e_f + e^{+\prime} \to \gamma_f + e_f$$

and the process

$$e_i + \gamma_i \to e_i + \gamma_i + \gamma_f + e_f + e^{+\prime} \to \gamma_f + e_f$$

The calculation leads to the *Klein-Nishina formula*

$$\sigma = 2\pi\left(\frac{e^2}{mc^2}\right)^2\left\{\frac{1+x}{x^2}\left[\frac{2(1+x)}{1+2x} - \frac{1}{x}\log(1+2x)\right] + \frac{1}{2x}\log(1+2x) - \frac{1+3x}{(1+2x)^2}\right\}$$

$$x = \frac{\hbar\omega}{mc^2} \tag{25-32}$$

which is in excellent agreement with experiment. At low frequencies this becomes

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2(1-2x) \tag{25-33}$$

and at high frequencies ($x \gg 1$) this reads

$$\sigma = \pi\left(\frac{e^2}{mc^2}\right)^2\frac{1}{x}\left(\log 2x + \tfrac{1}{2}\right) \tag{25-34}$$

Thus the Compton cross section, too, drops off at high energies. At energies above a few MeV, the dominant absorptive process is *pair production*.

It is a remarkable fact that a photon at high enough energies, $\hbar\omega > 2mc^2$ can "materialize" into an electron and a positron (Fig. 25-2). The latter can be

[6] The fact that (25-31) looks like an off-diagonal version of the second-order energy shift is, of course, no accident.

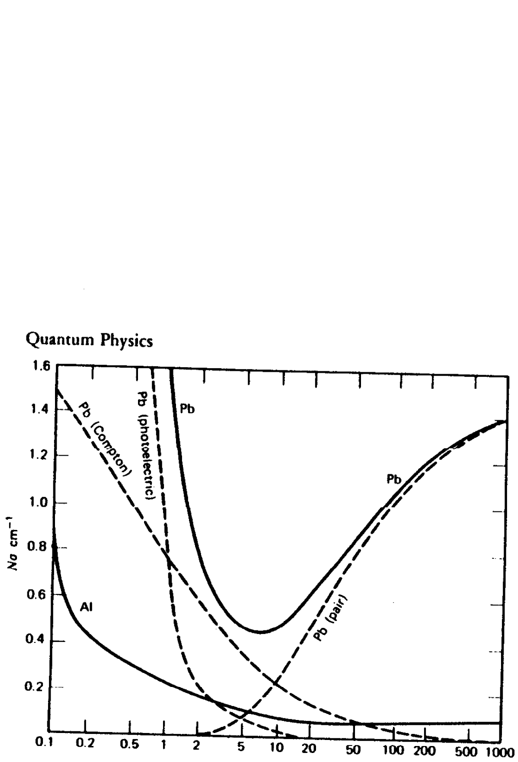


**Fig. 25-2.** Total absorption coefficient for lead and aluminum as a function of energy, in units of the electron rest energy (0.51 MeV). The photoelectric cross section for Al is negligible on the scale depicted here.

properly called an "antielectron"; it has the same mass as the electron and the same spin, but its charge and magnetic moment have the same value with *opposite sign* as those for the electron, and the nonrelativistic coupling with the electromagnetic field is obtained by the replacement of $\mathbf{p}$ by $\mathbf{p} - e\mathbf{A}(\mathbf{r},t)/c$. Such a materialization can only occur in the presence of a third particle, a nucleus, for example, since energy and momentum conservation cannot hold for the process

$$\gamma \rightarrow e + e^+$$

To see this without going through a long kinematical calculation, consider the inverse process $e + e^+ \rightarrow \gamma$ in the center of mass frame. The electron and positron have equal and opposite momenta, so that the final state has energy $2(m^2c^4 + p^2c^2)^{1/2}$ and momentum 0. A photon of energy $E$ must carry momentum $E/c$. If there is a nucleus present, it can absorb momentum and energy (for a massive nucleus this will be very small, $p^2/2M$), so that it becomes possible to balance energy and momentum.

The calculation of

$$\gamma + \text{nucleus} \rightarrow e + e^+ + \text{nucleus}$$

is beyond the scope of this book. The theory of quantum electrodynamics that



is used in these calculations also shows that we can transfer particles from one side of the equation to the other, provided we change the transferred particles to their antiparticles. Thus one predicts that

$$\text{nucleus} + e^\pm \rightarrow \text{nucleus} + e^\pm + \gamma$$

should also occur, with a matrix element very closely related to that of pair production. This is in agreement with experiment, and the last process is responsible for *cosmic ray showers.*

An incident $\gamma$ ray of very high energy (it may come from the decay $\pi^0 \rightarrow 2\gamma$, with the $\pi^0$ produced when a primary cosmic ray proton hits a nucleus at the top of the atmosphere) will make a pair, with each member carrying roughly half the original energy. Each member can produce a photon, as indicated above,[7] and the end products can make further photons and pairs. Showers coming from extremely high energy events occurring at the top of the atmosphere can cover areas of several square miles! Less spectacular showers in counters are used to identify photons or electrons. An incident particle that is charged, but much heavier, will be deflected less, and will therefore radiate less.

Detailed calculations show that energy lost in material through these processes follows the law

$$E(x) = E_{\text{inc}}\, e^{-x/L} \tag{25-35}$$

where the "radiation length" is given by

$$L = \frac{(m^2c^2/\hbar^2)\, A}{4Z^2\alpha^3 N_0 \rho \log (183/Z^{1/3})} \tag{25-36}$$

where $N_0 = 6.02 \times 10^{23}$ is Avogadro's number, $m$ is the electron mass, $A$ is the atomic weight, $Z$ is the charge of the nucleus, and $\rho$ is the density of the material in grams per cubic centimeter. The "pair production length" is given by

$$L_{\text{pair}} = \frac{9}{7} L \tag{25-37}$$

The formula is not good for very low $Z$. Typical values of $L$ are

| | |
|---|---|
| Air | 330 m |
| Al | 9.7 cm |
| Pb | 0.53 cm |

*Bremsstrahlung* is the dominant energy loss mechanism for electrons at high energies. At low energies ionization dominates. Lack of space keeps us from discussing this essentially classical effect.

[7] This process is called *Bremsstrahlung*, and can be understood classically; a charge deflected in the Coulomb field of the nucleus is accelerated, and hence radiates.