MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Lecture Notes 8
PROBLEMS OF THE CONVENTIONAL (NON-INFLATIONARY) HOT BIG BANG MODEL

INTRODUCTION:

By the 1970s the conventional hot big bang model was well established. It was supported strongly by the observation of Hubble expansion, by the existence of a thermal background of microwave radiation, and by the measured abundances of the lightest isotopes of atomic nuclei. Nonetheless the model suffered from at least two serious problems — the horizon/homogeneity problem and the flatness problem — which form the subject of this set of lecture notes. In the next set of lecture notes we will discuss a third problem — the magnetic monopole problem — which arises if one assumes that particle physics at very high temperatures is described by a type of theory called a grand unified theory.

THE HORIZON/HOMOGENEITY PROBLEM:

The horizon problem is the difficulty in explaining the large-scale uniformity of the observed universe. This large-scale uniformity is most evident in the microwave background radiation. This radiation appears slightly hotter in one direction than in the opposite direction, by about one part in a thousand — but this nonuniformity can be attributed to our motion through the background radiation. Once this effect is subtracted out, using best-fit parameters for the velocity, it is found that the residual temperature pattern is uniform to about one part in 10^5. Uniformity in temperature is not necessarily mysterious, as any isolated system will evolve over time towards a state of thermal equilibrium, which is state of uniform temperature. For example, when we take a hot slice of pizza out of the oven, it begins immediately to cool toward room temperature. It turns out, however, that the standard processes of thermal equilibration cannot, in the context of the conventional hot big bang model, explain the uniformity of temperature in the universe. The problem is that in this model the universe evolves much too quickly to allow this uniformity to be achieved by the usual processes by which a system approaches thermal equilibrium.

In order to see this, we will not need to know anything about the details of thermal transport processes. We will use only the fact that no physical process can cause matter, energy, or information to move faster than the speed of light. Thus, no process can carry energy beyond the "horizon distance," which was defined in Lecture Notes 4 as the present distance of the furthest particles from which light has had time to reach us, since the beginning of the universe. The issue of horizons was introduced into cosmology by W.

Rindler in 1956,* and the horizon problem is described (without using the words "horizon problem") in two well-known textbooks: S. Weinberg, *Gravitation and Cosmology*, J. Wiley and Sons (1972), and C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, W.H. Freeman & Co. (1973).

In Lecture Notes 4 we found that this horizon distance is given by 3ct for the case of a matter-dominated $k = 0$ universe, and in Lecture Notes 6 we showed that it is 2ct for the case of a radiation-dominated $k = 0$ universe. We also showed in Lecture Notes 6 that the universe became matter-dominated at about 50,000 years after the big bang, and that the cosmic microwave background radiation decoupled from the rest of matter at $t_d \approx 370,000$ years after the big bang. Thus, the radiation decoupled well after the universe became matter-dominated, so to a good approximation the horizon distance at this time is given by $\ell_h(t_d) \approx 3ct_d \approx 1,100,000$ light-years. (I am using the convention that a subscript "d" denotes the value of a given quantity at the time of decoupling, and a subscript "0" will denote its value at the present time.)

For comparison, we would like to calculate the distance, at time t_d, between our own galaxy (or, more precisely, the matter which will later become our galaxy) and the site of emission of the cosmic background radiation that we are now receiving. To do this, we can make use of some previous results. In Lecture Notes 6 we learned that the cosmic microwave background radiation was emitted (or more precisely, decoupled) when the temperature was 3000 °K. Since the current temperature is about 2.7 °K, and since $aT = constant$ as the universe expands, it follows that the redshift at the time of decoupling is given by

$$1 + z = \frac{a(t_0)}{a(t_d)} = \frac{3000°\text{K}}{2.7°\text{K}} \approx 1100 , \tag{8.1}$$

where I have also made use of Eq. (2.15) to relate $1 + z$ to the ratio of the scale factors. Knowing $1+z$ we can find the present distance between our galaxy and the site of emission of the radiation, using the result of Problem 3, Problem Set 2. You found there that for a matter-dominated $k = 0$ universe, the present physical distance of an object seen at redshift z is given by

$$\ell_p(t_0) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] . \tag{8.2}$$

Using $H_0 = 67.7$ km-s^{-1}-Mpc^{-1}, one finds that $H_0^{-1} \approx 14.4 \times 10^9$ yr and $\ell_p(t_0) \approx 28.0 \times 10^9$ light-yr. That is, the region of emission of the cosmic background radiation that we are presently observing is a spherical shell of matter at just a little bit less than

* W. Rindler, "Visual horizons in world-models," *Monthly Notices of the Royal Astronomical Society*, Vol. 116, pp. 662–677 (1956), http://adsabs.harvard.edu/abs/1956MNRAS.116..662R

the present horizon distance, $\ell_h(t_0) \approx 2cH_0^{-1}$. But physical distances vary with time as $a(t)$, so the physical radius of this shell of matter at the time of decoupling t_d is given by

$$\begin{aligned} \ell_p(t_d) &= \frac{a(t_d)}{a(t_0)}\ell_p(t_0) \\ &\approx \frac{1}{1100} \times 28.0 \times 10^9 \text{ lt-yr} \approx 2.55 \times 10^7 \text{ lt-yr} . \end{aligned} \tag{8.3}$$

Thus, at the time of emission of the cosmic background radiation, the region of emission was a spherical shell with a radius many times larger than the horizon distance. Specifically, the radius was $\ell_p(t_d)/\ell_h(t_d) \approx 2.55 \times 10^7$ lt-yr/1.1×10^6 lt-yr ≈ 23 times larger than the horizon distance.

To state the problem most clearly, suppose that one detects the cosmic microwave background in a certain direction in the sky, and suppose that one also detects the radiation from precisely the opposite direction. At the time of emission, the sources of these two signals were separated from each other by about 46 horizon distances. Thus it is absolutely impossible, within the context of this model, for these two sources to have come into thermal equilibrium by any physical process.

This problem is not a genuine inconsistency of the conventional hot big bang model — if the uniformity is assumed in the initial conditions, then the universe will evolve uniformly. The "problem" is that one of the most salient features of the observed universe — its large-scale uniformity — cannot be explained by the conventional model; it simply must be assumed as an initial condition. The suggestion then is not that the conventional model is wrong, but rather that it is incomplete.

The calculation described above depended on our approximation that the universe was matter-dominated at all relevant times, which is a rather crude approximation. Nonetheless, since 46 is so far from one, we can be confident that this problem will not go away with a more careful calculation.

THE FLATNESS PROBLEM:

A second problem of the conventional hot big bang model is known as the flatness problem — it refers to the difficulty in understanding why the present value of Ω (the ratio of the mass density ρ to the critical mass density ρ_c) is close to 1. Today we know that Ω_0 is equal to 1 at least to within about 1/2% — more precisely, by combining their own data with data from other experiments, the Planck team* concluded that

$$\Omega_0 = 0.9993 \pm 0.0037 \tag{8.4}$$

* N. Aghanim et al. (Planck Collaboration), "Planck 2018 results, VI: Cosmological parameters," Table 4, Column 5, arXiv:1807.06209.

at the 95% confidence level. Historically, however, the flatness problem was already severe even in 1980, when we only knew that Ω_0 was somewhere in the range

$$0.1 < \Omega_0 < 2 \quad \text{(circa 1980)}. \tag{8.5}$$

The key fact is that the value $\Omega = 1$ is a point of unstable equilibrium, something like a pencil balancing on its point. The word "equilibrium" implies that if Ω is ever ***exactly*** equal to one, it will remain equal to one forever — that is, a flat ($k = 0$) universe remains a flat universe. However, if Ω is ever slightly larger than one, it will rapidly grow toward infinity; if Ω is ever slightly smaller than one, it will rapidly fall toward zero. Thus, in order for Ω to be anywhere near 1 today, the value of Ω in the early universe must have been extraordinarily close to one.

Like the horizon problem, this problem is not a genuine inconsistency of the conventional model. If one is willing to assume that the value of Ω in the early universe was extraordinarily close to one, then the model will describe how the universe evolves to have a value of Ω today within the accepted range. The problem is again the lack of explanatory or predictive power of the model — the extraordinary closeness of Ω to unity in the early universe cannot be explained, but must simply be assumed as an initial condition. The mathematics behind the flatness problem was undoubtedly known to almost anyone who has worked on the big bang theory from the 1920's onward, but apparently the first people to consider it a problem in the sense described here were Robert Dicke and P.J.E. Peebles, who published a discussion in 1979.†

To work out the evolution of Ω, we need only recast some relations that we have already derived. The key relation is the first-order Friedmann equation for the evolution of the scale factor, derived in Lecture Notes 3:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} , \tag{8.6}$$

where the overdot represents a derivative with respect to t. Recalling that $\rho_c = 3H^2/8\pi G$ (see Eq. (3.33)), one can divide both sides of the equation by H^2 to give

$$1 = \frac{\rho}{\rho_c} - \frac{kc^2}{a^2H^2} , \tag{8.7}$$

which can be rewritten as

$$\Omega - 1 = \frac{kc^2}{a^2H^2} , \tag{8.8}$$

† R.H. Dicke and P.J.E. Peebles, "The big bang cosmology — enigmas and nostrums," in **General Relativity: An Einstein Centenary Survey**, eds: S.W. Hawking and W. Israel, Cambridge University Press (1979).

where we recalled that

$$\Omega \equiv \frac{\rho}{\rho_c} \,. \tag{8.9}$$

But the evolution of a and H are already understood. For a matter-dominated $k = 0$ universe, we know from Lecture Notes 3 that $a \propto t^{2/3}$, and therefore $H = \dot{a}/a = 2/(3t)$. It follows that

$$\Omega - 1 \propto \left(\frac{1}{t^{2/3}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t^{2/3} \quad \text{(matter-dominated).} \tag{8.10}$$

For a radiation-dominated $k = 0$ universe, on the other hand, we know from Lecture Notes 6 that $a \propto t^{1/2}$, so $H = 1/(2t)$. This gives

$$\Omega - 1 \propto \left(\frac{1}{t^{1/2}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t \quad \text{(radiation dominated).} \tag{8.11}$$

We can now trace the evolution of Ω backward in time. From the Planck limit on Ω_0 listed in Eq. (8.4), we can conclude that

$$|\Omega_0 - 1| < .01 \,. \tag{8.12}$$

We could have written $|\Omega_0 - 1| < 0.005$, but for simplicity we will consider only integer powers of ten. Again for simplicity, we will assume that the universe can be described in terms of a matter-dominated era and a radiation-dominated era, both nearly flat so that Eqs. (8.10) and (8.11) apply, with a sharp transition between the two. The transition occurs at about 50,000 years after the big bang, while we estimate the current age of the universe as 13.8×10^9 years. Using Eq. (8.10), we conclude that the value of Ω at 50,000 years is given by

$$(\Omega - 1)_{t=50{,}000\ \text{yr}} \approx \left(\frac{50{,}000}{13.8 \times 10^9}\right)^{2/3} (\Omega_0 - 1) \approx 2.36 \times 10^{-4}\, (\Omega_0 - 1) \,. \tag{8.13}$$

Let us now calculate the value of Ω at 1 second after the big bang. One second is a particularly interesting time, because it is the earliest time for which we have direct evidence that the conventional hot big bang model seems to be working. The processes which lead to nucleosynthesis begin at about $t = 1$ sec, and the predictions derived from big bang nucleosynthesis calculations are in good agreement with observations.

To find the value of Ω at one second, begin by noting that

$$\frac{1\ \text{sec}}{50{,}000\ \text{yr}} = \frac{1\ \text{sec}}{50{,}000\ \text{yr}} \times \frac{1\ \text{yr}}{3.16 \times 10^7\ \text{sec}} = 6.33 \times 10^{-13} \,.$$

Combining Eqs. (8.11) and (8.13), we find that

$$\begin{aligned} (\Omega - 1)_{t=1\ \text{sec}} &\approx 6.33 \times 10^{-13}\, (\Omega - 1)_{t=50{,}000\ \text{yr}} \\ &\approx 1.49 \times 10^{-16}\, (\Omega_0 - 1) \,. \end{aligned} \tag{8.14}$$

Using Eq. (8.12), we conclude that

$$|\Omega - 1|_{t=1\ \text{sec}} < 10^{-18} \,. \tag{8.15}$$

Thus, at one second after the big bang, the value of Ω must have been equal to one to an accuracy of 18 decimal places! The flatness problem is the statement that the conventional hot big bang model provides no explanation of how the value of Ω came to be tuned so precisely. Note that if we had put ourselves back into the setting of 1980, using Eq. (8.5) instead of (8.4), we still would have reached the extraordinary conclusion that Ω at one second must have equaled 1 to an accuracy of 16 decimal places.

As we will see shortly, the horizon and flatness problems provide much of the motivation of the inflationary universe model, which gives a simple resolution to both of them.