MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

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Lecture Notes 10

THE NEW INFLATIONARY UNIVERSE

INTRODUCTION:

The new inflationary universe is a scenario in which the mass density of at least a small patch of the early universe becomes dominated by the potential energy of a scalar field, in a state which is sometimes called a *false vacuum.* This peculiar form of energy leads to a negative pressure, and hence a repulsive gravitational force, driving the region into a period of exponential expansion, during which it expands by many orders of magnitude — hence the name "inflationary". The word "new" refers to a modification of my original proposal[1] which was suggested independently by Linde[2] and by Albrecht and Steinhardt.[3] They suggested a new mechanism by which the exponential expansion phase could be ended, solving some crucial problems that existed in my original proposal. The inflationary model is very attractive because it offers possible solutions to the horizon/homogeneity problem, the flatness problem, and the magnetic monopole problem, which were discussed in Lecture Notes 8 and 9. It also predicts that the universe should be flat to high accuracy, a fact which has now been verified to an accuracy of 0.4%.[4] In addition, inflationary models give predictions for the properties of the small ripples that are observed in the cosmic microwave background (CMB) radiation. As we will discuss at the end of these notes, the predictions of the simplest inflationary models are beautifully in agreement with what has been measured. If inflation is correct, it would mean that particle physics mechanisms are responsible for the production of essentially all the matter, energy, and entropy in the observed universe.

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[1] A. H. Guth, "The inflationary universe: A possible solution to the horizon and flatness problems," *Physical Review D*, vol. 23, pp. 347–356 (1981), available at http://prd.aps.org/abstract/PRD/v23/i2/p347_1, or in its original preprint form at http://slac.stanford.edu/pubs/slacpubs/2500/slac-pub-2576.pdf.

[2] A. D. Linde, "A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems," *Physics Letters B*, vol. 108, pp. 389–393 (1982), available at http://www.sciencedirect.com/science/article/pii/0370269382912199.

[3] A. Albrecht and P. J. Steinhardt,"Cosmology for grand unified theories with radiatively induced symmetry breaking," *Physical Review Letters,* vol. 48, pp. 1220–1223 (1982), available at http://prl.aps.org/abstract/PRL/v48/i17/p1220_1, or with an MIT certificate at http://prl.aps.org.libproxy.mit.edu/abstract/PRL/v48/i17/p1220_1.

[4] P. A. R. Ade et al. (Planck Collaboration), "Planck 2015 results, XIII: Cosmological parameters," Table 5, Column 6, Astronomy & Astrophysics vol. 594, article A13 (2016), arXiv:1502.01589.

SCALAR FIELDS AND THE FALSE VACUUM:

The (original) inflationary universe scenario was developed to solve the magnetic monopole problem, but it quickly became clear that the scenario might solve all three of the problems discussed in Lecture Notes 8 and 9. The scenario contained the basic ingredients necessary to eliminate these problems, but unfortunately the scenario also contained one fatal flaw: the exponential expansion was terminated by a phase transition that occurred by the random nucleation of bubbles of the new phase, very similar to the way that water boils. It was found that this violent boiling would lead to a grossly inhomogeneous universe that looks nothing like our universe. This difficulty — which came to be called the *graceful exit problem* — was summarized in the original paper[1] (in a section credited to Erick Weinberg and Harry Kesten as well as me), and was later discussed in detail by Erick Weinberg and me[5] and by Stephen Hawking, Ian Moss, and John Stewart.[6] Fortunately, the graceful exit problem is completely avoided in a variation known as the new inflationary universe, developed independently by Andrei Linde[2] (then at the Lebedev Physical Institute in Moscow, now at Stanford) and by Andreas Albrecht and Paul Steinhardt.[3] (Albrecht and Steinhardt were both at the University of Pennsylvania at the time of their discovery; now Albrecht is at UC Davis, and Steinhardt is at Princeton.)

At the end of these notes I will also briefly describe chaotic inflation, a version of inflation proposed by Linde in 1983.[7] There are now hundreds of versions of inflation, but they are essentially all variants of new or chaotic inflation.

In order for the new inflationary scenario to occur, the underlying particle theory must contain a scalar field ϕ. The potential energy function $V(\phi)$, which represents the potential energy per unit volume, must have a plateau. This plateau is usually taken to be at $\phi \approx 0$, and $\phi = 0$ is usually assumed to be a local maximum of $V(\phi)$. $V(\phi)$ must be very flat in the vicinity of $\phi = 0$. In the example shown below, V is assumed to depend only on $|\phi|$.

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[5] A. H. Guth and E. J. Weinberg, "Could the universe have recovered from a slow first order phase transition?" *Nuclear Physics B,* vol. 212, pp. 321–364 (1983), available at http://www.sciencedirect.com/science/article/pii/0550321383903073, or with an MIT certificate at http://www.sciencedirect.com.libproxy.mit.edu/science/article/pii/0550321383903073.

[6] S. W. Hawking, I. G. Moss, and J. M. Stewart, "Bubble collisions in the very early universe," *Physical Review D,* vol. 26, pp. 2681–2693 (1982), available at http://prd.aps.org/abstract/PRD/v26/i10/p2681_1, or with an MIT certificate at http://prd.aps.org.libproxy.mit.edu/abstract/PRD/v26/i10/p2681_1.

[7] Andrei D. Linde, "Chaotic Inflation," Physics Letters, vol.~129B, pp.~177-181 (1983).

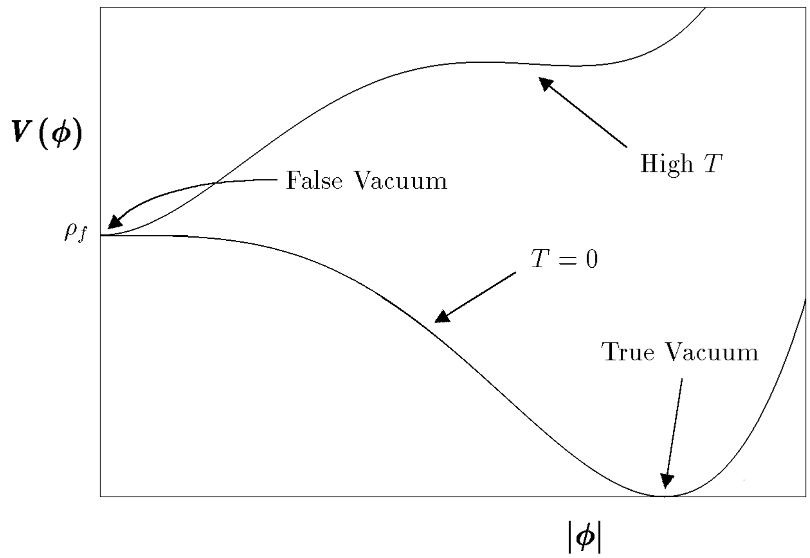

Figure 10.1: The potential energy function (labeled "$T = 0$") and the high-temperature finite-temperature effective potential (labeled "High T") for a scalar field that could drive new inflation.

To discuss the issues one at a time, I will first discuss the physical properties of a scalar field of the type described in the previous paragraph, and then we will consider the role that such a field might play in the early universe.

In most theoretical models of this type, one finds that at high temperature T the thermal equilibrium value of ϕ lies at $\phi = 0$. At high temperatures the field will actually fluctuate wildly, but in most theoretical models the ***average*** value is predicted to be zero. A potential energy function of this general form is shown as Figure 10.1. The curve labeled "High T" is a graph of what is called the finite-temperature effective potential, which is actually a graph of the free energy per unit volume; it will not be important for us to know exactly what free energy is, but to interpret the graph we should keep in mind that the free energy is minimized in the thermal equilibrium state. For these purposes we can treat the energy density (and pressure) of the true vacuum as zero, even though we learned in Lecture Notes 7 that they are apparently not. Taking $\Omega_{\text{vac}} = 0.691$ and $h_0 = 0.677$ from Table 7.1 of Lecture Notes 7 and using Eq. (3.34) from Lecture Notes 3 for the critical mass density, we find that the vacuum energy density of our universe is about

$$\begin{aligned} \rho_{\text{vac}} &= \Omega_{\text{vac}}\rho_c \approx 0.691 \times 1.88 \times (0.677)^2 \times 10^{-26}\ \text{kg/m}^3 \\ &= 5.95 \times 10^{-27}\ \text{kg/m}^3 = 5.95 \times 10^{-30}\ \text{g/cm}^3\ . \end{aligned} \tag{10.1}$$

We will soon see that this number is totally negligible compared to the huge energy densities that we expect for early universe inflation.

The scalar field ϕ that drives the inflation was originally taken to be the Higgs field of a grand unified theory, but it now seems very unlikely that his could work. The Higgs fields are required to have relatively strong interactions in order to induce spontaneous symmetry breaking, which is why the Higgs fields were introduced in the first place. These interactions generically lead to large quantum fluctuations in the evolution of the field, which in turn lead to unacceptably large inhomogeneities in the mass density of the universe. Most inflationary models assume, therefore, the existence of another scalar field, similar to the Higgs field but much more weakly interacting. This field is usually called the *inflaton*.[8]

So, in thermal equilibrium at high temperatures, one expects the scalar field to have a mean value around zero. If the system cools, the thermal excitations will disappear, and the scalar field will find itself in a state of essentially zero temperature, with $\phi \approx 0$. This state is called the false vacuum, and its peculiar properties are the driving force behind the inflationary model.

The false vacuum is clearly unstable, as ϕ will not remain forever at a local maximum of $V(\phi)$. However, if $V(\phi)$ is sufficiently flat, then the time that it takes for ϕ to move away from $\phi = 0$ can be very long compared to the time scale for the evolution of the early universe. Thus, for these purposes the false vacuum can be considered metastable. Furthermore, while ϕ remains near zero, the energy density remains fixed near $V(0)$, and cannot be lowered even if the universe is expanding. It is this property that motivates the name, "false vacuum." To a particle physicist, the vacuum is defined as the state of lowest possible energy density. The adjective "false" is used here to mean "temporary," so a false vacuum is a state which temporarily has the property that its energy density cannot be lowered.[9]

Since the false vacuum has $\phi = 0$ and no other excitations, the mass density has a fixed value which is determined by the potential energy function $V(\phi)$. For a typical

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[8] While it had been thought for many years that the inflaton could under no circumstances be a Higgs field of any sort, in 2008 Fedor Bezrukov and Mikhail Shaposhnikov proposed that the Higgs field of the standard model of particle physics could serve as the inflaton, if one assumed that in addition to its known interactions, it also has "non-minimal" interactions with gravity — i.e., interactions beyond what is required by the equivalence principle. See F. Berukov and M. Shaposhnikov, "The Standard Model Higgs Boson as the inflaton," Physics Letters B, vol. 659, pp. 703–706 (2008), arXiv:0710.3755 [hep-th].

[9] Historically, the phrase "false vacuum" was first used to refer to a state in which the scalar field was at a local minimum of the potential energy function, so the state could decay only by quantum mechanical tunneling. Here I have stretched the definition a bit, using the phrase to describe a scalar field which, although still quite stable, is near a local maximum of the potential energy function.

grand unified theory, this value can be estimated in terms of the GUT energy scale $E_{\rm GUT} \approx 10^{16}$ GeV by using dimensional analysis:

$$\rho_{\rm f} \approx \frac{E_{\rm GUT}^4}{\hbar^3 c^5} = 2.3 \times 10^{84}\,{\rm kg/m}^3 = 2.3 \times 10^{81}\,{\rm g/cm}^3 \ . \tag{10.2}$$

(Thus the energy density of our vacuum, estimated in Eq. (10.1), is smaller by more than 100 orders of magnitude.)

The pressure p of the false vacuum is completely determined by the fact that, on the time scales of interest, its energy density cannot be lowered. To see that a constant energy density implies a negative pressure, remember the conservation of energy equation derived in Problem 4 of Problem Set 6:

$$\dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) \ . \tag{10.3}$$

If $\dot\rho = 0$, this equation implies immediately that

$$\boxed{p = -\rho_{\rm f} c^2 \ .} \tag{10.4}$$

(We used this same argument in Lecture Notes 7, when we were discussing vacuum energy density and the cosmological constant.)

To understand this result from first principles, think of an imaginary piston that is filled with false vacuum and surrounded by ordinary true vacuum, as shown below, in Fig. 10.2:

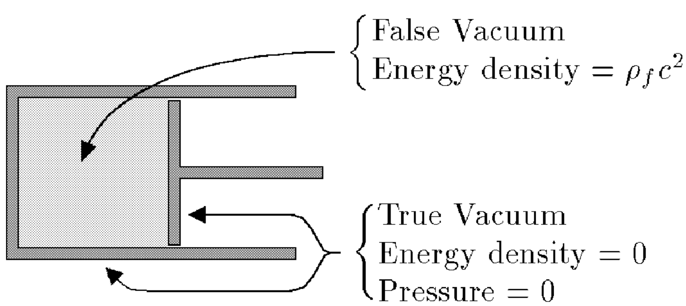

Figure 10.2: A piston used for a thought experiment to show that the pressure of a false vacuum state is the negative of its energy density.

Since this is a thought experiment, we can imagine that the "true vacuum" outside the piston genuinely has zero energy density and zero pressure. If one prefers not to be so imaginative, the energy density and pressure of our vacuum are in any case totally negligible on the scales that are relevant here. Suppose now that the piston is pulled out so that the volume of the chamber increases by ΔV. We assume that the walls of the box are designed to guarantee that the region inside remains completely filled with false vacuum. The energy of the system then increases by $\rho_{\rm f} c^2 \Delta V$, and therefore the agent that moved the piston must have done precisely this amount of work.

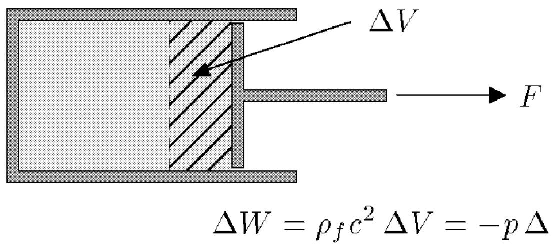

Figure 10.3: The piston of the thought experiment is pulled out, enlarging the chamber. The energy density of the false vacuum inside the chamber is fixed, so the energy in the chamber goes up. The energy must come from the agent that pulled on the piston. For the agent to do positive work, the pressure inside the chamber must be negative.

Since the pressure on the outside is zero, the agent must be pulling against a negative pressure, which would oppose the motion. Quantitatively, since the work done is $-p\Delta V$, it follows that $p = -\rho_{\rm f} c^2$, confirming the previous result.

The large negative pressure creates a gravitational repulsion, exactly as we discussed in Lecture Notes 7 in the context of a cosmological constant. The gravitational repulsion can be seen in the second order differential equation for a, the second order Friedmann equation,

$$\ddot{a} = -\frac{4\pi}{3} G \left(\rho + \frac{3p}{c^2} \right) a \ , \tag{10.5}$$

which implies that both the pressure and the energy density normally contribute to the slowing of the cosmic expansion. For the false vacuum, however, the large negative pressure leads to $\rho + 3p/c^2 < 0$, and it follows that $\ddot{a}$ is ***positive***. The false vacuum creates a gravitational repulsion which causes the growth of the scale factor a to accelerate. It is this repulsion which will drive the colossal expansion of the inflationary scenario. The equations are the same as those for a cosmological constant, except that the false vacuum energy density disappears when the scalar field rolls off the hill in the potential energy diagram, while the vacuum energy associated with a cosmological constant is permanent.

THE NEW INFLATIONARY UNIVERSE:

We can now go through the new inflationary scenario step by step. The starting point of a cosmological scenario is, unfortunately, still somewhat a matter of taste and philosophical prejudice. Some physicists find it plausible to assume that the universe began in some highly symmetrical state. Many others, however, consider it more likely that the universe began in a highly chaotic state, since the number of chaotic configurations is presumably much larger. One advantage of the inflationary scenario, from my point of view, is that it appears to allow a wide variety of starting configurations.

We can begin by discussing what would happen if the early universe were in thermal equilibrium, at least in the sense of having regions of approximately horizon size in which thermal equilibrium held. In that case, inflation could begin if the universe was hot ($kT > 10^{16}$ GeV) in at least some of these regions, and if at least one of these hot regions were expanding rapidly. In the hot regions, thermal equilibrium would imply $<\phi> = 0$, where $<\phi>$ denotes the mean value of the field ϕ as it undergoes its thermal fluctuations. Rapid expansion would cause these regions to cool, and the scalar field would settle down to a cool state in which the field is trapped on the plateau of the potential energy hill. The expansion must be rapid enough so that the cooling of the scalar field occurs before the region recollapses under the influence of gravity.

Thermal equilibrium would make things simple, but we said earlier that the inflaton field must interact very weakly, to avoid generating overly large quantum fluctuations. For such a weakly interacting field, a fairly straightforward calculation of collision rates shows that the mean time between collisions would be long compared to the age of the universe at the onset of inflation. Thus there is no compelling reason to assume thermal equilibrium, although — in the absence of a theory that fixes the initial conditions — one could assume anything one wants. For inflation to start, the minimal assumption would be that there existed at least some regions of high energy density with $<\phi> \approx 0$, and that at least one of these regions was expanding rapidly enough so that ϕ became trapped in the false vacuum.

The above paragraphs describe the new inflationary universe with a hot beginning, but there are certainly other possibilities. Linde has also proposed the idea of chaotic inflation,[7] in which inflation is driven by a scalar field which is initially chaotic but far from thermal equilibrium. In this scenario inflation happens while the scalar field rolls down a gentle hill in the potential energy diagram, so the potential energy diagram need not have a plateau. Alexander Vilenkin[10] (of Tufts University) and Linde[11] have separately investigated speculative but attractive scenarios in which the universe is created by

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[10] A. Vilenkin, "The Birth of Inflationary Universes," Physical Review D, vol. 27, p. 2848 (1983). With an MIT certificate, click here.

[11] A. D. Linde, "Quantum creation of the inflationary universe," Lettere al Nuovo Cimento, vol. 39, pp. 401-405 (1984) With an MIT certificate, click here.

a quantum tunneling event, starting from a state of absolutely nothing. In these models the universe enters directly into a de Sitter phase. In a similar spirit James Hartle (of the University of California at Santa Barbara) and Stephen Hawking (of Cambridge University) have proposed a unique quantum wave function for the universe,[12] incorporating dynamics which leads to an inflationary era.

Although a wide variety of scenarios have been proposed to describe the onset of inflation, an important feature of inflation is that all these scenarios lead to similar if not identical predictions. Once inflation starts, the colossal expansion dilutes away the evidence of how it began. Later I will discuss the phenomenon of *eternal inflation,* which carries this idea of dilution to an extreme. We will see that for almost all inflationary models, once inflation starts, it never stops. Instead it goes on producing "pocket universes" forever. This eternal aspect of inflation presumably erases all traces of how inflation began, and it also obviates the question of whether the conditions leading to inflation are likely. As long as the probability that inflation can start is nonzero, and as long as there is no other mechanism that can compete, it appears (at least to this author) that there are no other questions about initial conditions that need to be answered. An ultimate theory of the origin of the universe would still be very interesting, intellectually, but most likely it would not affect in any way the consequences of inflation.

To continue with the description of the new inflationary scenario, we assume that there exists a region which is sufficiently homogeneous, isotropic, and flat to be described by a flat Robertson–Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^{\,2} \ , \tag{10.6}$$

and the equation of motion becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho \ . \tag{10.7}$$

The solution is given by

$$\boxed{a(t) = \text{const} \times e^{\chi t} \ ,} \tag{10.8}$$

where

$$\boxed{\chi = \sqrt{\frac{8\pi}{3} G\rho_\text{f}} \ .} \tag{10.9}$$

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[12] J. B. Hartle and S. W. Hawking, "Wave function of the universe," Physical Review D, vol. 28, p. 2960 (1983). With an MIT certificate, click here.

This exponential expansion is of course the hallmark of the inflationary model. (For our parameters, $\chi^{-1} \approx 10^{-37}$ sec.) Such a space is called a de Sitter space.

We of course cannot expect to find a region of the early universe that is exactly homogenous, isotropic, and flat, so it is important to know that it is enough to come close. As long as a region meets these criteria approximately, the behavior will be governed by what has been called the *cosmological no-hair conjecture*,[13] which holds that the region will evolve so that it locally resembles exact de Sitter space. As long as $p = -\rho c^2 =$ *constant*, which will hold as long as the scalar field ϕ is near its false vacuum value, the space will start to expand and any initial particle density will be diluted. Any initial distortion of the metric is stretched (i.e., redshifted) until it is no longer locally detectable. This behavior can be proven quite generally in a linearized perturbation analysis, and has also been seen to hold in some specific solutions with large perturbations. There is no proof that the early universe must contain regions that start inflating, but it seems very plausible.

An important property of a de Sitter region, which helps to ensure its durability, is the presence of *event horizons*. These are different from the horizons that we have been discussing since Lecture Notes 4, which are technically called *particle horizons*, and refer to the possibility that two objects can be far enough apart so that light from one object would not have had enough time since the big bang to reach the other. The event horizon of de Sitter space can be seen by calculating, in the metric described by Eqs. (10.6) and (10.8), the coordinate distance that light can travel between times t_1 and t_2:

$$\Delta r(t_1, t_2) = \int_{t_1}^{t_2} \frac{c}{a(t)}\,\mathrm{d}t = \frac{c}{\mathrm{const}} \int_{t_1}^{t_2} e^{-\chi t}\,\mathrm{d}t = \frac{c}{\mathrm{const}\,\chi}\left[e^{-\chi t_1} - e^{-\chi t_2}\right] . \tag{10.10}$$

The point is that this distance is limited even as $t_2 \to \infty$. Note that

$$\lim_{t_2 \to \infty} a(t_1)\,\Delta r(t_1, t_2) = c\chi^{-1} . \tag{10.11}$$

Physically, this means that if two objects at rest in these coordinates are separated by a physical distance more than $c\chi^{-1}$, a light pulse emitted by one object will never reach the other. This in turn means that if a de Sitter region is large compared to $c\chi^{-1}$, then the effect of inhomogeneities from outside the region cannot penetrate into the region any further than a shell of thickness $c\chi^{-1}$. Once the de Sitter region is large compared to $c\chi^{-1}$, it is impervious to outside influences.

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[13] R. M. Wald, "Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant," *Physical Review D*, vol. 28, pp. 2118–2120 (1983), available at http://prd.aps.org/abstract/PRD/v28/i8/p2118_1, or with an MIT certificate at http://prd.aps.org.libproxy.mit.edu/abstract/PRD/v28/i8/p2118_1.

As the inflating region continues to exponentially expand, the mass density of the inflaton field is fixed at ρ_f . Thus, the total energy of the inflaton field is increasing! If the inflationary model is right, the energy of the inflaton field is the source of essentially all the matter, energy, and entropy in the observed universe.

This creation of energy seems to violate our naive notions of energy conservation, but we must remember that there is also an energy associated with the cosmic gravitational field— the field by which everything in the universe is attracting everything else, thereby slowing down the cosmic expansion. Even in Newtonian mechanics one can see that the energy density of a gravitational field is ***negative***. To see this, note that the gravitational field is strengthened as one brings masses together from infinity, but the potential energy of the system is lowered as objects are brought together under the influence of the attractive force. Thus the stronger field corresponds to a lower energy. A good analogy is the electrostatic field, since Coulomb's law is very similar to Newton's law. By calculating how much work needs to be done in pushing charges to create a specified configuration of a static electric field, it is possible to show that the energy density stored in an electric field is given by

$$u_{\text{electrostatic}} = \frac{1}{2}\epsilon_0 \left|\vec{E}\right|^2 \tag{10.12a}$$

or

$$u_{\text{electrostatic}} = \frac{1}{8\pi} \left|\vec{E}\right|^2 \ , \tag{10.12b}$$

depending on what units you are using. The calculation for Newtonian gravity is essentially identical, giving

$$\boxed{u_{\text{Newton}} = -\frac{1}{8\pi G} \left|\vec{g}\right|^2 \ .} \tag{10.13}$$

The sign difference arises from the sign difference in the force law: two positive charges repel, while two positive masses attract. In the context of inflation, the energy stored in the gravitational field becomes more and more negative as the universe inflates, while the energy stored in "matter" (everything except gravity) becomes more and more positive. The total energy remains constant, and very small— perhaps it is exactly equal to zero.

After the region has undergone exponential expansion for some time, inflation must somehow end, at least in the region that is going to describe our visible universe. The scalar field is in an unstable configuration, perched at the top of the hill of the potential energy diagram of Fig. 10.1. It will undergo fluctuations due to thermal and/or quantum effects. Some fluctuations begin to grow, and at some point these fluctuations become large enough so that their subsequent evolution can be described by the classical equations of motion. I will use the term "coherence region" to denote a region within which the scalar field is approximately uniform. The coherence regions are irregular in shape, and their initial size is typically of order $c\chi^{-1}$. Note that $c\chi^{-1}$ is only about 10^{-14} proton diameters; the entire observed universe will evolve from a region of this size or smaller.

The scalar field ϕ then "rolls" down the potential energy function shown in Fig. 10.1, obeying the classical equations of motion derived from general relativity. As long as the spatial variations in ϕ are small, these classical equations take the form

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \ . \tag{10.14}$$

(The derivation of Eq. (10.14) is a straightforward application of general relativity, but it is a little beyond the scope of this course.) If the initial fluctuation is small, then the flatness of the potential for $\phi \approx 0$ will ensure that the rolling begins very slowly. Note that the second term on the left-hand-side of Eq. (10.14) is a damping term, helping to slow down the speed of rolling. As long as $\phi \approx 0$, the mass density ρ remains about equal to $\rho_{\rm f}$, and the exponential expansion continues. The expansion occurs on a time scale χ^{-1}, while the time scale of the rolling is much slower. This "slow roll" of the scalar field is the crucial new feature in the ***new*** inflationary universe.

For the scenario to work, it is necessary for the length scale of homogeneity to be stretched from $c\chi^{-1}$ to at least about 10 cm before the scalar field ϕ rolls off the plateau of the potential energy diagram. This corresponds to an expansion factor of about 10^{28}, which requires about 65 time constants (χ^{-1}) of expansion. The expected duration of the expansion depends on the precise shape of the scalar field potential, and models have been constructed which yield much more than the minimally required amount of inflation.

When the ϕ field reaches the steep part of the potential, it falls quickly to the bottom and oscillates about the minimum. The time scale of this motion is a typical GUT time of $\hbar/E_{\rm GUT} \approx 7 \times 10^{-41}$ sec, which is very fast compared to the expansion rate. The scalar field oscillations are then quickly damped by the couplings to the other fields, and the energy is rapidly converted into a thermal equilibrium mixture of particles. (From a particle point of view, the scalar field oscillations correspond to a state of spinless particles, just as an oscillating electromagnetic field corresponds to a state of photons. The damping of the scalar field is just the field theory description of the decay of these particles into other kinds of particles.) The release of this energy reheats the region back to a temperature which can be of order $kT \approx 10^{16}$ GeV, or can be much lower, depending on the strength of the interactions. The universe is continuing to expand and cool as the gas of particles approaches a state of thermal equilibrium, so the reheat temperature is low if this process of thermalization is slow, and high if it is quick.

From here on the standard scenario takes over. The era of inflation has set up precisely the initial conditions that had previously been assumed in standard cosmology. You can check that a region of radius ≈ 10 cm, at a temperature $kT \approx 10^{16}$ GeV, will become large enough by the time T falls to 2.7 K to encompass the entire observed universe.

CHAOTIC INFLATION:

While I have described the new inflationary model, because I think it is the simplest version to understand, there are now many variants of inflationary models. One very important variant is known as *chaotic inflation*,[7] invented by Andrei Linde in 1983. Linde realized that in fact inflation does not require a plateau in the potential energy diagram, but can in fact happen with a potential energy function as simple as

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \ , \tag{10.15}$$

which in fact describes a non-interacting particle of mass m. If the field ϕ is started at a large enough value, then sufficient inflation can occur as the scalar field rolls towards $\phi = 0$. Linde initially proposed that the scalar field could start at a large value in some places due to "chaotic" initial conditions. Later he showed that quantum fluctuations can cause these models to also undergo eternal inflation, which will be discussed below, so the question of initial conditions is perhaps irrelevant.

SOLUTIONS TO THE COSMOLOGICAL PROBLEMS:

Let me now explain how the three problems of the standard cosmological scenario discussed in Lecture Notes 8 and 9 are avoided in the inflationary scenario. First, let us consider the horizon/homogeneity problem. The problem is clearly avoided in this scenario, since the entire observed universe evolves from a single coherence region. This region had a size of order $c\chi^{-1}$ at the time when the fluctuation began to grow classically. This size is much smaller than the sizes that are relevant in the standard model at these times, and the region therefore had plenty of time to come to a uniform temperature before the onset of inflation. As long as there are about 65 or more time constants of exponential expansion, then the exponential expansion causes this very small region of homogeneity to grow to be large enough to encompass the observed universe.

The flatness problem is avoided by the dynamics of the exponential expansion of the coherence region. As ϕ begins to roll very slowly down the potential, the evolution of the metric is governed by the mass density ρ_{f}. Assuming that the coherence region (or at least a small piece of it) can be approximated by a Robertson-Walker metric, then the scale factor evolves according to the standard Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} \ , \tag{10.16}$$

where $k = +1$, -1, or 0 depending on whether the region approximates a closed, open, or flat universe, respectively. (There could also be perturbations, but the cosmological no-hair theorem guarantees that they would die out quickly.) In this language, the flatness problem is the problem of understanding why the kc^2/a^2 term on the right-hand-side is so extraordinarily small, compared to the other terms. But as the coherence region expands exponentially, the mass density ρ remains very nearly constant at $\rho_{\rm f}$, while the kc^2/a^2 term is suppressed by at least a factor of $\left(10^{28}\right)^2 = 10^{56}$. Since the equation must continue to hold, the term on the left-hand side must remain nearly constant, like the mass density. This provides a "natural" explanation of why the value of the kc^2/a^2 term immediately after the phase transition is smaller than that of the other terms by a tremendous factor.

Except for a very narrow range of parameters, this suppression of the curvature term will vastly exceed that required by present observations. This leads to the prediction that the kc^2/a^2 term of Eq. (10.16) should remain totally negligible until the present era, and even far into the future. This implies that the value of Ω today is expected to be equal to one with a high degree of accuracy.

The inflationary prediction that $\Omega = 1$ seemed to be at odds with observation until 1998, with the discovery of the dark energy. Astronomers never found enough matter to make up a critical mass density, although there was always some room for uncertainty. Some inflationary theorists constructed versions of inflation that could lead to an open universe; this could be arranged by choosing the parameters to that inflation proceeds for just long enough to solve the flatness problem, but not so long that it flattened the universe completely.

But the situation changed dramatically in 1998 with the Supernova Type Ia measurements, which indicated the presence of a cosmological constant or a very slowly evolving scalar field that could simulate a cosmological constant. In either case, the total energy in this new component of the universe is just what is needed to complete the inventory for a flat universe. The best current estimate of Ω_0 is based on the Planck satellite data[4] for the anisotropies of the cosmic microwave background radiation, combined with several other astronomical observations, giving $\Omega_0 = 0.9992 \pm 0.0040$.

Finally, we turn to the monopole problem. Recall that in the standard scenario, the tremendous excess of monopoles was produced by the disorder in the Higgs field (i.e., by the Kibble mechanism). There is no known way to prevent the Kibble mechanism from operating, but as long as inflation occurs after or during the process of monopole formation, the monopoles will be diluted enormously. During inflation the volume of the coherence region increases by a factor of about $\left(10^{28}\right)^3 = 10^{84}$ or more, which is enough to convert the monopole glut into a situation where no monopoles will be seen.

RIPPLES IN THE COSMIC MICROWAVE BACKGROUND

After subtracting a contribution attributed to the motion of the Earth through the cosmic microwave background, the temperature of the CMB appears to be uniform in all directions to an accuracy of about 1 part in 100,000. Nonetheless, at the level of 1 part in

100,000, there are anisotropies (i.e., non-uniformities) that have by now been measured to high precision.

The huge stretching of inflation tends to smooth everything out. Any density of particles that might be present before inflation is diluted away, so that during inflation the energy density is dominated by the energy density of the false vacuum state. If there was any curvature in space itself before inflation began, the effect of inflation is to stretch out those curves. As one stretches a sphere the surface gets flatter and flatter, and the same is actually true for any curved space. So, when inflation is described in the context of classical general relativity, the result of inflation would be an almost completely smooth space. There was a period of about a year, in the very early days of inflation, when this appeared to be a serious problem. If inflation left the universe almost completely smooth, then there would be no way for galaxies to form.

But quantum mechanics came to the rescue. The idea that quantum mechanics might be responsible for the structure of the universe goes back at least to Andrei Sakharov, the Russian nuclear physicist and political activist, who put forward the idea in 1965.[14] In 1981 Mukhanov and Chibisov[15] revived Sakharov's idea in a modern context, studying the density perturbations generated in a closely related model proposed by Alexei Starobinsky[16] in 1980. The original work on density perturbations arising from scalar-field-driven inflation centered around the Nuffield Workshop on the Very Early Universe, Cambridge, U.K., June-July 1982, organized by Gary Gibbons and Stephen Hawking. There was much animated discussion and disagreement during the workshop, but in the end everyone agreed on the answer. There were four papers[17] that came out of the workshop, laying the foundations for calculating density perturbations arising from inflation.

The important feature of quantum mechanics in this context is that it is intrinsically probabilistic. So, while the classical approximation of inflation theory predicts a

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[14] A. D. Sakharov, "The initial stage of an expanding universe and the appearance of a nonuniform distribution of matter," Zh. Eksp. Teor. Fiz. 49, 345 (1965) [JETP Lett. 22, 241 (1966)].

[15] V. F. Mukhanov and G. V. Chibisov, "Quantum fluctuations and a nonsingular universe," Pis'ma Zh. Eksp. Teor. Fiz. 33, 549 (1981) [JETP Lett. 33, 532 (1981)].

[16] A. A. Starobinsky, "A new type of isotropic cosmological models without singularity," Phys. Lett. B, vol. 91, p. 99 (1980) .

[17] S.W. Hawking, "The development of irregularities in a single bubble inflationary universe," Physics Letters B, vol. 115, p. 295 (1982), (with an MIT certificate, click here); A. A. Starobinsky, "Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations," Physics Letters B, vol. 117, p. 175 (1982); A. H. Guth and S.-Y. Pi, "Fluctuations in the new inflationary universe," Physical Review Letters, vol. 49, p. 1110 (1982) (with an MIT certificate, click here); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, "Spontaneous creation of almost scale-free density perturbations in an inflationary universe," Physical Review D, vol. 28, p. 679 (1983) (with an MIT certificate, click here).

completely smooth universe, the quantum theory implies that the matter density will be almost uniform, but due to quantum uncertainties the density will be a little higher than average in some places, and a little lower than average in others. These uncertainties are just the ripples that are needed to allow galaxy formation to proceed, and they are just what is needed to compare with observations of the ripples, starting in 1992. Of course quantum uncertainties are not usually significant on macroscropic scales, so it seems bizarre that quantum fluctuations can be responsible for the large-scale structure of the universe. This is made possible, however, by the extremely rapid expansion during inflation, which stretches the quantum fluctuations from very short length scales, where we expect them to be strong, to macroscopic and even astronomical length scales.

Inflation is of course not a unique theory, since we do not know exactly what the inflaton field is, or exactly what equations of motion it obeys. We don't even know that inflation was driven by a single inflaton, as there may have been two or more. Thus, the detailed predictions for density perturbations arising from inflation are model-dependent, meaning that different assumptions about the inflaton will lead to different predictions. Nonetheless, there is a wide class of "simple" inflationary models which give very similar predictions for the spectrum of the density fluctuations. The word "spectrum" here has pretty much the same meaning it would have for sound waves: the perturbations can be broken up into components with definite wavelengths, and the "spectrum" is a description of how the intensity varies with wavelength. (For sound waves we might be more likely to use frequency rather than wavelength, but for cosmological density perturbations we have no choice but to use wavelength — we don't see oscillations, and we expect oscillations only in some cases.) For the ripples on the CMB, the wavelength is measured in degrees, not in meters, since we are seeing a pattern on the sky as a function of polar angles θ amd ϕ.

The "simple" inflationary models that give similar results are more technically called single field slow-roll models, and they are characterized by the facts that there is a single inflaton, and that, during the period when relevant density perturbations are created, both $H = \dot{a}/a$ and $\partial V/\partial\phi$ are nearly constant and the $\ddot{\phi}$ term of Eq. (10.14) is small compared to the other two terms. The overall magnitude of the density perturbations, on the other hand, depends on more of the details of the inflaton potential energy function, so at present there is no inflationary prediction for the magnitude.

The ripples in the CMB are measured most easily from space, although ground-based measurements can also be significant, especially at very short angular wavelengths, for which high angular resolution is needed. So far there have been three satellite experiments that have been completely dedicated to measuring the properties of the CMB. The first was the Cosmic Background Explorer (COBE), launched by NASA in 1989, 15 years after planning began in 1974. In January, 1990, the COBE group announced their first measurements of the CMB spectrum, showing that it agreed beautifully with the expected black-body spectrum (recall Figure 6.5 in Lecture Notes 6). In April of

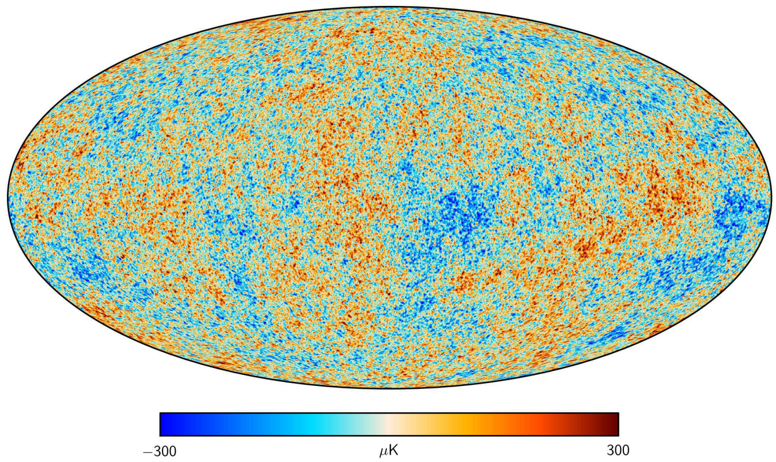

Figure 10.4: The cosmic microwave background radiation as detected by the *Planck* satellite.[18] After correcting for the motion of the Earth, the temperature of the radiation is nearly uniform across the entire sky, with average temperature $T_{\rm cmb} = 2.726$ K. Tiny deviations from the average temperature have been measured; they are so small that they must be depicted in a color scheme that greatly exaggerates the differences, to make them visible. As shown here, blue spots are slightly colder than $T_{\rm cmb}$ while red spots are slightly warmer than $T_{\rm cmb}$, across a range of $\Delta T/T_{\rm cmb} \sim 10^{-4}$.

1992, the team announced the first measurements of anisotropies in the CMB. The 2006 Nobel Prize in Physics was awarded to John Mather and George Smoot for their work on the COBE mission. The second CMB satellite mission was the Wilkinson Microwave Anisotropy Probe (WMAP), launched by NASA in 2001. The WMAP was 45 times more sensitive, with 33 times the angular resolution of its COBE satellite predecessor. The third CMB satellite was Planck, launched in 2009 by the European Space Agency. The resolution of Planck was about $2\frac{1}{2}$ times better than WMAP, with higher sensitivity and also measurements in 9 frequency bands, compared to 5 for WMAP.

Figure 10.4 shows the microwave sky, as seen in the 2015 data release from the Planck satellite. The radiation is almost completely uniform, but the tiny variations are shown in a false-color image, with the temperature color-code shown by the bar at the bottom. This picture is illustrative, but it is hard to learn anything just by looking at it.

Figure 10.5[19] shows a spectrum computed from the 7-year data release of WMAP

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[18] R. Adam et al. (Planck Collaboration), "Planck 2015 results, I: Overview of products and scientific results," Figure 9, Astronomy & Astrophysics vol. 594, article A1 (2016), arXiv:1502.01582 [astro-ph.CO]

[19] A. H. Guth and D. I. Kaiser, "Inflationary cosmology: Exploring the Universe from the smallest to the largest scales," Science, 11 Feb 2005, vol. 307, pp. 884-890 (2005). With an MIT certificate, click here. Also available at arXiv:astro-ph/0502328.

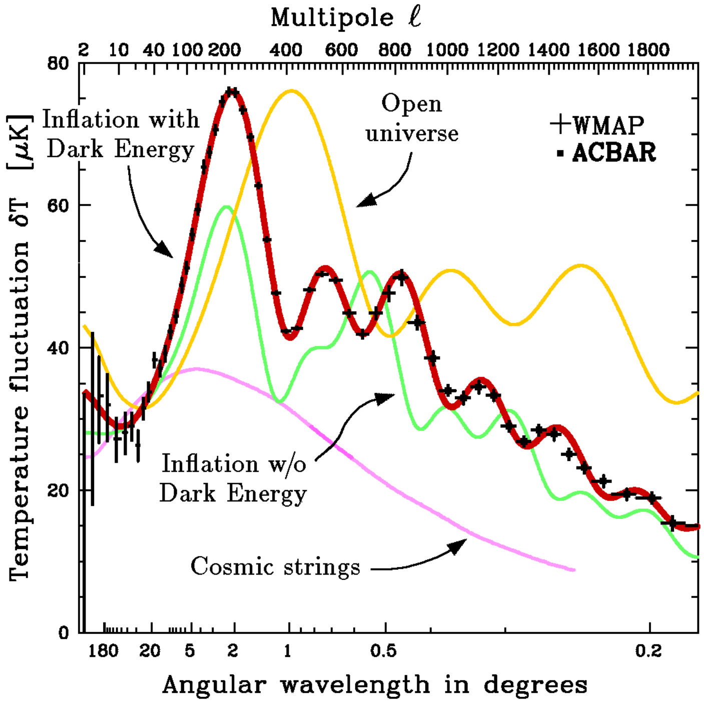

Figure 10.5: Comparison of the WMAP 7-yr and ACBAR observational measurements of the temperature fluctuations in the CMB with several theoretical models, as described in the text.

for long angular wavelengths, and an experiment called ACBAR for shorter angular wavelengths. The graph was constructed by Max Tegmark, to be used in a summary of inflation written by David Kaiser and me.[19] The vertical axis shows the strength of the fluctuations, in microkelvin, and the horizontal axis shows the angular wavelength, with the longest wavelengths on the left. (For those who are familiar with spherical harmonics, the decomposition into angular wavelengths is accomplished by an expansion in spherical harmonics $Y_{\ell m}(\theta, \phi)$, and the vertical axis represents the strength at each ℓ. The angular wavelength is taken as $360°/\ell$.) The graph shows a comparison between different theories. The red line shows the predictions for an inflationary model with $\Omega_{\rm vac} = 0.72$; the yellow line describes an open universe, with $\Omega_m = 0.30$ and $\Omega_{\rm vac} = 0$; the green line describes an inflationary model without dark energy, meaning that $\Omega_m = 1$, $\Omega_{\rm vac} = 0$; the purple line shows the prediction of a completely different mechanism for the generation of density perturbations, called cosmic strings.[20] Cosmic strings were mentioned in passing on

[20] The curve for cosmic strings was taken from U.-L. Pen, U. Seljak, and N. Turok, “Power spectra in global defect theories of cosmic structure formation,” Physical Review Letters, vol. 79, pp. 1611–1614 (1997), or with an MIT certificate, click here. Also available at arXiv:astro-ph/9704165.

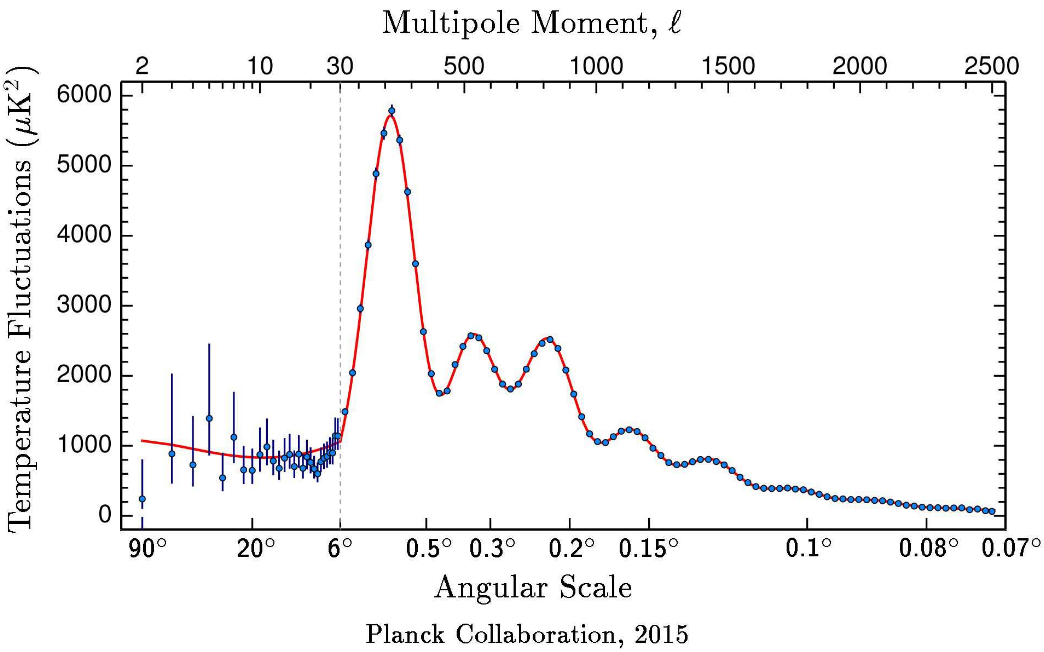

Figure 10.6: The measurements of the CMB temperature fluctuations by the Planck satellite, 2015 data release. Taken from Ref. [4], Figure 1.

p. 13 of Lecture Notes 9; they are linelike topological defects, in contrast to monopoles which are pointlike defects. They could create density perturbations through the random processes involved in their formation, and prior to the careful CMB measurements they were considered a viable theory for the origin of density perturbations. Now, however, they are clearly ruled out.

The error bars on the graph are clearly much larger on the left, at large angular wavelengths, but there is a simple explanation. For perturbations with an angular wavelength of 0.2° there are a huge number of samples on the sky, but for angular wavelengths such as 180° there are very few.

Figure 10.6 shows a more recent graph of the spectrum of the CMB, showing the data from the 2015 data release of the Planck satellite project. The red line shows a theoretical curve from a best-fit simple inflationary model, described in Table 4, Column 1 of Ref. [4]: $\Omega_m = 0.315$, $\Omega_{\rm vac} = 0.685$, and $H_0 = 67.3$ km-s^{-1}-Mpc^{-1}. It is actually a six-parameter fit to the data, where the overall height of the curve is one of the remaining parameters that is fit. There is also a parameter τ that describes a small amount of absorption of CMB photons on the way to the Earth — the fraction that arrive is $e^{-\tau}$, where $\tau = 0.078$; and finally there is a parameter $n_s = 0.966$, which describes a small deviation from the simple approximation that H and $\partial V/\partial\phi$ are constant during the period in which the presently observed density perturbations were created. As one can see, the fit is excellent.

In the words used by the Planck team, “The Planck results offer powerful evidence in favour of simple inflationary models.”

ETERNAL INFLATION:

We will not have time to fully discuss the mind-boggling implications of this feature, but the basic facts are pretty straightforward. As the scalar field rolls off the potential energy plateau shown in Fig. 10.1, we must remember that in a full quantum mechanical treatment there will always be some probability that the scalar field will remain at the top of the hill. Approximate calculations show that this probability falls off exponentially with time, with a time constant that is similar to, but maybe a factor of 100 slower than, the time constant of the exponential expansion. This means that if an observer stayed at any one point of the inflating region, it is highly probable that she would see inflation end in a very short amount of time, perhaps 10^{-35} second. However, if we were to calculate how the total volume of false vacuum changes with time, we would find that the growing exponential of the expansion dominates over the falling exponential of the decay, so the total volume of false vacuum grows exponentially in time! Once inflation starts we expect it never to stop, but instead it will continue forever. The decay of the false vacuum (the transition of the scalar field to the true vacuum value) does not happen globally, but instead pieces of the false vacuum undergo the decay and produce huge regions of inhabitable space that can be called *pocket universes.* An infinite number of pocket universes are produced. The collection of the infinite number of pocket universes is called the *multiverse.*

Eternal inflation is easiest to understand for new inflation, but it can happen also in chaotic inflation. In 1986 Linde[21] showed that as the inflaton field “rolls” down a hill in the potential energy diagram, it is possible for quantum fluctuations to drive it up the hill often enough for the volume of the inflating region to increase with time, rather than decrease.

Can we see these other universes? Almost certainly not, although in principle other pocket universes could reveal their presence by colliding with ours. Such collisions could show up as circular distortions in the cosmic microwave background. Astronomers have in fact looked for such patterns,[22] but have not found any persuasive evidence for them.

Is this discussion physics or metaphysics? That’s debatable, but in my opinion it is physics, albeit very speculative physics at this stage. First, it seems to be an

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[21] A. D. Linde, “Eternal Chaotic Inflation,” Modern Physics Letters A, vol. 1, issue 2, p. 81 (1986).

[22] S. M. Feeney, M. C. Johnson, D. J. Mortlock, and H. V. Peiris, “First observational tests of eternal inflation,” Physical Review Letters, vol. 107, article 071301 (2011). With an MIT certificate, click here. Also available at arXiv:1012.1995 [astro-ph.CO].

almost unavoidable consequence of inflation, which itself makes a number of testable predictions, and has been very successful. Second, it now appears that the possibility of a multiverse may have relevance to perplexing problems in fundamental physics, such as the cosmological constant problem discussed at the end of Lecture Notes 7. The problem, we recall, was that the vacuum energy density of our universe, measured by its acceleration, is vastly smaller (120 orders of magnitude!) than naive estimates from particle physics. The multiverse offers a possible (although controversial) explanation for this situation. According to string theory, there is no unique vacuum state, but instead a colossal number, perhaps 10^{500} or more, of long-lived metastable states, any one of which could serve as the vacuum for a pocket universe.[23] This set of possible vacua is often called the "landscape" of string theory. Even if string theory is not right, it is still possible that nature allows a huge number of different vacuum-like states. Each vacuum-like state would have its own energy density, expected to be typically of the order of the "Planck scale," the energy density that one can construct from the fundamental constants G, $\hbar$, and c. By dimensional analysis, one finds that the only way to construct an energy density from these quantities is

$$\rho_{\text{Planck}} \equiv \frac{c^5}{\hbar G^2} = 5.16 \times 10^{96}\ \text{kg/m}^3 = 5.16 \times 10^{93}\ \text{g/cm}^3\ . \qquad (10.17)$$

On Problem Set 8, Problem 5, you found an estimate for the energy density of the vacuum fluctuations of the electromagnetic field, which was of this order of magnitude. Vacuum energy densities can be positive or negative, so a natural expectation is that the energy densities of the possible vacua would range roughly from minus the Planck scale to plus the Planck scale. But if they are anything like evenly spread, there would be a fantastic number (maybe $[(5.95 \times 10^{-30}\ \text{g/cm}^3)/(2 \times 5.16 \times 10^{93}\ \text{g/cm}^3)] \times 10^{500} \approx 6 \times 10^{376}$) of vacua with energy densities as small as what we observe, although they would still be incredibly rare in the full set of $\sim 10^{500}$ vacua.

To explain why we might be living in such a rare type of vacuum, the argument invokes a selection effect associated with the fact that we are living beings. This selection effect is often called the "anthropic principle." We expect most of the pocket universes in the multiverse to have vacuum energies with a magnitude of the order of the Planck scale, but such pocket universes would fly apart (if $\rho_{\text{vac}} > 0$) or implode (if $\rho_{\text{vac}} < 0$) on a time scale of order 10^{-44} sec. (To find the time scale, calculate χ^{-1} for χ given by Eq. (10.9), with ρ_{f} replaced by ρ_{vac}.) It is therefore easy to believe that no life will exist in such typical pocket universes. The complexity of life requires time to evolve, so we expect life

[23] R. Bousso and J. Polchinski, "Quantization of four form fluxes and dynamical neutralization of the cosmological constant," Journal of High Energy Physics, vol. 2000, 06, 006, arXiv:hep-th/0004134. See also R. Bousso and J. Polchinski, "The string theory landscape", Scientific American, vol. 291, p. 78–87 (2004). With an MIT certificate, click here.

to form only in those rare types of vacuum in which the vacuum energy densiy is extremely close to zero. In 1998 H. Martel, P. R. Shapiro, and Steven Weinberg[24] estimated how large the vacuum energy density could be for it to still be possible for matter to condense out of the background into mass concentrations large enough to form observers. They found that under these assumptions, life would form only in those pocket universes in which the vacuum energy density were of the same order of magnitude as the current critical density. This result makes the selection effect explanation look very plausible, but we must keep in mind (1) that the Martel-Shapiro-Weinberg calculation ignored the possibility of life forms very different from ourselves, and (2) the calculation ignored the fact that other parameters of the laws of physics, and not just the cosmological constant, could be different in different pockets.

A PERSONAL SUMMARY:

I hope that you have enjoyed our journey into the current status of cosmology. I personally find it mind-boggling that we can use the big-bang theory to calculate the abundances of the light chemical elements, and even more mind-boggling that we can theorize about the behavior of the universe at 10^{-37} seconds after its beginning. It is mind-boggling that the structure of the universe could have arisen from quantum uncertainties, and astounding that such a wild idea can lead to a fit with the data as good as Figure 10.6.

It is absolutely incredible how far physics has taken us in the quest to understand the universe, but at the same time it is incredible how many key questions remain unanswered. The baryonic matter that we understand comprises only about 5% of the total energy of the universe. What is the dark matter, which makes up 26% of the universe? If the dark energy is really vacuum energy, why is the energy density so much smaller than particle theorists would expect? And if inflation is right, what exactly is the inflaton, and what is the detailed description of its dynamics?

I find it amazing how much we understand about cosmology, and equally amazing how much we don't.

[24] H. Martel, P. R. Shapiro, and S. Weinberg, "Likely Values of the Cosmological Constant," The Astrophysical Journal, vol. 492, pp. 29-40 (1998), arXiv:astro-ph/9701099.