Massachusetts Institute of Technology
Physics Department

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Repairing Yesterday's Lecture

I'm afraid I got carried away about the need to be careful about being consistent in perturbation theory. Here is a correct discussion.

Remember that we use the unusual normalization

$$\begin{aligned} |n\rangle &\equiv |n^0\rangle + |\delta n\rangle \\ \langle n^0|n\rangle &= 1 \end{aligned} \tag{1}$$

and then "renormalize" at the end of the calculation,

$$|\bar{n}\rangle = \frac{|n\rangle}{\sqrt{1+\langle\delta n|\delta n\rangle}} \tag{2}$$

Suppose you have computed the *first order* correction to an energy eigenstate. Let λ be the small parameter. Then

$$|n\rangle = |n^0\rangle + \lambda \sum_{m\neq n} c_m |m^0\rangle + \mathcal{O}(\lambda^2)$$

Note that the terms of order λ^2 and beyond *do not contain the any contributions proportional to the original state* $|n^0\rangle$. This is required by the normalization condition eq. (1). Then the norm of $|n\rangle$ is given by

$$\langle n|n\rangle = 1 + \lambda^2 \sum_{m\neq n} |c_m|^2 + \mathcal{O}(\lambda^3)$$

and the conventionally normalized state $|\bar{n}\rangle$ is given by

$$|\bar{n}\rangle = (1 - \frac{\lambda^2}{2}\sum_{m\neq n}|c_m|^2)|n^0\rangle + \lambda\sum_{m\neq n} c_m|m^0\rangle + \mathcal{O}(\lambda^2)$$

Contrary to what I said in lecture, the term of order λ^2 in the normalization of the bare state, $|n^0\rangle$ is significant because higher order terms in the perturbative expansion will not generate any corrections of order λ^2.

Despite my misstatement about this particular example, the warning still holds: be careful to make sure you do not take seriously terms in perturbation theory that are beyond the order to which you have computed.