

Massachusetts Institute of Technology
Physics Department

Physics 8.322
Quantum Theory II

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Summary of WKB Connection Formulas

Here is a summary of the connection formulas for the two cases corresponding to turning points on the right and left of an allowed region:

I. Forbidden region to the right (turning point at $x_0 = a$).

- If the wavefunction is known to be exponentially falling in the forbidden region, then its phase and amplitude are known in the allowed region:

$$\frac{1}{\sqrt{\kappa(x)}} \exp \left[-\frac{1}{\hbar} \int_a^x \kappa(x') dx' \right] \Rightarrow \frac{2}{\sqrt{p(x)}} \cos \left[\frac{1}{\hbar} \int_x^a p(x') dx' - \frac{\pi}{4} \right] \quad (1)$$

- A wavefunction 90° out of phase in the allowed region continues into a growing exponential in the forbidden region as follows:

$$\frac{1}{\sqrt{p(x)}} \cos \left[\frac{1}{\hbar} \int_x^a p(x') dx' + \frac{\pi}{4} \right] \Rightarrow \frac{1}{\sqrt{\kappa(x)}} \exp \left[\frac{1}{\hbar} \int_a^x \kappa(x') dx' \right] \quad (2)$$

II. Forbidden region to the left (turning point at $x_0 = b$).

- If the wavefunction is known to be exponentially falling in the forbidden region, then its phase and amplitude are known in the allowed region:

$$\frac{1}{\sqrt{\kappa(x)}} \exp \left[-\frac{1}{\hbar} \int_x^b \kappa(x') dx' \right] \Rightarrow \frac{2}{\sqrt{p(x)}} \cos \left[\frac{1}{\hbar} \int_b^x p(x') dx' - \frac{\pi}{4} \right] \quad (3)$$

- A wavefunction 90° out of phase in the allowed region continues into a growing exponential in the forbidden region as follows:

$$\frac{1}{\sqrt{p(x)}} \cos \left[\frac{1}{\hbar} \int_b^x p(x') dx' + \frac{\pi}{4} \right] \Rightarrow \frac{1}{\sqrt{\kappa(x)}} \exp \left[\frac{1}{\hbar} \int_x^b \kappa(x') dx' \right] \quad (4)$$