Survey of Strong Interactions (lecture notes)

The strong interaction, as its name suggests, is the strongest force presently known in Nature. Nevertheless its existence was only discovered in the twentieth century, since in ordinary circumstances it acts only short distances (basically, within atomic nuclei). Ironically, the fact that the strong force is so powerful, together with the fact that it can cancel among several bodies, is the basic reason for its hidden character. Unbalanced strong forces create an unstable situation, which ordinarily cannot be maintained.

At subnuclear distances a conflict arises between canceling the strong force, by enforcing oppositely (color) charged particles to be on top of one another, and the quantum-mechanical cost of giving the relative position a definite value (i.e., zero). The point is that minimizing the relative position, according to Heisenberg's uncertainty relation, brings in large momenta, and therefore extracts an energetic price. Compromise between these two effects determines the size, and the mass, of protons and neutrons. (Alternative "local minima" in the energy landscape are other, metastable baryons.) The forces of nuclear physics are residuals, resulting from the fact that the color forces are imperfectly canceled.

The song of the previous paragraph should sound familiar: it is a reprise, with ornamentation, of atomic physics. The existence of atoms (i.e., of sensible ground states) arises from compromise between attractive electric forces, that want cancelation, and quantum resistance to localization, as originally postulated by Bohr. Alternative compromises give excited states, and the forces of chemistry are residuals.

Color, Confinement, and Asymptotic Freedom

The original suggestion for the color quantum number came out of quark model spectroscopy. The kind of problem it solved is most simply illustrated by the Δ^{++} resonance. This is a relatively light spin-$\frac{3}{2}$ resonance, that appears very prominently in (by today's standards) low-energy nucleon-nucleon or pion-nucleon collisions. It was discovered by Fermi and his group at Chicago, around 1950! In the quark model, we assign it the content uuu. Since it is a low-lying state, we expect the wave functions to be $s - wave$, and then the quark spins must be in a totally symmetric state, to get $J = \frac{3}{2}$. But this means the three u quarks all occupy the same state, mocking Pauli (and Fermi). One way out is to assume that the quarks carry an additional hidden quantum number, and that they are in a totally antisymmetric state with respect to that quantum number. With a three-valued color index, we can form the totally antisymmetric combination

$$q^\alpha q^\beta q^\gamma \ \propto \ \epsilon^{\alpha\beta\gamma} \tag{1}$$

and thereby respect Fermi statistics.

An early hint that something deeper might be at work here was the basic phenomenon of *confinement*. The quark model recognized two body plans: three-quark bound states (baryons), and quark-antiquark bound states (mesons). This was sufficient to account very nicely for the low-lying hadron spectrum, as we'll review in more detail later. It begs the question why individual quark states are never observed, or two-quark states. The appearance of the ϵ symbol in Eqn. (1) suggested at least a less arbitrary statement of the rule. For invariance of $\epsilon^{\alpha\beta\gamma}$ is a fundamental feature of the group $SU(3)$, acting on the color indices. So one could state the confinement rule as a group-theoretic command: Only color singlet states are allowed! (What about $SO(3)$? That wouldn't forbid diquark states!)

That formulation of the confinement problem suggests a dynamical role for the color quantum number. If color is some kind of charge, that generates fields, then one can minimize the field energy by having color singlets.

Much more convincing and specific ideas about the dynamical role of color came from quite a different direction. As we'll explore in depth in Study 1 (deep inelastic scattering), experiments probing the interior of protons revealed that their constituents (partons) interact weakly with one another at short distances and disperse little energy-momentum as radiation when then are struck. This behavior posed a paradox: If the basic interactions are feeble, both as forces at short distances and as sources of disruptive radiation, why is the strong interaction strong? The answer is *asymptotic freedom.* A general consequence of quantum field theory is that the effective strength of a an interaction can vary with distance or (taking it to Fourier space) momentum, basically due to vacuum polarization. Usually the result is to screen the interaction, as nominally empty space – actually a medium, full of spontaneous activity – acts like a dielectric medium. But in nonabelian gauge theories one can have the opposite effect, antiscreening. Antiscreening arises because the spin-spin magnetic force between gluons, with their large color magnetic moments, is attractive between like charges and spin alignments. Note that because we are dealing with a highly relativistic situations, magnetic forces are on the same level as (repulsive) electric ones, and for gluons they dominate.

Antiscreening, in this context, is called asymptotic freedom. It imples that a feeble "seed" interaction, such as a feeble field sourced by a color charge, gets amplified, by space itself, to become stronger far away. It also means, when we take Fourier transforms, that radiation involving significant deflection of a moving charge – that is, large momentum transfer, or "hard" radiation – is rare. Soft radiation, on the other hand, can be copious. Hard radiation in momentum space reflects short distance behavior in position space, and is turned off by asymptotic freedom, while soft radiation reflects long distance behavior, and is common.

Antiscreening is unusual. Only nonabelian gauge theories, among known quantum field theories, support asymptotic freedom.

This uniqueness, for theoretical physics, is a gift from heaven. Our two lines of thought converge beautifully: We can get asymptotic freedom, and identify the hinted dynamical role for color, by promoting the color symmetry to a *local* $SU(3)$ gauge theory. Implementing local color symmetry gives us quite specific new force-carrying particles, the gluons, and prescribes their interactions with quarks and with each other uniquely, up to one overall coupling constant. That ideal, symmetry-fulfilling operation gives us a unique candidate theory: Quantum ChromoDynamics, or QCD.

By now there are mountains of quantitative evidence that QCD is in fact the correct theory of the strong interaction.

Perturbative QCD

Asymptotic freedom suggests, at least heuristically, the *jet paradigm* that has come to dominate experimental high energy physics. Its leading idea is as follows. Suppose that we produce, either through electromagnetic or weak currents, or directly in strong interactions, a rapidly moving gluon, quark, or antiquark – let's call it a parton. The parton cannot persist as an isolated particle (confinement). Indeed, it will usually emit lots of soft radiation – in effect, splitting into several partons all moving in the same direction, with none carrying off significant energy or momentum into a different direction. Those partons can re-organize themselves, by further soft radiation if necessary, into color singlet hadrons. But if we follow the flow, not of individual particles, put of overall energy and momentum, not much has happened! The collimated spray of hadrons that the original parton has triggered inherits its total energy and momentum, in a collimated spray, or jet.

Thus although we do not see quarks, antiquarks, or gluons as isolated particles, we can reconstruct them, in favorable cases, from flows of energy and momentum in the hadrons we do observe. We can compare the observed energy-momentum patterns in jets – their probability distributions in number, energies, and angles – to those we predict, directly from Feynman graph calculations in QCD, for the underlying partons. That is how much of the "mountains of evidence" I mentioned previously have been accumulated. Having validated the theory on test cases, we can use it to anticipate results of future experiments. Nowadays, people commonly speak of "QCD background" to more exploratory work, such as searches for Higgs boson(s) or supersymmetric partners.

The paradigmatic example of the jet paradigm is e^+e^- annihilation. This is, of course, a practical thing to study, and a succession of great accelerators culminating (so far) in the Large Electron-Positron collider (LEP) at CERN, have explored it in great detail at ever higher energies. The annihilation can proceed through a virtual photon γ or Z, or "on resonance" with a sort-of-real Z. These virtual or unstable particles couple directly to quark-antiquark pairs with known couplings, that do not fall off rapidly with momentum. So they are a predictable source, with the Standard Model, of quark and antiquark jets. Those partons occasionally (at a predictable overall rate, and with predictable energy and angular distributions) occasionally radiate hard gluons, making three-jet events; more rarely, but also predictably, one has four, five, or even six jets to study. Running the accelerator supplies us with many billions of events, so that we can compare the predictions with reality in great detail. In doing so, we are basically checking the fundamental interactions of QCD, the vertices that go into the Feynman graphs – i.e., the terms in the Lagrangian! – directly.

As one tiny illustration, the overall rate of two-jet events gives remarkably direct evidence for the color quantum number, which underlies an unsubtle factor of three in the rate.

We also have the *Coulombic paradigm* in heavy quark - heavy antiquark spectroscopy. The leading idea here is to invoke the Jesuit Credo

> It is more blessed to ask forgiveness than permission.

In that spirit, we assume that we can use a small effective coupling, treated to lowest order, and hope for the best (checking self-consistency). At the level of one-gluon exchange, QCD generates a simple Coulomb-like potential. It gives us a "hydrogenic" spectrum of states, whose characteristic size is the appropriate "Bohr radius". Now that radius is inversely proportional to the reduced mass of the system, and so if we are dealing with a heavy quark and a heavy antiquark, the size will be small. And if the size is small, it was correct to assume the strong interaction acts feebly (asymptotic freedom)! So our approximation is blessed, and our audacity forgiven.

Ideas and calculations to exploit the simplicity of hard radiation and short distance behavior in QCD have become extremely sophisticated over the years. The challenge is to find situations and choices of observables that are insensitive to the much more difficult, and as a practical matter incalculable, soft radiation and long distance effects. Those effects can never be turned off, but only evaded. Systematic evasion is the subject known as "perturbative QCD". It has developed a heavy technical armamentarium, a voluminous technical literature, and a corps of devoted practitioners doing useful, impressive work.

Non-perturbative QCD

The color concept allows us to give an intuitive explanation of confinement. As sketched already above, the basic idea is that color non-singlets source non-trivial gluon fields. We can refine that now, with the basic notion of gluon paramagnetism, running asymptotic freedom backwards ("infrared slavery"). The point is that non-trivial gluon fields are not just unfavorable energetically, but dangerously unfavorable, because they are self-amplifying.

Refining that thought, the gluon disturbance emanating from a color source will tend to organize itself into a flux tube, because the gluons attract one another. So if there is some characteristic that prevents the tube from ending, and preserves its form, we will get a *linear dependence* of the energy on distance, i.e. the length of the flux tube connecting source and nominal anti-source. That means a constant force, at large distances, and of course directly that it takes an infinite amount of energy to separate the source and anti-source completely.

In fact there is such a characteristic, namely the *triality* of a source. The triality of a color charge is an additive quantum number, defined modulo three, that is one for a quark, minus one for an antiquark, and zero for a gluon. (Mathematically, it is the value of the number of upper minus lower indices in the $SU(3)$ tensor characterizing the source. It is defined modulo three, because applying the invariant tensor $\epsilon^{\alpha\beta\gamma}$ can rearrange the absolute counting, but not the counting modulo three.) Since the Hamiltonian of QCD has triality zero, no action of that Hamiltonian can change the triality of a state. A unit of triality emanating from a quark continues to exert its Gauss' law influence indefinitely, and keeps a constant flux tube going, until it terminates on a source that can absorb the triality.

There is direct evidence for this picture from heavy quark - heavy antiquark spectroscopy. As we discussed previously, the short-distance potential is Coulombic. At larger distances, the flux tube picture suggests that the potential becomes linear. Combining the two, we get an excellent quantitative account of bottom-antibottom and (less accurately) charm-anticharm spectroscopy.

For light quarks, the *bag picture*, pioneered at MIT, provides a modern version of the quark model, that is remarkably successful semi-quantitatively. The idea is to implement confinement as a boundary condition, that terminates quark fields at the surface of a "bag". The bag has a characteristic tension, determined phenomenologically, and used to fit all hadrons. Within the bag, the quarks (or gluons) interact perturbatively, as suggested by asymptotic freedom. We'll say a bit more about this model later.

For accurate work in non-perturbative QCD, the only successful approach remains direct numerical solution of the equations (despite heroic efforts to develop alternatives). The good news is that this approach, known as lattice gauge theory, works brilliantly! I've appended a remarkable Figure to these notes, that reports the results of first-principles calculations of hadron masses, directly from the equations, using various clever algorithms and harnessing enormous computer power.

There is a million-dollar prize by the Clay Institute for a proof of a specific, weak version of confinement in an idealized version of QCD. That is a challenging mathematical problem, but as a practical matter physicists have got far beyond it. Our numerical work not only shows that there are no quarks in the spectrum, but computes the actual spectrum – accurately, and with few per cent precision!

Frontiers and Questions

I'll conclude this survey with a brief enumeration of what I see as important frontiers and open questions in QCD

1. Extending the applications of perturbative QCD, both by bringing in new processes and by calculating known applications to higher order. We've already discussed this. It is very important for supporting exploratory work at high-energy accelerators.
2. Using QCD to get microscopically based foundations for nuclear physics. Once validated, these foundations would allow us to do a better job of modeling supernova explosions, neutron stars, and other astrophysical situations where nuclear and hypernuclear physics difficult to access in terrestrial laboratories arises. For example, it's a

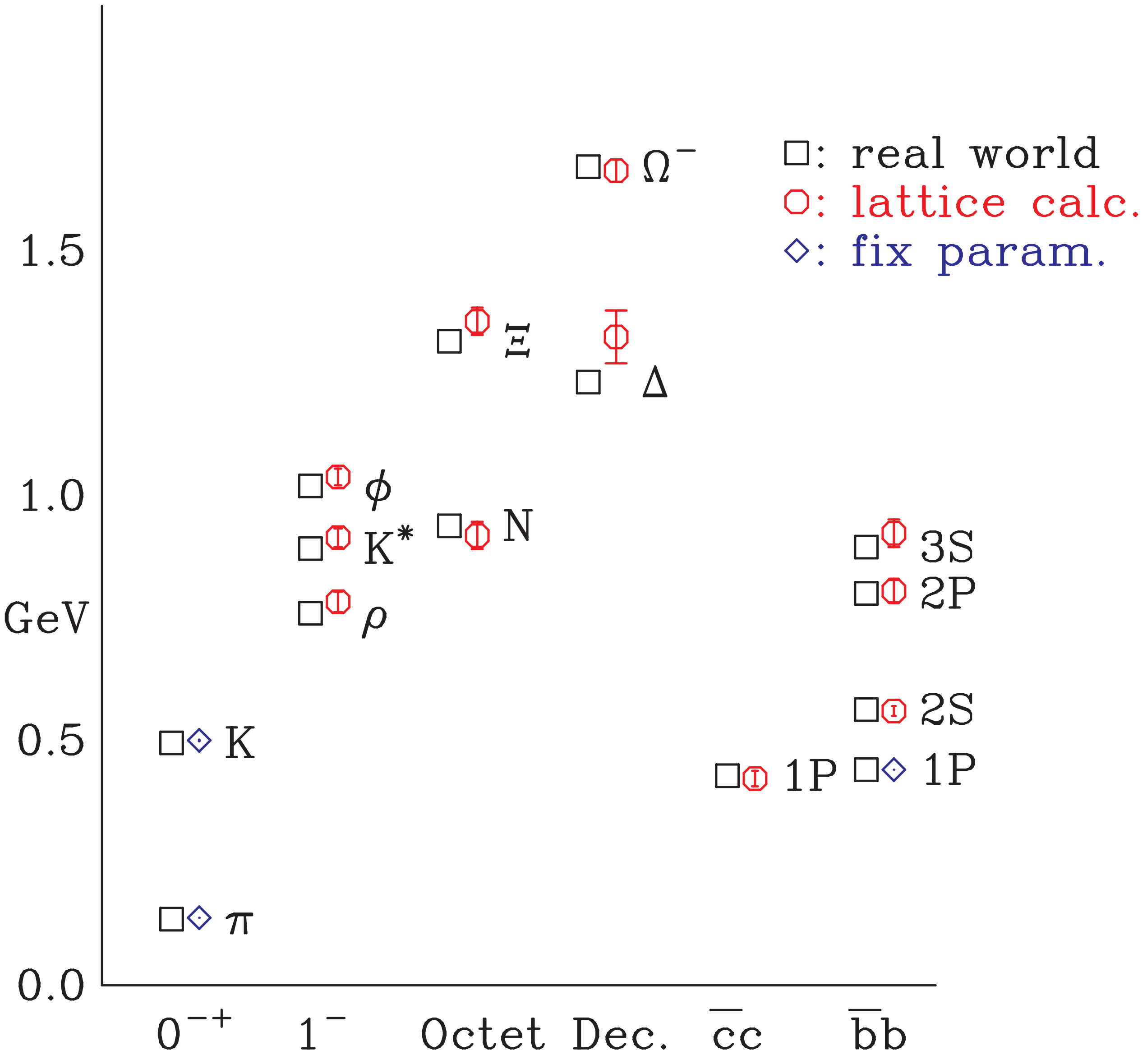
scandal that the nuclear equation of state at high pressure remains very poorly determined, leading to gross uncertainties in predictions for the neutron star mass-radius equation.

3. Although lattice gauge theory validates QCD in its non-perturbative aspect, and calculates a wide variety of hadron properties successfully, its application to dynamical processes, such as scattering processes, is at present severely limited. Can one develop more flexible algorithms, to deal with dynamics? Quantum chemists have made progress on problems of this kind, using sophisticated variational techniques.

4. There are striking, simple regularities in the description of meson and baryon states with large angular momentum. I won't pause here to describe this "Regge theory" in detail, other than to note that it supports a mass versus angular momentum relation of the form $M^2 \propto J$, which works remarkably well. In any case, no one has made a convincing connection between microscopic QCD and the Regge theory phenomenology, which is another scandal.

5. The behavior of quark-gluon matter under extreme conditions of temperature and baryon number density is interesting both theoretically and for the description of experiments at heavy ion colliders, neutron stars, and the early universe. Lattice gauge theory gives definitive results for thermodynamics at high temperature, but has difficulties handling dynamics or non-zero baryon number (chemical potential). All other presently known approaches to these problems are crude, unreliable, or of strictly limited applicability (inclusive or).

So although in some ways QCD is a mature subject – the basic equations are known, and they aren't negotiable! – many big, ripe, challenging problems remain open, and there's plenty of useful work to be done.


□: real world
○: lattice calc.
◇: fix param.
GeV
1.5
1.0
0.5
0.0
0^{-+}
1^{-}
Octet
Dec.
$\overline{c}c$
$\overline{b}b$
π
K
ρ
K^*
ϕ
N
Ξ
Δ
Ω^-
1P
1P
2S
2P
3S