6.S979 Lecture 11 Take 2

$$C_{\text{classical}} \subsetneq C_q \subsetneq C_{qs} \subsetneq C_{qa} \subsetneq C_{qc}$$

$$C_{qa} = \overline{C_q} = \overline{C_{qs}}$$

Today:

$C_q \neq C_{qs}$

Slofstra '16, '17: $C_{qs} \neq C_{qc}$, $C_{qs} \neq C_{qa}$

Coladangelo Stark '18: $C_q \neq C_{qs}$ $(\Rightarrow C_q \neq C_{qa})$

C_q, C_{qs} are not closed

Tool: Tilted CHSH inequality

A_0, A_1, B_0, B_1 observables

$[A, B] = 0$

$$\underbrace{\beta}_{\in [0,1)} A_0 + (A_0 B_0 + A_0 B_1 + A_1 B_0 - A_1 B_1) \preceq \sqrt{8 + 2\beta^2} \cdot I$$

(pf: degree 3 NPA proof)

<u>Fact:</u> Tilted CHSH self-tests

$$|\psi\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle = |EPR_\alpha\rangle$$

$$\sin 2\theta = \sqrt{\frac{4-\beta^2}{4+\beta^2}} \qquad \alpha = \tan\theta$$

$$A_0 = Z, \qquad A_1 = X$$

$$B_{0/1} = \cos\mu\, Z \pm \sin\mu\, X$$

function of θ

$$|\psi\rangle = \cos\theta\left(|00\rangle + \alpha\,|11\rangle\right) = |EPR_\alpha\rangle$$

Defining a separating correlation

$$C_q = \bigcup_{d=1,2,\dots} \{ \langle\psi| A\otimes B |\psi\rangle : |\psi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d \}$$

$\neq$

$$C_{qs} = \{ \langle\psi| A\otimes B|\psi\rangle : |\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$
$$A \text{ acts on } \mathcal{H}_A$$
$$B \text{ acts on } \mathcal{H}_B \}$$

State space

$$\ell_2 \otimes \ell_2$$

Basis $\{ |ij\rangle, \; i,j \geq 0 \}$

$$\{ |0\rangle, |1\rangle, |2\rangle, \dots \}$$

$$|\psi\rangle = \sum_{i=0}^{\infty} \psi_i |i\rangle \qquad \sum_i |\psi_i|^2 < \infty$$

$\| |\psi\rangle \|$

$$|\psi\rangle = \sqrt{1-\alpha^2} \sum_{i=0}^{\infty} \alpha^i |i\, i\rangle \qquad \langle\psi|\psi\rangle = 1 \checkmark$$

$\alpha \in (0,1)$

$$\sum_{i=0}^{\infty} (\alpha^i)^2 = 1 + \alpha^2 + \alpha^4 + \dots$$
$$= \frac{1}{1-\alpha^2}$$

$\alpha\,|EPR_\alpha\rangle$

$\text{span}\{|00\rangle, |11\rangle\}$

$$|\psi\rangle = \sqrt{1-\alpha^2}\begin{pmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \\ \alpha^5 \\ \vdots \end{pmatrix} \begin{matrix} |00\rangle \\ |11\rangle \\ |22\rangle \\ |33\rangle \\ |44\rangle \\ |55\rangle \\ \vdots \end{matrix}$$

Even:

$$|\psi\rangle \propto |EPR_\alpha\rangle \oplus \alpha^2 |EPR_\alpha\rangle \oplus \alpha^4 |EPR_\alpha\rangle \oplus \cdots$$

"direct sum"

Odd:

$$|\psi\rangle \propto \frac{1}{\sqrt{c}}|00\rangle \oplus \sqrt{\frac{1-c}{c}}\left(|EPR_\alpha\rangle \oplus \alpha^2 |EPR_\alpha\rangle \oplus \alpha^4 |EPR_\alpha\rangle \cdots\right)$$

$\sqrt{1-\alpha^2}$

$\sqrt{1-\alpha^2} =$

Correlation $p(a, b | x, y)$

$x \in \{0, 1, 2, 3\}$

$y \in \{0, 1, 2, 3, 4\}$

$a \in \{0, 1, 2\}$

$b \in \{0, 1, 2\}$

Even correlations

$x, y \in \{0, 1\}$

$$|\psi\rangle = |EPR_\alpha\rangle \oplus \alpha^2 |EPR_\alpha\rangle \oplus \dots$$

Alice & Bob use binary measurements (no outcome 2)

$$\tilde{A}_{0/1} = \begin{pmatrix} A_{0/1} & & \\ & A_{0/1} & \\ & & \ddots \end{pmatrix} = A_{0/1} \oplus A_{0/1} \oplus \dots$$

$$\tilde{B}_{0/1} = \begin{pmatrix} B_{0/1} & & \\ & B_{0/1} & \\ & & \ddots \end{pmatrix}$$

a \ b	0	1	2
0	$CHSH^{\alpha}_{x,y}$		0
1			0
2	0	0	0

Odd case

$$(x, y) \in \{2, 3\}$$

measure which branch you're in

$$|\psi\rangle = \ldots \underbrace{|00\rangle}_{\text{give up and return } 2} \oplus \ldots \underbrace{\left(|EPR_\alpha\rangle \oplus \alpha^2 |EPR_\alpha\rangle \oplus \ldots \right)}_{\text{tilted CHSH}}$$

shift x to $\{0,1\}$

$$\tilde{A}^{x}_{a \in \{0,1\}} = \begin{pmatrix} 0 & & & \\ & A^{x-2}_a & & \\ & & A^{x-2}_a & \\ & & & \ddots \end{pmatrix}$$

$$\tilde{A}^{x}_{2} = \begin{pmatrix} I & 0 & & \\ & 0 & & \\ & & \ddots & \end{pmatrix}$$

a\b	0 1	2
0 1	$\frac{c-1}{c}\,CHSH^{\alpha}_{x-2,\,y-2}$	
2		$\frac{1}{c}$

Consistency case

$(y = 4)$

$$B^{4}_{a} = \begin{pmatrix} A^{0}_{a} & & & \\ & A^{0}_{a} & & \\ & & A^{0}_{a} & \\ & & & \ddots \end{pmatrix}$$

A^0_a // $|a\rangle\langle a|$

Even blocks

$x = 0, \quad y = 4$

$a \backslash b$	0	1	2
0	$\frac{1}{c} \cdot \frac{1}{1-\alpha^4}$		0
1		$\frac{1}{c} \cdot \frac{\alpha^2}{1-\alpha^4}$	0
2	0	0	0

$\langle EPR_\alpha | Z \otimes Z | EPR_\alpha \rangle$

$x = 2, \quad y = 4$

Alice does Z on odd blocks

Bob does Z on even blocks

$a \backslash b$	0	1	2
0	0	$\frac{1}{c}\left(\frac{1}{1-\alpha^4} - 1\right)$	0
1	$\frac{1}{c}\frac{\alpha^2}{(1-\alpha^4)}$	0	0
2	$1/c$		0

It remains to show that this correlation $\notin C_q$

(i.e. $\nexists$ finite dim. state achieving it)

Wrong but beautiful argument:

Using self-testing for $CHSH_\alpha$

Even correlations

$\Rightarrow \exists\, \Phi$ s.t.

1) $\Phi(|\psi\rangle) = |EPR_\alpha\rangle \otimes |aux\rangle$

Odd correlations (+ consistency)

$\Rightarrow \exists\, \Phi'$ s.t.

2) $\Phi'(|\psi\rangle) = \frac{1}{\sqrt{c}}|00\rangle \oplus \sqrt{\frac{c-1}{c}}\left(|EPR_\alpha\rangle \otimes |aux'\rangle\right)$

this is false!

$\Rightarrow |\psi\rangle$ is inf. dim.

Pf: Assume not. Then by Schmidt decomposition

$$|\psi\rangle = \sum_i \sigma_i \, |u_i\rangle |v_i\rangle$$

$\sigma_i > 0$ "Schmidt coefficients"

(*) unique

Also

(*) Local Φ doesn't change Schmidt coeffs.

1) $|\psi\rangle \xrightarrow{\Phi} (|00\rangle + \alpha |11\rangle) \otimes |aux\rangle$

$$\text{"} \quad \sum_{i=1}^{k} \tau_i |u_i\rangle |v_i\rangle$$

$$\mathrm{Sch}(|\psi\rangle) = \underbrace{\{\tau_1, \alpha\tau_1, \tau_2, \alpha\tau_2, \ldots, \tau_k, \alpha\tau_k\}}_{\text{even } \#}$$

2) $|\psi\rangle \xrightarrow{\Phi'} \Big(c_1 |00\rangle + c_2 \big(|11\rangle + \alpha |22\rangle \big) \otimes |aux\rangle \Big)$

$$|aux\rangle = \sum \tau_i' |w_i'\rangle |v_i'\rangle$$

$$\mathrm{Schm}(|\psi\rangle) = \Big\{ \underbrace{c_1,\ c_2\tau_1',\ c_2\alpha\tau_1',\ c_2\tau_2',\ c_2\alpha\tau_2',\ c_2\tau_3',\ c_2\alpha\tau_3',\ \ldots}_{\text{odd } \#} \Big\}$$

$\Rightarrow |\psi\rangle$ is not finite dim

The actual proof:

2) $\Phi'(|\psi\rangle) = \frac{1}{\sqrt{c}} |00\rangle \otimes |\text{aux}''\rangle \oplus \sqrt{\frac{c-1}{c}} \left(|\text{EPR}_\alpha\rangle \otimes |\text{aux}'\rangle \right)$

Open questions:

1) Reduce # inputs and outputs

4, 5, 3, 3

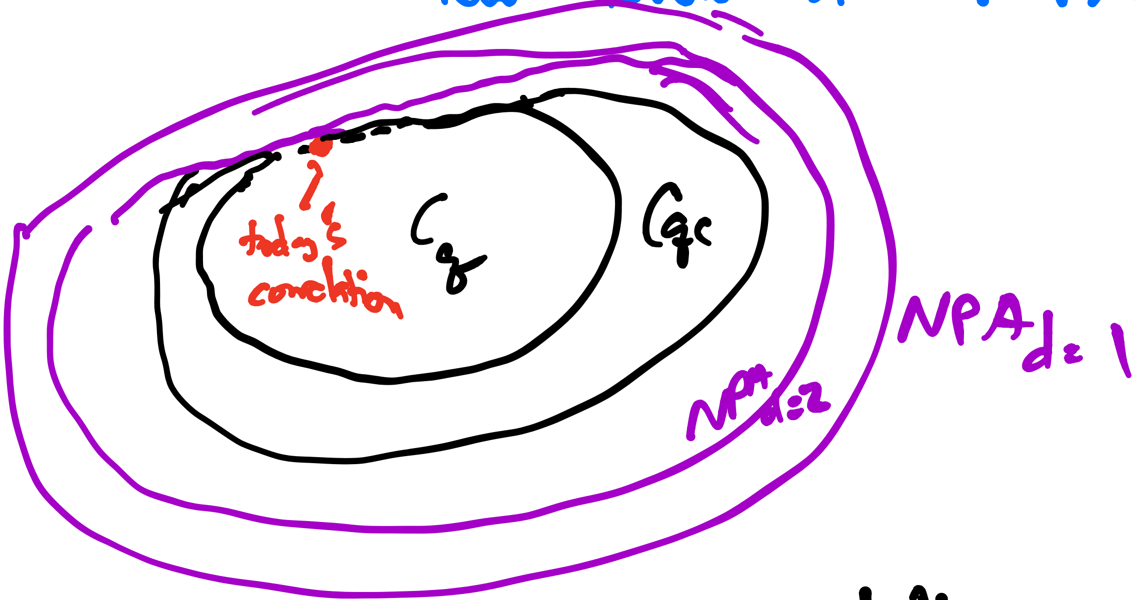
$I_{\underbrace{33}_{\#\text{ inputs}}\underbrace{22}_{\#\text{ outputs}}}$

Conjectured to separate C_{qs} from C_q

Uses higher levels of NPA

2) Tilted CHSH is exactly characterized by low levels of NPA

Q: Can this correlation be tightly characterized by low levels of NPA?



If so, ex. of correlation where

<u>outer</u> approx. converges <u>exactly</u> rapidly

<u>inner</u> approx. don't converge exactly for any d