Beyond DCF Analysis in Real Estate Financial Modeling: Probabilistic Evaluation of Real Estate Ventures

by

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Submitted to the Program in Real Estate Development in
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ABSTRACT
This thesis introduces probabilistic valuation techniques and encourages their usage in the real estate industry. Including uncertainty and real options into real estate financial models is worthwhile, especially when there is an elevated level of unpredictability surrounding the investment decision.

Incorporating uncertainty into real estate *pro formas* not only provides different results over deterministic models, it changes the angle of attack to real estate valuation problems. When uncertainty is taken into account, the focus shifts from simply maximizing financial returns, to modeling and managing uncertainty to make better *ex ante* finance and design decisions. The ability to add optionality in probabilistic financial modeling can enhance returns by curtailing losses during downturns and taking advantage of upside conditions.

A step-by-step example is carefully crafted to demonstrate the simplicity with which uncertainty, Monte Carlo Simulations and Real Options may be included into real estate *pro formas.* The example is entirely Excel based and is separated into three parts with each progressively increasing in complexity. SimpleCo Tower establishes the familiar Discounted Cash Flow *pro forma* as a starting point. ModerateCo Tower describes how uncertainty and Monte Carlo simulations can be incorporated into a *pro forma* while illustrating the effect of non-linearity on financial models. ChallengeCo Tower reveals how real options can add value to an investment and how it should not be overlooked.

The case study illustrates how the techniques outlined in this thesis can add significant value to real estate decisions without much added effort or investment in expensive software. The case study also shows how the use of real world data to model uncertainty can be put into practice.

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CHAPTER 1: Introduction

"May the odds be ever in your favor."

- Suzanne Collins, author of *The Hunger Games*

In the blockbuster science fiction novel and movie series, *The Hunger Games*, Collins describes a dystopian society in which a handful of teenagers are engaged in an ultra-competitive battle to the death. This competitive environment could draw comparisons to the arena of real estate investing, where deals are won or lost by razor-thin margins. While Collins' quote suggests that nothing can be done about one's odds in the world of her book, this is not the case with real estate. With knowledge of probabilistic valuation methods, real options, and economics, real estate professionals can effectively improve the odds in their favor.

1.1 Thesis Purpose

The world of corporate finance was introduced to Monte Carlo methods approximately 50 years ago, significantly altering the valuation approach for derivatives. In contrast, there has not been wide-spread adoption of stochastic valuation techniques in real estate finance despite the positive track record of Monte Carlo Simulations in corporate finance (Marshall & Kennedy, 1992). The benefits of probabilistic valuation techniques for real estate have been widely documented since the early 1990's (Baroni, Barthélémy, & Mokrane, 2006; Farragher & Savage, 2008; Louargand, 1992). Yet, the real estate industry still relies on sensitivity analyses for their risk assessment of real estate investments. Farragher and Savage's 2005 survey of 32 intuitional investors and 156 developers showed that only 2% of these firms utilize Monte Carlo Simulation techniques.

> **Key Terms: Stochastic vs Deterministic**
>
> A stochastic or probabilistic model relies on probability to obtain its values for future states of the system.
>
> A deterministic model has no randomness involved in generating its future output values.

The message from academia is not getting through to industry. The rejection of probabilistic techniques by real estate professionals is due, in large part, to the inability of academics to present a compelling argument for probabilistic financial modeling. Academic theses tend to be excellent at defining what concepts are, but have difficulty with coaching the application of theoretical concepts to the real world. The fragmentation of the research, which occurs because of the multi-disciplinary nature of the subject matter, inhibits acceptance of stochastic techniques because the true benefits are not realized together in a sweeping overall view from the start of the process, all the way to the end. With roots in engineering, mathematics, economics and finance, the concepts presented in this thesis have never been presented together before.

This thesis advocates for the use of probabilistically-based valuation in the real estate industry by:

- organizing research from multiple disciplines,
- demonstrating the great numerical and strategic advantages of stochastic modeling,
- clarifying how little additional effort is required to achieve those advantages, and
- emphasizing the applicability of concepts described above to real world problems.

This thesis attempts to mend the disconnect between academia and industry by focusing on the effective presentation of ideas and application of modern pedagogical theory.

1.2 Format of Presentation

This thesis is structured to appeal to a wide range of real estate professionals. The major, big picture arguments for implementing probabilistic strategies may be of greater importance to executives and managers, while an analyst may want to understand the finer points of modeling uncertainty and real options in Excel. The chapters in this thesis vary in their level of detail. Chapters 2, 3, and 4 walk through a simplified example that incorporates elements of probabilistic valuation at a broad level to demonstrate the main points of this thesis. Discussion in these chapters will tend to be more qualitative. For those looking for a greater detail, the appendix describes how the ideas presented can be implemented into Excel, step by step. Additionally, an Excel workbook of every example is available for real estate practitioners to explore every cell. Chapters 5 and 6 show how the probabilistic concepts translate to the real world, with a detailed case study of 2 World Trade Center to bookend the thesis in chapter 7.

1.3 Current Industry Practice: Excel, Argus and Discounted Cash Flow Analysis

The tools of the trade for analyzing income producing properties are Microsoft Excel and Argus. Over the last decade, the ability to work with Excel has become essential in the business world, especially for graduates of business schools. Despite the prevalent usage of Excel in the workplace, generally very few features of the program are used by professionals. Excel users are largely unaware of the computing power available to them and resort to using a handful of common finance calculator functions. However, this is not the fault of professionals, as the user experience, beyond basic calculator functions, becomes unintuitive and frustrating to those not familiar with computer programming. Good coaching and constant practice is required to develop skills beyond basic calculator functions in Excel and this thesis addresses this by providing easy-to-follow examples.

Argus is software designed to save real estate professionals time by allowing the input of information through a graphical user interface (GUI). A *pro forma* is generated by Argus once all the information is imputed. Argus, in particular, is useful for organizing lease information and producing rent rolls, a task that is tedious when the analysis is performed manually in Excel. Argus allows real estate analysts to assess the financial feasibility of a deal quicker, enabling a firm to inspect a greater volume of deals. Unfortunately, Argus does have a few drawbacks. While the *pro forma* is exportable to Excel, Argus does not export the formulas which it uses to calculate its numbers, essentially making Argus a "black box"; the inner workings and logic of the program cannot be inspected. Reliance on the automation which Argus provides to real estate analysts could erode human performance, as practice from working with the nuts and bolts of a real estate *pro forma* is reduced. A similar argument is made over automation in aircraft cockpits, as reports, such as Sarter & Woods (1994), express concerns over the ability of pilots to react to non-normal situations. Another issue is the inflexibility of Argus to adapt to a wide range of real estate ventures. Argus is great at modeling "cookie-cutter" projects, but its effectiveness is reduced when it's used to model complex real estate projects.

The main method of valuation for income producing real estate is the Discounted Cash Flow (DCF) approach. While the direct capitalization method (using cap rates) is also widely used, the absolute reliance on one year's net operating income relegates the direct capitalization method to quick back-of-the-napkin analyses. The DCF approach involves projecting future years of cash flow and discounting them using a risk-adjusted discount rate to arrive at the Net Present Value of the project.

DCF *pro formas* are taught in introductory real estate and corporate finance courses in universities around the world. There are slight variations in the way the DCF approach is taught from school to school; this does little to deter the widespread usage of DCF *pro formas.*

The deterministic DCF approach does possess limitations, however. First, the analysis of uncertainty is very limited in DCF models. The discount rate reflects the level of risk in a project, but this method oversimplifies risk by relying on single discount rate when there are multiple sources of uncertainty. Also, the discount rate doesn't take into account the asymmetry between upside and downside risk – generally, downside events matter more to investors than upside events. Thirdly, it ignores the effect of options or possible changes which may occur to the real estate over the life of the investment as owners and managers have flexibility to respond to changes in the economy by making decisions that affect future cash flows. Despite its pitfalls, the DCF approach is well understood at all levels of experience in the real estate industry which makes it a good starting point to discuss probabilistic valuation techniques from. The basic DCF *pro forma* is highlighted in Chapter 2.

1.4 Reluctance to Adopt New Techniques and Reliance on Intuition

Why has the adoption of probabilistic valuation techniques, such as Monte Carlo Simulations, not occurred in the real estate industry? Byrne (1996) suggests that both the small teams and the entrepreneurial nature of the real estate industry prevents the full acceptance of probabilistic methods in financial modeling. But shouldn't the entrepreneurial spirit of the industry translate into an insatiable appetite to find an edge to get ahead of the competition?

Without a doubt real estate teams are small. Whether the teams are based in the largest investment banks or in the largest multi-national developers, only a few analysts and even fewer managers are involved in the decision-making process in any given real estate investment. The heavy workload on open deals could crowd out time available to spend on improving processes, thus perpetuating the status quo. The notion that real estate firms are not embracing stochastic valuation techniques because they are small should be rejected. Real estate firms focus on efficiency and are likely to adopt new methods, processes, or technology if the cost-benefit rationale makes sense to them. Incorporating uncertainty into the financial analysis of real estate ventures is a "low hanging fruit" and represents a major improvement in analytics with very little effort or cost.

Real estate has always been perceived as less sophisticated compared to other asset classes such as stocks or bonds. This perception was largely due to private nature of real estate transactions and the lack of data available for economic analysis. While the market for stocks has been developing since the 1600's, real estate equity as a securitized asset only began trading in the 1960's. Without reliable data to guide finance decisions, real estate professionals depended on their instincts and intuition to remain solvent during recessions.

As any experienced professional knows, our instincts do fail us from time to time. Part of the reason why uncertainty is overlooked is because it involves seeing financial losses as a possibility. Negativity bias is a psychological phenomenon that may explain what happens when we see losses or experience negative moments (Baumeister, Bratslavsky, Finkenauer, & Vohs, 2001). A common example of this effect is the anti-anticipation and stress of receiving a large restaurant bill, which is further exacerbated if the actual bill amount is unknown. Humans tend try to avoid these negative experiences that shake our confidence even if great benefits are possible.

Previously published research advocating for the use of probabilistic valuation techniques were missing a key component: data from a sufficient number of market cycles to describe the behavior of market factors and uncertainty. With over 50 years of data available, the time is ripe for real estate to explore scientific approaches. Appropriate usage of real estate data from indices are discussed further in chapters 5 and 6.

The techniques described in this thesis will not eliminate the need for good instincts in real estate, but rather, they will enhance decision-making by providing different perspectives on real estate problems.

1.5 The Role of Modern Pedagogy in this Thesis

The struggle of university researchers to connect with learners is well documented (Seymour, 2008). The academic tenure system is cited as a major reason why teaching and communication have taken a back seat, as professors are encouraged to push out publications more than developing teaching skills. If professors have difficulty keeping students in their classrooms engaged, what hope do they have in trying to engage readers in a one-way medium? Indeed, scholarly articles seem to be more effective in communicating ideas to other academics, but what about the rest of society?

The main intent of this thesis is present probabilistic concepts to real estate professionals with a high level of clarity. Often times, authors of scholarly articles enter into auto-pilot mode and deliver their ideas based on their own experience as learners or casual observations. For this thesis, special attention is paid to pedagogy to prevent a researcher-centered teaching approach and move towards a learner-centered approach.

As you might imagine, there is no scarcity of research on how adults learn. Described below are two major theories in modern pedagogy which guide the manner of presentation for concepts introduced in this thesis. The first theory is of mental models, or schemas. Child psychologist Jean Piaget proposed a process in which children use their interactions with the world to develop models of objects and patterns of action (Lang, 2008). It turns out that what we know already about the world greatly influences how we encounter new experiences; our existing models are under constant revision. When adults are met with new experiences or ideas, they work to fit these new elements in to patterns which they already understand. There are two learning processes which can occur when a person encounters a new experience: assimilation and accommodation. Assimilation occurs when a person takes in a new idea by making the idea fit to into their existing models. On the other hand, accommodation occurs when the new idea does not fit into any pre-existing models and changes are made to a person's existing models to take in the new information.

Awareness of both these processes is crucial to effective delivery of the ideas presented in this thesis. In some scenarios, assimilation needs to occur which requires the presenter to help the audience connect to pre-existing knowledge. An example of this occurring in this thesis is the use of the familiar Discounted Cash Flow *pro forma* as a starting point for more complex feature additions. For scenarios in which accommodation is likely to occur, clarity is vital to cease the perpetuation of common misconceptions and pitfalls. Clarity is emphasized when presenting the Flaw of Averages in Chapter 3.

Bloom's Taxonomy is the second pedagogical theory that is applied in this thesis. Bloom's Taxonomy is a framework developed by Benjamin Bloom in 1956 to categorize learning objectives. The framework divides educational objectives into three domains: cognitive, affective, and psychomotor (Krathwohl, 2002). Skills in the cognitive domain include those of knowledge and critical thinking. The affective domain include skills relating to emotion, while the psychomotor domain focuses on skills with physical tools, such as hammers. The cognitive domain is most relevant for the concepts presented in this thesis. In the revised Bloom's Taxonomy, Krathwohl presents 6 levels of processes in the cognitive domain. From lowest complexity to highest, they are: remember, understand, apply, analyze, evaluate and create. If the goal is to teach professionals how to create their own simulations, the corresponding discussions and examples should match that goal in detail and complexity. Since chapters in this thesis vary in their objectives (some professionals might only want to go up to 'understand' level, while other will want to 'create'), careful attention is paid to maintain consistency in cognitive levels. Mismatched objectives and discussions lead to frustration for readers. The appendices and chapters 5, 6, and 7 cater to readers who want to reach the 'create' level, while next 3 chapters reside at the 'understand' level. We begin gently by walking through the deterministic discounted cash flow *pro forma.*

CHAPTER 2: SimpleCo Tower – A Deterministic Example

The first example, SimpleCo Tower, is a simplified discounted cash flow *pro forma* of the kind analysts typically use to financially model commercial real estate transactions. SimpleCo Tower is a 10 story office tower with a floor plate of 17,000 sf. A financial model is created to evaluate the purchase of the building for a price of $17 million. There are three major sections to a *pro forma:* the assumptions, the cash flow projections, and the outputs.

Figure 1: SimpleCo Tower Sketch

A visual representation of the 10-story office tower, SimpleCo Tower.

2.1 Assumptions of a Deterministic Model

SimpleCo Tower Assumptions	
Purchase Price	$100 /gsf
Gross Floor Area	170,000 sf
Efficiency	90%
Office Rent	$30 /sf
Rent Growth Rate	3%
Expense Growth	3%
Stabilized Vacancy	5%
Expenses	$15 /sf
Capital Expenditures	10% of NOI
Terminal Cap Rate	11.00%
OCC/Discount Rate	12.50%

Figure 2: SimpleCo Tower Assumptions

This chart can be viewed in the SimpleCo Excel file on the CD.

The assumptions are a set of parameters with which the financial model most abide by. Some assumptions are physical (such as floor area and efficiency), while others are economic (such as rent and discount rate.) Estimating the assumptions accurately is important because they drive all numbers in the *pro forma.*

2.2 Projecting Cash Flows for SimpleCo Tower

The cash flow projection section of the *pro forma* projects many line item several years into the future. Variables in the formulas are often linked or referenced to the assumptions on this page. The cash flow projection organizes the revenues and costs associated with a particular property and calculates the net cash inflows/outflows for each year of property ownership. In SimpleCo Tower, the Property before Tax Cash Flow (PBTCF) is calculated without the effects of income tax or leverage.

(in 000's)	Year	1	2	3	4	5	6	7	8	9	10	11
Potential Gross Income		$4,590	$4,728	$4,870	$5,016	$5,166	$5,321	$5,481	$5,645	$5,814	$5,989	$6,169
Vacancy		$230	$236	$243	$251	$258	$266	$274	$282	$291	$299	$308
Effective Gross Income		$4,361	$4,491	$4,626	$4,765	$4,908	$5,055	$5,207	$5,363	$5,524	$5,689	$5,860
Operating Expenses		$2,550	$2,627	$2,705	$2,786	$2,870	$2,956	$3,045	$3,136	$3,230	$3,327	$3,427
Net Operating Income		**$1,811**	**$1,865**	**$1,921**	**$1,978**	**$2,038**	**$2,099**	**$2,162**	**$2,227**	**$2,293**	**$2,362**	**$2,433**
Capital Expenditures		$181	$186	$192	$198	$204	$210	$216	$223	$229	$236	
CF From Operations		$1,629	$1,678	$1,729	$1,781	$1,834	$1,889	$1,946	$2,004	$2,064	$2,126	
Reversion (Purchase and Sale)	-$17,000										$22,120	
PBTCF	**-$17,000**	**$1,629**	**$1,678**	**$1,729**	**$1,781**	**$1,834**	**$1,889**	**$1,946**	**$2,004**	**$2,064**	**$24,246**	

Figure 3: SimpleCo Tower Pro Forma

The cash flow projections are shown for SimpleCo Tower. This chart can be viewed in the SimpleCo Excel file on the CD.

2.3 Return Measures: NPV and IRR

The output section of a DCF *pro forma* calculates the objective return measures. In the world of finance, no return measure is as prevalent as Net Present Value (NPV), or its sibling the Internal Rate of Return (IRR).

Net Present Value and IRR

The time value of money principle is the most fundamental in finance. Cash flow today is worth more than cash flow in the future because of interest earning potential. Future cash flows are discounted to arrive at an equivalent value today called the Present Value (PV).

Net Present Value is the sum of the PVs of all future cash inflows and outflows of a project.

The Internal Rate of Return (IRR) is the discount rate which makes NPV equal 0.

For SimpleCo Tower, our NPV at a 12.5% discount rate is -$135,000 and the IRR is 12.37%.

SimpleCo Tower is a deterministic model. For each unique set of assumptions there is one sole outcome. The output (NPV in this case) is *determined* by the input assumptions to the exact cent. There is no uncertainty in the model because a set of assumptions always lead to a sole output return measure. Pressing the “F9” key recalculates formulas in Excel, but doing so will never change the NPV in the SimpleCo Tower *pro forma.*

The NPV can change if an assumption is manually altered. The effect on NPV of a change in an assumption can be recorded in a sensitivity analysis. Utilizing a data table in Excel, the change in NPV can be seen when one or two variables change (a sensitivity analysis is performed on the rent growth rate in section 3.3). Unfortunately, this analysis is limited to two variables and the real world usually doesn't "hold all else constant". What alternatives are out there for financial modeling?

CHAPTER 3: ModerateCo Tower - Incorporating Uncertainty into a Financial Model

Figure 4: ModerateCo Tower Sketch
A visual representation of ModerateCo Tower, a 10-story office tower that is physically identical to SimpleCo Tower.

Most real estate professionals are familiar with the techniques described in the SimpleCo Tower *pro forma* because the deterministic DCF model is taught in many introductory finance courses around the world. ModerateCo Tower expands on the SimpleCo Tower *pro forma* by adding uncertainty to one of the assumptions, the rent growth rate. Everything else about ModerateCo is the same as SimpleCo.

3.1 Uncertainty in the Rent Growth Rate of ModerateCo Tower

The SimpleCo Tower example assumed that the rent growth rate was 3% per year. Based on the averaging of historic rent growth rates, 3% is a common assumption among real estate professionals. When the rent growth rate is subject to uncertainty, it is acknowledged that the true rent growth rate is unknown and varies within a range. Excel's random number function is used to simulate uncertain behavior. Appendix A goes through step-by-step how uncertainty was built in to the ModerateCo Tower financial model.

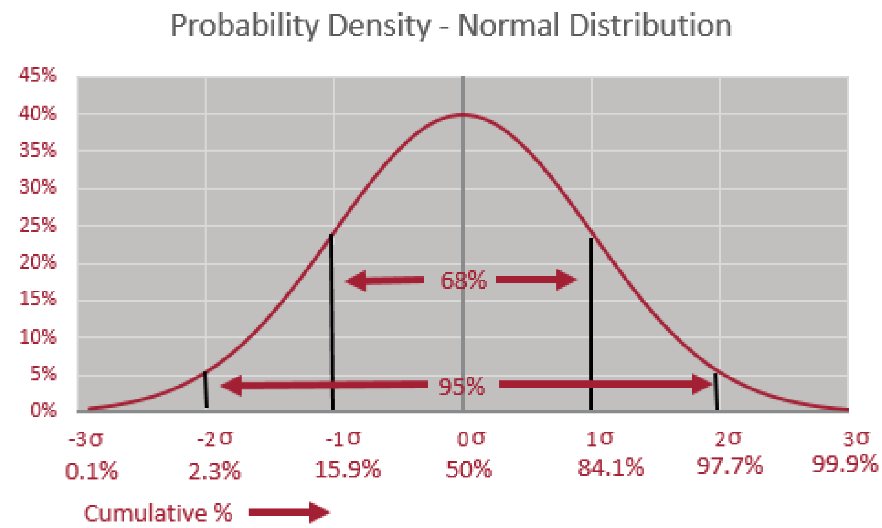


Figure 5: Normal Distribution Curve Used to Model Rent Growth

Also known as a 'Gaussian Distribution', a random variable is 'normalized' according to this distribution. The RAND function in Excel fetches a random number between 0 and 1 and is centralized towards the mean. For example, if the number comes out to be .159, it will be placed -1 standard deviation from the mean. 68% of values (.159 to .841) will fall within 1 standard deviation of the mean.

In ModerateCo, the uncertainty is created as a symmetrical Normal distribution around a mean. Using 3% as the mean for the rent growth rate, there should be an equal chance for the growth rate to appear above or below 3%.

3.2 Monte Carlo Simulations and Expected NPV

Once the financial model has input assumptions that randomly change, the corresponding output NPVs can be recorded many times using a Data Table in Excel. The process of running a model for a specified number of iterations is simply called a Monte Carlo Simulation. The NPV will vary from simulation to simulation because the input variables are always changing. In the case of ModerateCo, the input variable, Rent Growth Rate, changes.

Monte Carlo: What's in a name?

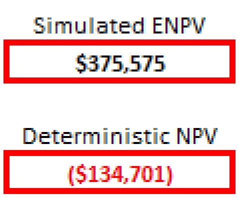
As a favorite hangout of Ian Fleming's fictional character James Bond, Monte Carlo is often associated with luxurious, mysterious and exotic living. Perhaps, Monte Carlo Simulations sound more foreign then they actually are.

"Monte Carlo" Simulation just refers to a simulation where the number of iterations are set by the user. For example, we run 5,000 iterations for ModerateCo Tower, not one more nor one less.

The input variable could be 2% leading to a certain NPV value, and in the next iteration, the rent growth rate could be 3.5%, leading to a higher NPV value.

After running 5,000 iterations of the model, ModerateCo Tower calculates the mean of the 5,000 NPVs to yield an Expected Net Present Value (ENPV). In this case, the mean of the simulated NPV (ModerateCo) will be consistently greater than the deterministic NPV (SimpleCo) even though 3% was used as the mean in ModerateCo. In other words, even if multiple sets of 5,000 simulations were ran, the simulated ENPV of ModerateCo will generally be significantly greater than the NPV of SimpleCo.

Figure 6: Results from the Monte Carlo Simulation ENPV versus Deterministic NPV



Interesting result! The simulated ENPV is an expected NPV because it is just an average of all the results in a Monte Carlo Simulation. In this case, ModerateCo's NPV was recorded 5,000 times and averaged to get an average of $375,575. The deterministic NPV is taken directly from the SimpleCo *pro forma.* This result can be viewed in the ModerateCo Excel file.

How could this difference occur? Shouldn't the SimpleCo NPV and ModerateCo ENPV be the same if we ran many simulations of ModerateCo?

Intuition may try to apply the Central Limit Theorem or Law of Large Numbers in this case. As the number of iterations of a random independent variable becomes very large, the variables will be normally distributed around the expected value (if using the NORM.INV function). In fact, there should be close to an equal number of occurrences of rent growth rate above and below the mean rent growth rate in ModerateCo since we are using a symmetrical normal distribution to model the uncertainty in the rent growth rate. While the input variable behaves this way with the expected value as its mean, this actually *does not* extend to the output NPV. The Flaw of Averages explains why.

3.3 The Flaw of Averages and Jensen's Inequality

First coined by Savage, Danziger, & Markowitz (2009), the Flaw of Averages is a major error that occurs when using averages in deterministic models instead of proper stochastic variables. De Neufville & Scholtes (2011) describe the Flaw of Averages as the widespread-but-mistaken assumption that evaluating a project around average conditions give a correct result.

The simple math behind the Flaw of Averages concept is based on Jensen's Inequality. In 1906, Danish mathematician Johan Jensen proved that:

> Jensen's Inequality
>
> The average of all the possible outcomes associated with uncertain parameters is generally not equal to the value obtained from using the average value of the parameters.

Basically what happens is a symmetrically distributed input variable leads to an asymmetric distribution of output values. When this occurs, the system or model is described as non-linear. The SimpleCo model is a perfect example because it's *pro forma* uses a 3% historic average for its rent growth. When the deterministic 3% is replaced with an input random variable symmetrical distributed around 3%, the output NPV value ends up significantly greater for ModerateCo over SimpleCo!

The source of non-linearity in this case is annual compounding. The same effect that makes compound interest (non-linear) greater than simple interest (linear) at the same rate generates the difference in returns between SimpleCo and ModerateCo.

For ModerateCo Tower, 3% is the mean growth rate, so a 2% growth rate and a 4% growth rate should occur with equal probability. Because the curve is convex (due to compounding), going up to 4% results in a greater upward NPV improvement [|2440-(-135)|= 2,575] than the NPV erosion of going down to a 2% growth rate [|-2,528-(-135)| =2,393]. Systems behave asymmetrically when upside and downside effects are not equal.

SimpleCo Tower Sensitivity Analysis

Rent Growth Rate	-$135
-2%	-$10,525
-1%	-$8,740
0%	-$6,819
1%	-$4,752
2%	-$2,528
3%	-$135
4%	$2,440
5%	$5,209
6%	$8,187
7%	$11,389
8%	$14,831

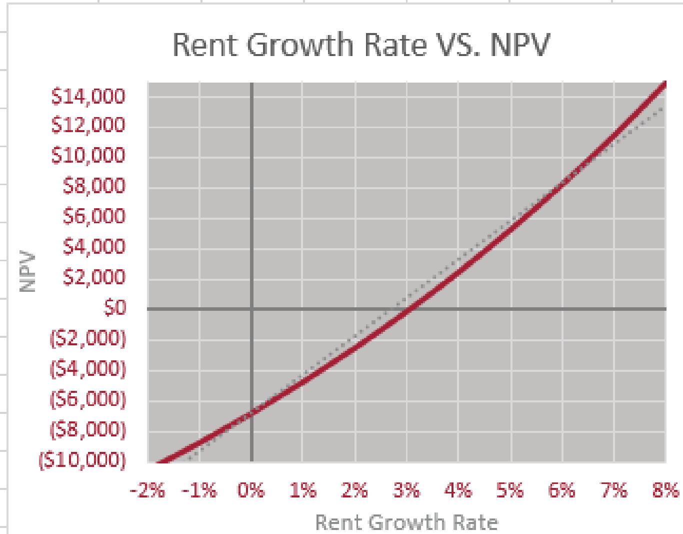


Figure 7: Non-linearity in the Rent Growth Rate

Let's say that we have two iterations of the model. In one iteration, the rent growth rate is 2%, and the other is 4%. Leading to a NPV result set of -2,528 and 2,440. If we average these two values, we get -44 which is higher than the result we would get at 3% of -$135! The difference becomes greater and greater as values further from the mean are used. This sensitivity analysis of the rent growth rate can be found under the SimpleCo *pro forma*, in the SimpleCo Excel file.

The Flaw of Average has a significant impact on NPV and could spell the difference between winning a bid and losing a bid, as exemplified by the SimpleCo and ModerateCo comparison.

3.4 A Different Approach to Real Estate Financial Analysis: Distributions and Risk Profiles

Incorporating uncertainty into real estate *pro formas* not only gives a different result over deterministic models (as per the Flaw of Averages), it changes the approach to real estate valuation problems. In the deterministic SimpleCo Tower case, the strategy is to lock in a set of *ex ante* assumptions based on the analyst's best forecast, find the single best value and hope for the best. When uncertainty is factored in to the analysis, the focus shifts to modeling and managing the uncertainty to make better finance and design decisions today. The single best expected value of NPV is no longer the sole objective in a stochastic model: range and distribution of outcomes become relevant.

Introducing a realistic level of randomness into financial models changes the framing of the valuation problem. Understanding the *likelihood* of losing or profiting become important once we introduce uncertainty into the analysis. The cumulative distribution function (CDF) of ModerateCo provides information on the probability of loss or profit scenarios. ModerateCo has about a 50% probability of having negative NPV and a 50% probability of a having a positive NPV. The downside probability is more limited than the upside probability, as illustrated by the long tail towards the right (more positive NPVs).

This scenario is a typical observation for real estate projects. Ideally, an analyst will want to manage the uncertainty by finding ways to limit the downside losses and accentuate the upside profits.

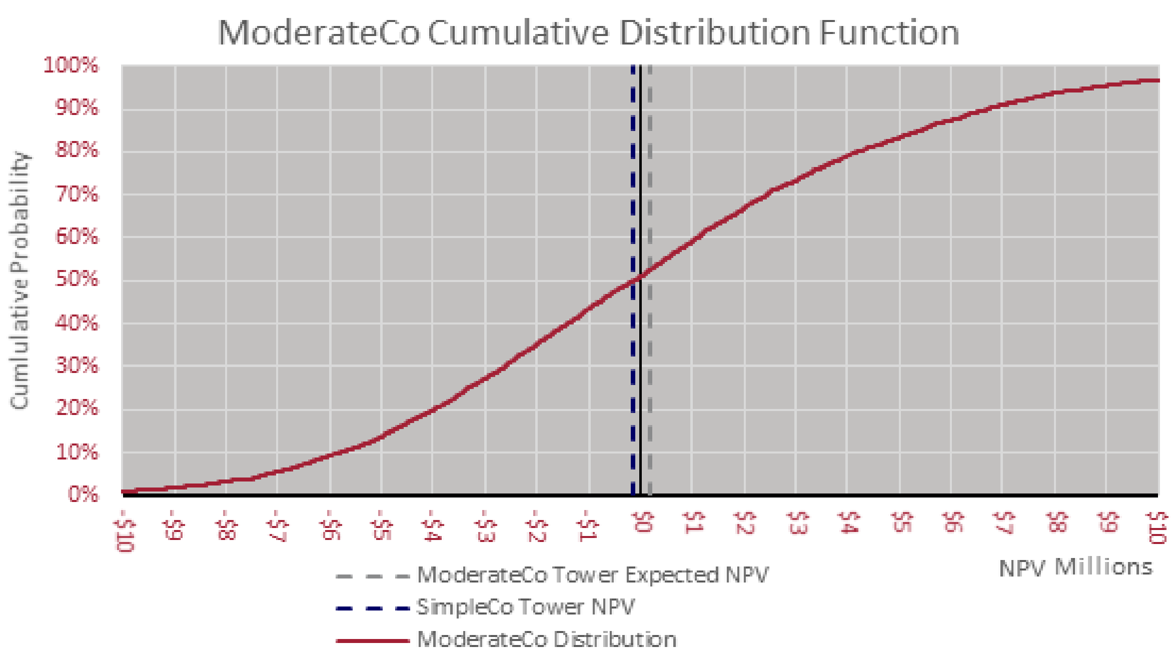


Figure 8: ModerateCo Tower Cumulative Distribution Function

On the CDF, the likelihood of NPV outcomes is displayed as well as the NPV for the deterministic SimpleCo Tower and the probabilistic expected NPV for ModerateCo Tower. This chart and its corresponding Monte Carlo Simulation is included in the ModerateCo Excel file, on the 'ModerateCo Distribution" tab.

Other useful measures that come out of this analysis of distributions include Value at Risk and Value at Gain. Value at Risk denotes how much loss could occur at a specified probability over a time frame. In the ModerateCo example, the Value at Risk (V10 number)

NPV is -$6 million. That is, there is a 10% probability that a negative $6 million NPV or worse will be incurred over the 10 year life horizon of the investment. On the other side, the "V90 NPV" is $7 million. This Value at Gain "V90" number can be read as: There is a 10% chance that the NPV for the project over the 10 year investment horizon will be over $7 million.

3.5 Static Input Variables versus Random Walks

The rent growth rate's behavior in the ModerateCo model is currently a static variable. Once a rent growth rate is randomly generated for a scenario, it remains the same for the life of the investment. Deterministic *pro formas* frequently model input assumptions as a static variable because the basis for their assumptions are from historic averages of long-term annual rates. Economic conditions change over the life of a long-lived investment and deterministic financial models are poor at modeling this behavior. Since ModerateCo's input variables are randomly generated and do not rely on historic averages, a change over time over can be modeled in to the annual rent growth rate.

Growth rates generally do not move independently from year-to-year with absolute randomness; rates tend to vary around the results from the preceding period. Pearson (1905) described this behavior as a "Random Walk".

A random walk modeled into a *pro forma* will allow an investment's profitability performance to decline and recover over the investment horizon. This up and down behavior is essential to the modeling of real options in the proceeding chapter.

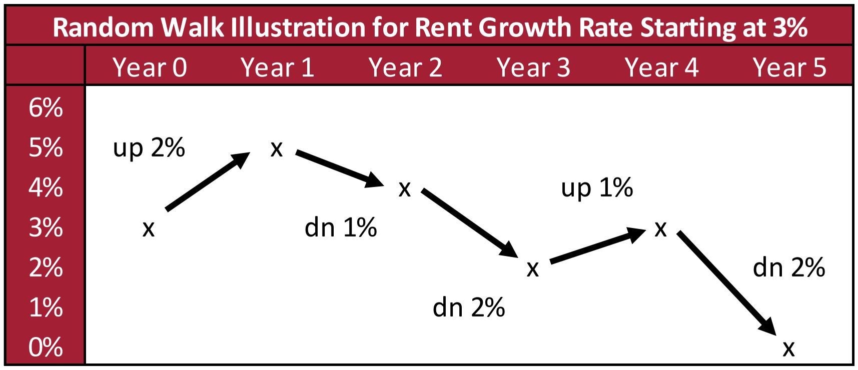


Figure 9: Random Walk Illustration

A visual representation of a year-to-year random walk evolution of the rent growth rate. This behavior can be exhibited by many different variables.

The additional variability translates into greater volatility in the results of the Monte Carlo simulation and amplifies effect of the Flaw of Averages.

The strong effect of the Flaw of Averages and Random Walk volatility should be enough motive to start modeling real estate using probabilistic techniques. The thesis continues to make the case for stochastic valuation of real estate in ChallengeCo Tower by using Real Options.

Results	ENPV	St. Dev.
ModerateCo	$192,043	$4,920,803
ModerateCo w/ Random Walk	$1,042,254	$9,240,832
SimpleCo Deterministic NPV	($134,701)	

Figure 10: Comparison of Returns between ModerateCo and SimpleCo

Random-walk behavior adds greater volatility to the model which inflates the effect of the Flaw of Averages even further. The Monte Carlo Simulations and results can be viewed in the ModerateCo Excel file.

CHAPTER 4: ChallengeCo Tower - Managing Uncertainty in Real Estate Projects

With distributions, we gather information that will aid us in finance decision-making. This chapter focuses on how to use this information advantageously. Real option analysis is utilized to explain the impact of adding flexibility into the design or financial models.

A Real Option is described specifically as a “right without an obligation”. Chapter 6 provides a detailed discussion on Real Options, but for now, a basic option to add more floors in the future to the on-going example of SimpleCo and ModerateCo towers will be described.

ChallengeCo does not stray much from the enduring example. The subject building is still a 10 story office building. However, ChallengeCo Tower is now an investment in a 10 story development project instead of a pre-existing stabilized office tower. Rather than a purchase price, we use a development cost to build the project. An option to build 10 additional floors in the future is examined further in the ChallengeCo Tower *pro forma* provided in the ChallengeCo Excel file.

Almost all input assumptions are subject to uncertainty using the same NORM.INV function described in ModerateCo. Additionally, the input assumptions will exhibit “random walk” behavior, with the preceding year’s value used as the mean for next year’s value. Each input assumptions will go through their own random walks, culminating into a specific NPV for a unique 10 year unique state of the world.

4.1 Real Option Analysis in ChallengeCo using IF Statements

Using IF statements in Excel, real options can be modeled with ease. Two pieces of information are required to model real options. Firstly, the “trigger” conditions need to be specified: What conditions need to occur before the option is exercised? Secondly, the exercise costs and other consequences of the option need to be identified: What is the effect if the option is actually exercised? Once these two pieces of information are detailed, the option can be modeled into the *pro forma*. The objective here is to model the option in such a way that the consequences of an exercised option are automatically displayed in the *pro forma* if the predetermined conditions occur. Then, a Monte Carlo Simulation examines the effects of the option on the NPV in comparison with an identical development without the option. Appendix D, discusses the use of "if statements" to model real options in greater detail.

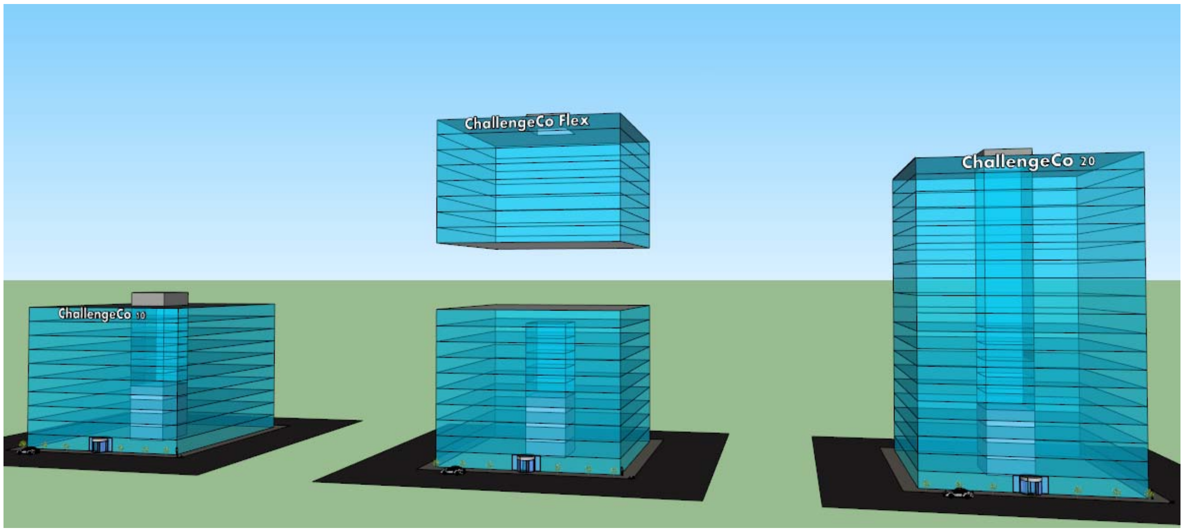


Figure 11: Three ChallengeCo Tower Options

The building on the left represents the Inflexible ChallengeCo project at 10 floors. It is physically identical to ModerateCo and SimpleCo Towers. In the middle shows a flexible design where an additional 10 floors can be built on top of the first phase of 10 floors. On the right is the inflexible 20 floor ChallengeCo Tower.

For ChallengeCo, three separate *pro formas* are created to show the difference in expected NPV and distributions. One *pro forma* calculates the NPV for a development project with a flexible design option built-in to the model to construct an additional 10 floors at a later date. The second *pro forma* calculates the NPV for a standard 10 story development with no option built-in. The third *pro forma* calculates the NPV for a 20 story development without an option built-in to the design.

4.2 ChallengeCo Tower's Result with a Real Option

The results from the Monte Carlo Simulations show that design flexibility can have a significant financial value. While the development with flexibility never dominates the two option-less alternatives (the flexible alternative distribution function is always to the left of either the 10 story or 20 story distribution function), the results show how the flexible alternative can be advantageous.

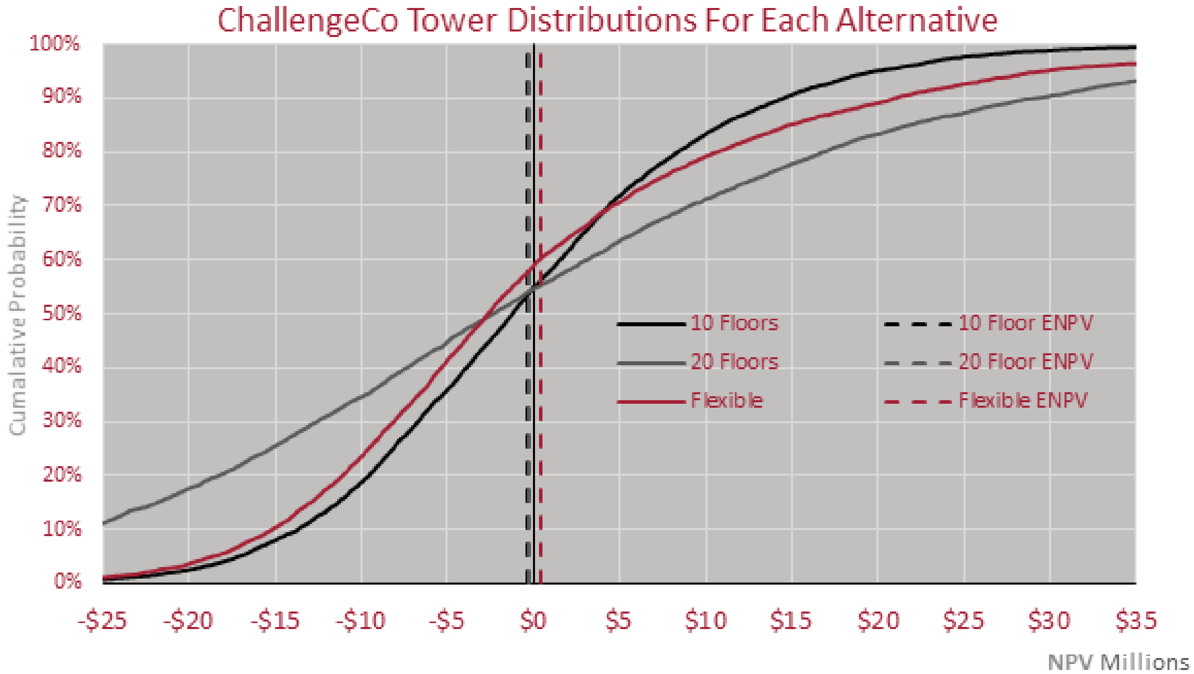


Figure 12: ChallengeCo Tower Expected NPVs

The CDF shows how the flexible design (in red) uses the real option to take advantage of upside conditions. At the low end, the flexible design does not exercise its option to expand, so its CDF curve closely follows the curve of the 10 floor inflexible design. If economic conditions are good, the flexible design begins to deviate from the 10 floor inflexible design by exercising its option to expand and follows closer to the 20 floor inflexible design curve to take advantage of the good economy. The Monte Carlo Simulations and CDFs can be found in the ChallengeCo Tower Excel file.

If economic conditions turn sour in the next 10 years, the option to expand is not exercised and the distribution function of the flexible alternative "hugs" the 10 floor inflexible option. In poor economic times, the flexible option will not perform as well as the 10 floor inflexible development because some extra construction costs are "sunk" into the initial construction cost of the flexible alternative (for example, constructing stronger columns to take the load of a possible 10 floor addition). On the other hand, the flexible alternative performs much better than the 20 floor inflexible development during a poor economy.

When economic conditions are good, the flexible alternative takes advantage of the upside by exercising its option to build more space. This is illustrated when the 10 floor inflexible alternative is compared with the flexible option above the $5 million NPV mark. The flexible alternative will deviate from the 10 floor inflexible alternative and capitalize on the opportunity of great market conditions.

The expected NPV of the flexible alternative is the greatest among the three alternatives for ChallengeCo Tower. For investors seeking to limit their downside exposure, while taking advantage of the upside as much as possible, flexibility can be a major win. Flexibility in design should not be overlooked when making investment decisions.

CHAPTER 5: Quantifying Uncertainty in the Real World

The example presented in Chapters 2 to 4 is simplified to underline the importance of including uncertainty and managing the risk that exists in real estate investing. In this chapter, the focus shifts to the execution of these techniques in the real world. To implement stochastic techniques into a real world financial model, the inputs that are subject to uncertainty must be quantified with a decent level of accuracy or the outputs cannot be trusted – the computer science axiom, "Garbage In, Garbage out" is appropriate here. Accuracy is important, but how precise or detailed should a financial model be? The real estate industry has embraced the DCF method which, as demonstrated by the Flaw of Averages in chapter 3, is generally less accurate and much less detailed than the simplest stochastic models. From this casual observation, perhaps professionals put more stock in accuracy (getting close to the actual number) rather than detail. There is a possibility of increasing the complexity of a financial model so much that the major fundamentals of the *pro forma* are washed out; losing sight of the "forest through the trees". Thus, it is important that increasing detail is not pursued at the expense of accuracy. The discussion herein is science based, but the implementation of these concepts remains an 'art' requiring real estate intuition.

5.1 Predictability in the Real Estate Market

In the 1960's, a powerful theory emerged called the Efficient Market Hypothesis (EMH) with contributions from economists such as Paul Samuelson and Eugene Fama (Malkiel & Fama, 1970; Samuelson, 1965). Their work explains how the stock market is so efficient and quick to adapt to new information that it is impossible to predict where the market is going, since future information is unpredictable. By extension, the stock market should behave as a complete random walk (Fama, 1995). Looking at historic trends is futile because future information occurs independently from what has already happened. Exchange traded funds (ETFs), which are funds which try to replicate the entire market, were created to make use of the EMH mantra, "active management of funds won't help". In fact, Fama & French (2010) show that 65% of actively managed high fee mutual funds did not beat passively managed ETFs.

In recent years, the EMH has been challenged by research that argues for a level of predictability existing in the markets. Lo & MacKinlay (1988) used new data to reject the idea of the markets behaving as a random walk. Jegadeesh & Titman (1993) discovered that momentum in the stock market can lead to above market returns; that is, relying on the short term tendency for a stock's price to go up if it was going up the last period. Shiller (1990) proposed that a mean-reverting behavior exists in the stock market due to investor irrationality. Alas, some level of predictability exists which can be used advantageously which keep financial analysts like the author of this thesis employed.

While weakened from modern empirics and the 2008 financial crisis, the EMH still affects the way investors' model future returns by cautioning market participants about how difficult it is to earn above-market returns. In addition, the EMH highlights the important role which uncertainty plays in the market.

Do these theories primarily focused on the stock market translate over to real estate? Yes and no. For variables in the office space market such as rent and vacancy, a level of predictability does exist due to patterns in momentum and cyclicality which are discussed in greater detail later in this chapter. In real estate asset markets, the EMH holds less weight compared to stock exchanges, because the cash flows of real estate asset (which are dependent on the space market) are fairly predictable. In general the EMH suffers from a lack of applicability to real estate because of the heterogeneity of real estate, the lack of public sales information, and time lags in the transaction process. On the other hand, Real Estate Investment Trusts (REITS) can behave similarly to the rest of the public capital markets. Generally, the more efficient a market is at integrating new information in asset prices, the less predictable it is. The predictive nature could even become endogenous to the price of an asset in a very efficient market. For example, when a new technique is developed and proven to be capable of making above-market risk adjusted returns, everybody will immediately copy the technique which becomes the new standard. The main takeaway from this section is that most real estate markets behave with both predictability and randomness at the same time and this should be reflected in the way financial models are created.

5.2 Real Estate Economics and the Stock Flow Model for Office Properties

The mechanics of how real estate prices move over time has been studied extensively and can be described using a Stock Flow Model. A Stock Flow model is simply any model which describes the process of how a durable stock of goods, such as real estate, increases and decreases and interacts over time with the flow of usage (i.e. leasing) of that stock of goods.

To start off, the Stock Flow Model draws on the familiar concept of supply and demand, with a few quirks, to correctly describe what occurs in real estate. Employment is the driver for rent of office space on the demand side. On the Supply side, the stock of office space is the main determining factor. The stock of office space is completely inelastic in the short-term because office buildings take time to build, When demand (employment) increases, the rent must increase because it takes time for new stock to arrive in the form of new construction. When employment fall, rents will fall by a greater percentage because of the durability of real estate capital leading to complete inelasticity in supply in the short run. New real estate stock is gradually introduced into the market to meet demand and because of this, rents and prices react quickly to changes in the demand, but stock does not. What triggers new construction? Asset prices – which are a function of rents and cap rates. Di Pasquale & Wheaton (1996) illustrates these relationships between construction, asset markets and space markets in the Four Quadrant model.

As the economy goes through its ups and downs, real estate prices and rents go up and down because demand changes without a quick response from the supply side due to the durability of real estate and lag to deliver new space. Eventually, increases in rents and prices promote new construction which gradually alleviates pressure on rents as the new space is delivered to market. Since there is a time lag in construction, it is rare that the exact amount of completions comes online and perfectly meets demand; there will be overbuilding and underbuilding which leads to real estate incurring its own cycle.

The variables required to create a stock flow model can be obtained by using linear regression techniques on a large result set of reliable historic data. Multiple linear regression attempts to quantify the relationship between a dependent variable and multiple independent variables. For example, a regression can be ran between the square footage of occupied space and employment, a major driver of office demand. If the numerical relationship can be predicted, forecasting employment (a source of demand) will lead to a prediction for occupied space.

In terms of complexity, uncertainty can be incorporated at different levels in a financial model. Some analysts will prefer to model uncertainty into rents directly, while others will prefer to add uncertainty to more primal sources such as employment growth and run these numbers through a stock flow model to arrive at rents. Please refer to Paradkar's (2013) thesis for a model which incorporates uncertainty using the stock-flow model.

5.3 Sources of Uncertainty in Real Estate

There are infinite possibilities when it comes to events or shocks that may influence real estate asset values and rents. Uncertainty can be driven by changes in the macro-economy and local economy. Technological innovation such as hydrologic fracking come out of the blue to effect office markets catering to the energy sector. Transportation infrastructure changes give rise to winners and losers in real estate. The endless list of potential shocks need to be simplified into a few sources before they can be quantified!

With the help of recent innovations in real estate indices, 7 important forms of uncertainty can be quantified: long-run market trend, long-run market cycle, market volatility, short-run inertia, individual asset specific volatility, individual asset pricing noise, and 'Black Swans".

Long-run Market Trend: This is the straight line appreciation trend which prevails over the long term in the real estate asset market. Research into residential real estate trends have found that over the super-long term (over the course of a century), prices appreciate close to the rate of inflation (Eichholtz, 1997). Growth in commercial real estate over the long haul has been found to be slightly less than inflation because of depreciation (Fisher, Geltner, & Webb, 1994; Wheaton, Baranski, & Templeton, 2009). With this knowledge, professionals may be tempted to input 1% or 2% because of the stability the Federal Reserve Bank offers for inflation in the United States, but keep in mind that investment horizons for commercial real estate tend to be shorter than 20 years.

Long-run Market Cycle: The long-run market cycle is the oscillating nature of real estate prices that is easily observable on a time-series graph. The peak-to-peak or trough-to-trough timing has been between 15 and 20 years in last few commercial real estate cycles. Wheaton (1999) explains how there are two ways in which the real estate market could manifest itself. One view is that real estate developers are completely rational and forward-looking while the other view is that developers are backward-looking (or 'myopic') when forecasting future supply and demand. When agents are rational and forward looking, they have a good understanding of how the market behaves with uncertainty, so prices reflect the present value of future cash flows and the uncertainty surrounding the cash flows. A practice which would classify as "myopic" behavior include extrapolating average historic rates forward in financial models. In Wheaton's simulations, he finds that both cases still (myopic or forward-looking) generate endogenous long-run cycles within real estate as developers struggle to forecast the exact amount of space to build.

Market Volatility: Zooming in a little bit from the Long-run Market Cycle level, there is volatility which exists month-to-month and year-to-year along the cycle preventing a smooth oscillating curve. Events that can influence this type of uncertainty include natural disasters or announcements by central banks. Any new discovery of information that provides a shock that the market takes time to adjust to are uncertainties related to market volatility.

Short-run inertia: Also called momentum, this is the tendency for prices that are rising to want to keep rising – or falling prices to keep falling. To measure inertia, auto-regressive techniques are employed which measures the level of influence a previous period's price movement has on the current period's price change. If the relationship is high between the prices for the two periods, it means that people are using the current period's price as a basis to forecast future periods. It is interesting to note that inertia is very weak in data involving REITs because of how efficient securities markets are. The frictions in private real estate markets, however, allow momentum to occur.

Individual Asset Volatility: If a financial model focuses in on a particular property, the model will be subject to idiosyncratic asset volatility, that is, risk which is specific to an individual asset that doesn't apply to the rest of the market. For example, a municipality might announce a new rapid transit station to be constructed next to an office tower which created an unexpected boost in the office property's value.

Individual Asset Pricing Noise: In the private real estate market, every professional will have differing opinions on the value of a property. If polled, the opinions of value for thousands of real estate experts might be scattered around the 'most likely' value, with a greater spread for more unique properties and less of a spread for properties with multiple comparables. Noise is the effect that these differing opinions have on the pricing of real estate. Appraisers take into account noise when they provide a range of prices that they believe a specific property can sell for.

Black Swans: In defining what Black Swans events are, Taleb(2007) states: "first, it is an outlier, as it lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility. Second, it carries an extreme impact. Third, in spite of its outlier status, human nature makes us concoct explanations for its occurrence after the fact, making it explainable and predictable." Any event with major impact on real estate values encompasses this risk. For example, a new renewable energy source (making combustion engines obsolete) suddenly discovered in a lab at MIT could have major "Black Swan" type ramifications for real estate.

Each type of uncertainty described above can be quantified on their own. Then a Monte Carlo Simulation outputs the effect of uncertainty as a whole on a real estate project. Modeling the effect of 7 types of uncertainty together without a Monte Carlo Simulation would be practically impossible.

5.4 Real Estate Indices

Real Estate indices provide some of the data from which the 7 forms of uncertainty can be extracted. There are many choices with regards to real estate indices, with each having their own advantages and disadvantages. There are three major types of real estate indices in the United States: appraisal-based, transactions-based, and stock market based (Geltner, 2014).

Appraisal-based indices use independent professional appraisals of properties to track real estate markets. They were the earliest forms of indices in real estate, so they tend to have longer histories. The major drawback with appraisal-based indices is that they are susceptible to a phenomenon called appraisal smoothing. Appraisers tend to use empirical information such as sales comparable to determine the current value of properties which develops a lag in their estimated value. This lag contributes to a smoothing effect on the index which reduces the apparent systematic risk in the real estate returns.

Transactions-based indices (TBI) use actual sales data of commercial real estate to track the market. To accomplish this, many of these indices monitor pairs of sales on properties to ensure that the changes reported are from an apples-to-apples comparison. TBIs are relatively new with data only stretching back to 2000 but they hold great promise because the underlying transaction price data not only quantifies market volatility reflected in the indices themselves, but also quantify individual asset idiosyncratic uncertainty using the residuals of the price regressions.

Stock market-based indices track the movement of publicly traded real estate investment trusts. Because each REIT generally specializes in one property type or another, they can be a great source of data when looking at a particular geographical area or industry. Keep in mind that REIT values do not perform the same as private real estate all the time. The efficiency of the stock market eliminates much of the inertia that would exist in the private market. In addition, there is evidence that the REIT market slightly leads the private real estate market (Barkham & Geltner, 1995).

5.4 Translating Data on Uncertainty into a *Pro Forma*

Quantifying uncertainty can be done in many of ways on Excel, but an emphasis should be placed on clarity and transparency as there are many moving parts to a stochastic *pro forma*. Chapter 6 runs through a case study in which the research presented in this chapter can be implemented in Excel. There are many modifications that can be made to *pro forma* for the case study in Chapter 6 and some of these possible variations are discussed below.

In the ModerateCo example in Chapter 3, a normal distribution is used to disperse the randomness around a mean, but there are other common methods for distributing uncertainty.

Uniform Distributions: The RAND function in Excel functions as a uniform distribution on its own. Every number between 0 and 1 has the same chance of appearing. If an analyst wants to model a random change in price next year between -10% and +10%, with every value having an equal chance of appearing, they can use the function = (RAND( )/5)-0.1. The divisor of 5 creates a 0% to 20% range, while subtracting 10% shifts the distribution down to create a -10% and +10% bound.

Normal (Gaussian) Distributions: In the ModerateCo and ChallengeCo examples, a normal distribution was used to disperse random variables. Normal distributions are familiar with most professionals with college degrees because these distributions are common place in introductory statistics courses. Normal distributions are often observed and nature and play an important role in science and business. In the standard normal distribution, sometimes referred to as a “bell curve”, only two unknowns are required to create a curve: mean and standard deviation. The mean is the average value of all the random numbers in the distribution, and in a symmetrical normal distribution, the mean will lie directly in the middle of the curve. The standard deviation (denoted by σ) is a measure of the spread of the distribution. The larger the standard deviation, the wider the range of numbers will be. In a standard normal distribution an empirical rule exists that states: 68% of values fall within one standard deviation from the mean, 95% within 2 standard deviations, and 99.7% within 3 standard deviations. This rule can also be referred to as the 68-85-99 rule or the 3 sigma rule. When setting up a normal distribution of random variables, keeping the 3 sigma rule in mind will help “fit” a distribution to observed volatility in an index. Excel has numerous function which relate to normal distributions but for modeling uncertainty, NORM.INV is the most used. This handy function fetches the number at a certain cumulative probability of a standard normal curve with a mean and standard deviation that a user can specify. For example, if the normal curve mean was 3% with a standard deviation of 1, a probability of 50% in the NORM.INV function will fetch a 3%. In the probability turned out to be 15.9%, the NORM.INV function will fetch a 2 (since a cumulative probability of 15.9% ends up a -1 σ).

Triangular Distributions: Sometimes the range and most likely number for a random variable is known. For these problems where there is not much information available, a triangular distribution can be used. A major benefit to triangular distributions is that the bounds are limited, compared to the theoretically infinite bounds of normal distributions. Triangular distributions are commonly used in business because not much information is required, yet allows for a "best guess" as the most likely number (or the mode). The mode does not need to be at the median between the two bounds, but if isn't, it becomes more difficult to model in Excel. For cases where the triangular distribution is not symmetrical, it is suggested to use an Excel add-in, such as @RISK software, to simplify formulas using their framework. As for symmetrical triangular distribution, imputing =rand()+rand()-1 in to excel will model a triangular distribution between -1 and 1, centered around 0. To move the center and mode of the distribution, add or subtract values. For example, =[rand()+rand()]+19 will model a distribution between 19 and 21, with 20 in the middle. To expand the bounds, multiply the RAND+RAND expression with a desired factor. For example, =4*[rand()+rand()] will yield a distribution between 0 and 8 centered around 4.

Other distributions are also possible, but it is suggested that a program such as @RISK software is used to keep formulas from becoming untidy. Long, elaborate formulas decrease transparency and increase difficulties when troubleshooting.

CHAPTER 6: Managing Uncertainty in the Real World

Quantifying the uncertainty in the inputs of a financial model was the focus in chapter 5. Now the focus will shift to a discussion on methods that manage uncertainty. The word "option" is frequently used to describe an alternative or a choice in everyday speech. The definition for an option used in this thesis is more specifically defined as "a right, but not the obligation, to buy (or sell) an asset under specified terms" (Luenberger, 1998). This chapter sheds light on how financial options generate value and discusses the applicability of financial option analysis to improve the financial performance of real estate projects.

6.1 The Basics of Financial Options

Options are a class of financial instruments widely known as derivatives. Derivatives are aptly named because their values are *derived* from other assets. Options can be conceived for stocks, bonds, commodities, foreign currency and other assets. In essence, options are contracts acquired at a cost that allow a party the right to purchase or sell an asset, without obligation, usually at a specified time and at a predetermined price. Just like the assets which these "contracts" are dependent on, options can be traded in private or on public derivative markets, such as the Chicago Board Options Exchange.

Two major types of financial options exist: call options and put options. Call options offer a party the right to *purchase* an asset for a predetermined price. Put options offer a party the right to *sell* an asset for a predetermined price. This predetermined price is called the strike price or exercise price.

Here is a scenario that demonstrates how a call option works:

Prospero Mining Company's stock price is $100 today and is undergoing an important geological study at one of their prospective mining sites in Canada. Portia, the rich savvy investor, only wants to invest in Prospero if the geological study finds gold; it would be disastrous for the company's stock price if gold is not found. However, Portia is also afraid that she might lose out if gold is found because the stock price of the Prospero Mining Company has the potential to double or triple! Portia's solution is to purchase a call option for the Prospero stock for $5 (known as the option premium) at an exercise price of $110.

For $5, Portia gains a right to buy Prospero Mining Company's stock at $110 sometime in the future. If Prospero Mining Company's geological study turns out positive, the stock price doubles, but Portia maintains the right to buy the stock at $110.

The mechanism for put options works the same as call options, except it is now a right to sell instead of buy. For example:

Antonio is a wheat farmer and he's worried that the market price of wheat may fall from its current price of $10 a bushel. If the price falls below $8, Antonio will not have enough income to get by this year and would need to sell his farm. Antonio buys put options from Claudius for an option premium of $1 per bushel with a strike price of $9. No matter what happens to the wheat price, Antonio will be able to survive because he has hedged his downside risk. If the price of wheat increases, Antonio will not exercise the option. If the price of wheat falls dramatically, Antonio is safe because of the put option.

To simplify the scenarios, the duration that an option is valid for was not discussed in Portia's or Antonio's example above. In reality, options vary in their exercise terms and expiration dates. The two most common types of options are American and European options. In typical American options, the holder of the option can exercise the option at any time before the expiration date; if an option is good for a year, the option holder can exercise it anytime within a year. In European options, the option holder can only exercise the option at the expiration date. Thus, the option holder of a European option has much less flexibility in exercising the option. Other exotic option types exist, but the vast majority of options are sold in an American or European style.

6.2 Sources of Value for Financial Options

Options provide risk mitigation by effectively operating as insurance for more costly assets. In the section 6.1 examples, Portia and Antonio were able to change their exposure to risk by purchasing options. The future may lead to positive or negative outcomes, but options allow investors to hedge against the risk of negative outcomes. There is no doubt that options can be very valuable, but what actually generates this value in options? Fundamentally, there are two drivers of value for an option: time and uncertainty.

In American options, the value increases as the expiration date is further into the future because it provides the option buyer more flexibility in the exercise timing. The longer the duration of the option, the greater the chances of the option present in a "in the money" state which increases the value of the option.

The value of a European option depends on the market prediction of what the environment will be like at the exercise date, since European options can only be exercised at one forward date and not before. All else being equal, the further into the future an exercise date is for a European option, the greater the uncertainty; leading to a higher option premium.

Options are only relevant because of uncertainty. In a world void of uncertainty, no one would buy or sell options because market participants would simply chose to buy or not buy the underlying asset with complete knowledge of what their investment return would be. An option's function is to protect an investor from risk, but with no risk to hedge against, there is no need for options. It is interesting to note that, as the level of uncertainty increases, the value of an option increases as well. Here, the uncertainty can be thought of in two components: the possibility of loss, and the magnitude of the potential loss.

The higher the probability of an option being exercised, the higher the option premium will be priced at by option writers. If there is near certainty that an option will be exercised, option writers will price their option very close to the strike price to ensure that there is a fair deal for both the buyer and seller in a competitive market.

The magnitude of loss relates to the volatility of an underlying asset's value. Assets with large price swings will not only have a higher possibility of being exercised, but also have the potential to overshoot the exercise price by a greater margin creating a larger loss for the option seller and a larger gain for the option buyer. While the purchasers of options are protected with a right but not an obligation, the writers of options are obligated to deliver on their contracts, exposing themselves to risk. In a competitive market (see the discussion on efficient markets in section 5.1), this risk will be priced *ex ante* into an option with available information, leaving little room for above-average risk adjusted returns.

6.3 The Valuation of Financial Options

Like other assets traded on a public exchanges, options are subject to competitive market dynamics, where the market collectively determines the price. How do market participants value options? There are three major techniques used to value options: Black-Scholes (mathematical) models, Binomial Pricing Model Models, and Monte Carlo Simulations.

Black and Scholes (1973) introduced the famous Black-Scholes formula to calculate the price of European Style Options using 5 known variables: strike price, stock price, time, volatility, and risk free rate. The model works on the assumption that options will be priced correctly in the market -- arbitrage opportunities (using replicating portfolios) quickly bring option prices back in line. In essence, Black and Scholes used this assumption to derive an equation for valuing European options. Unfortunately, the Black-Scholes formula is tremendously cumbersome to work with for American Options because of the possibility of exercise before the expiration date of an option.

First proposed by Cox, Ross, & Rubinstein (1979), Binomial Tree Models quickly became a favorite among analysts trying to model American Options because of the model's intuitive simplicity. The Binomial Tree model is created by formulating different scenarios which could occur to an underlying asset over time and recording them into a lattice structure. Each level of the tree represents a period of time, with two routes (up or down routes) available for the asset's price to follow at each node (Veronesi, 2010). Probabilities are assigned to each path, but normally, analysts define each branch as having a 50% chance of realization to simplify the model. A new asset price is assigned for each branch in the tree, leaving a visual representation of a varying price of an underlying asset over time. Based on the forecasted values (at discrete time intervals), the option premium is calculated starting from the end values and gradually computing the option values all the way back to the present.

A major limitation of the Binomial Tree Pricing Model is the inability to incorporate multiple sources of uncertainty. All uncertainty must factored into the price and probabilities which the model operator employs at each node. To overcome these restrictions, analysts have started to harness the power of computing technology to simulate thousands of scenarios in a matter of seconds, dwarfing the limited number of possibilities which can be modeled in a Binomial Tree. Using Monte Carlo Simulations to model the value of options allows for vast customization of uncertainty. To model option premiums using Monte Carlo method, an analyst first sets up a model in which an underlying asset's price is subject to uncertainty over time. IF statements (see Appendix D) are used to mimic the logic in option exercise decisions made based on the simulated asset value. Once the model is in place, many iterations are performed, with the final effect of the option in each scenario being recorded and analyzed.

A Monte Carlo Simulation acts as a "black-box" or a short-cut where difficult partial difference equations is normally required find an option's value. Like a stew cooking in a large pot, an analyst can continue throwing different factors of uncertainty, pay offs, probabilities and other nuances into the financial model. The computer does all the heavy lifting with Monte Carlo Simulations! It is no secret that this thesis advocates for the use of Monte Carlo Methods because of all of its advantages for very little effort.

6.4 Real Options

Like their financial counterparts, a real option is defined simply as a right without obligation. While financial options are tied to securities such as stocks or bonds, real options pertain to business decisions and are often, but not always, associated with tangible assets such as real estate or machinery. Incorporating design flexibility into a real estate project is an example of real option. The ability to alter the design of a structure to meet future conditions is a choice which can be made in the future. Yet, there is no obligation to exercise the option if conditions do not support a change to the structure.

Many principles of financial options translate over to real option analysis, but there are a few key differences. Real options are more valuable when there is greater uncertainty looming over business decisions and they tend to run in perpetuity with American style option exercise terms. Real options are not sold on public options exchanges, so arbitrage opportunities to correct prices of real options do not exist. Furthermore, real options may not be derived from anything at all, making each real option very unique to their situation. With no asset to serve as a basis for its value, real options are not really derivatives; they are more like business decisions which can be made in the future. With financial options, the underlying asset's price effects the value of the option, but for real options, the factors that determines value are not always tradable commodities. These factors could be intangibles such as demand-based measures like rent or vacancy.

In a real option, the cost to implement flexibility is often independent of economic conditions. To put it in another way, financial options have the expectations of the future priced into it by the market, making it difficult to make a profit. Typically, this is not usually the case with real options because no price correction mechanism exists. For example, let's say a small-cap development firm, Lear Developments, is planning to build a three story parking garage and wants to embed the flexibility to expand the garage at a future date by building another three stories on top of the existing structure. The option premium in this case is the extra construction cost to add the flexibility and the cost to build the three additional stories. Capulet Construction builds the parking garage and sends Lear an invoice based on the current labor and building material prices. Capulet Construction does not care about what profit Lear will earn; they just want to build the parking garage, collect their fee, and move on to another construction project. There is a great opportunity for Lear to make a positive NPV on the project by exploiting the disconnect between the construction cost and the economy; the real option premium is not related to the potential payoff!

Financial options tend to have simple terms, but real options can involve many interrelated qualities. Rather than a simple exercise price, real options could have a grand criteria that needs to be satisfied before it is exercised. In these situations, the Black-Scholes Model and Binomial Tree model cannot quantify the value of a real option. The level of complexity required by real options analysis makes the Monte Carlo Simulation the tool of choice when dealing with real options.

Financial options are valued by directly inputting unknowns, such as strike price and exercise period, into equations to arrive at the option premium. Real options are not as straight forward. Because of their intricacy, it can be near impossible to directly compute the value of a real option. Monte Carlo simulations offer a work-a-round solution by allowing analysts to find the NPV of a project without an option and comparing it to the NPV of the same project with an option in place. Conceptually, the option value would be the difference between the two NPVs.

The ability to model different real options into a real estate financial model is an option in itself! The NPV values which are calculated from a real options analysis are never binding. Real option modeling enjoys asymmetric outcomes; all of the real options that were not worthwhile are not undertaken, while "home run" options are pursued further. There is nothing to lose when modeling options, but potentially a lot to gain.

Sometimes, real options are already free to implement. The freedom to walk away from a property with an 'underwater' loan prevents further losses, effectively capping downside outcomes. In a financial model which incorporates uncertainty, this real option of walking away from an underwater loan improves expected NPV, yet costs nothing. Therefore, modeling this real option into a financial model can provide that extra edge that is difference from winning and losing a competitive land bid.

CHAPTER 7: Two World Trade Center Case Study

We developed a short case study to connect and apply the concepts presented in this thesis to a realistic situation. The scenario created for this thesis is inspired by an actual case study done on the World Trade Center site in New York City (Queenan, 2013). While the story is fictitious, the assumptions are made to be as realistic as possible with basis from legitimate sources.

7.1 Scenario Background

Wall Street has seen better days. 5 years after one of the deepest recessions in US history, the blame game is in full flight. Recovery has been excruciatingly slow and the popular thing for politicians to do is to pick apart Wall Street. No one seems sure about the future state of the financial sector.

After much difficulty with leasing One World Trade Center and Three World Trade Center in the thick of the financial crisis, Goldstein Properties and The Port Commission are hoping to unload the development rights to the 2.3 million square feet office piece of Two World Trade Center. Arden Forest Properties is interested in developing the blue chip office tower and brought in Timon Capital to be the money partner.

Figure 13: World Trade Center Site Plan (PANYNJ, 2013)

Two World Trade Center sits on the most North Eastern parcel in the site. One WTC, 3 WTC, and 4 WTC are all office towers.

Not surprisingly, Timon is very concerned about the future of the office market in Manhattan. In an effort to put their business partner at ease, Arden Forest creates a *pro forma* to see how the development would perform with uncertainty in the market and address the downside possibilities. Michael Cassio, a senior analyst at Arden Forest, wonders if this is a good opportunity to use his internship experience last summer at a derivatives desk in Chicago. Although Michael has only seen the value created by financial option for an investor, he suggests implementing an option in this development to see what would happen.

Figure 14: 2WTC Rendering (PANYNNJ, 2013)

A rendering of 2WTC as viewed from the 9/11 Memorial.

Two World Trade Center is part of a major mixed-use development consisting of 5 office skyscrapers plus retail, cultural, and transportation infrastructure. Sitting at the North East corner of the site, Two World Trade Center will be the second tallest building of the World Trade Center complex. As with all the other WTC sites, The Port Commission will look after the construction of the foundations because of the intricate network of tunnels below connecting to new WTC transportation hub south of the 2WTC site.

Goldstein has already committed to developing the retail podium at the foot of 2WTC, but now they going through a bid process for the development rights to the office portion of the property.

Michael Cassio knows that this will be a very competitive bid for a rare world-class property. The dollars at stake are much higher with a landmark skyscraper in the world's most prominent financial center. The slightest miscalculation on the *pro forma* can cost Arden Forest billions. Michael hopes that his secret weapon, real options analysis, will help Arden Forest win the site without exposing them to risk without compensation.

7.2 Creating a Detailed Stochastic *Pro Forma* for 2 World Trade Center

Michael begins with a big picture strategy for the *pro forma*: quantify uncertainty in the office market, create a real options model, and evaluate the *pro forma* by performing a Monte Carlo Simulation. Michael understands that office employment growth is the main driver of real estate demand, but there is simply not enough time to compile the data he needs for the bid that needs to be submitted in just a few days. Plus, he's not comfortable creating a real estate stock-flow model because he didn't read section 5.2 of this thesis or Sarwesh Paradkar's award winning MSRED thesis. Mr. Paradkar shows how the stock flow model can be implemented with a little extra data on employment, vacancy, and real estate stock (Paradkar, 2013). As far as uncertainty goes, Michael decides to directly forecast office rents.

7.3 Projecting Rents, Cap Rates, Construction Costs and Operating Expenses

Initial Rent

First off, Michael needs to know what the average office lease would go for in 2 WTC if it were leasing today. Luckily, there are many lease comparables within the same complex! One World Trade Center and 4 World Trade Center are asking for $75 per square for in gross rent for space despite an abundance of vacant space in the Downtown submarket (Levitt, 2013). Not all is lost as the entrance of One World Trade Center to the market has gradually increases office rents downtown to an average of $60 per square foot (Kozel, 2013). Michael decides that $65 per square foot is a reasonable rent for 2 WTC, since it will be a brand new Class A building. On the other hand, Michael doesn't want to put rents near $75 per square foot because 2 WTC needs to entice tenants away from other competing WTC office towers.

Leading with $65 per square foot, we can follow along with Michael's work in the 'Projections' tab of the 2 World Trade Center *pro forma*. We will gradually add layers of uncertainty to our rent forecasting.

Long-Run Trend

This was an easy one for Michael. The long-run trend for office properties has been well researched in terms of asset price and rents. Office properties don't beat the rate of inflation over the long-run (Eichholtz, 1997; Fisher et al., 1994; Wheaton et al., 2009). In fact, real estate will do slightly worse because of depreciation in the property. Since the US Federal Bank has a stated goal to keep inflations at around 2%, Michael chooses to use 2% as the long-run prevailing trend and varies it using a normal distribution. By setting a half-range of 1%, Michael uses 1% divided by 3 as the standard deviation in the NORM.INV function, which makes it a 99.7% probability that the long-run trend will lie between 1% and 3%. The initial rate now exhibits long-run trending by adding the percentage increase year after year.

Market Volatility

For market volatility, Michael uses gross rent data to find the historic volatility. In the assumptions Excel file, there is a sheet called volatility which details how to find the volatility of rents. In this case, we have quarterly data of rents and we first covert the rents into a percentage change from quarter to quarter. Next, the standard deviation is found on the quarterly changes using the ST.DEV function. Lastly, we translate the quarterly volatility to an annualized number but multiplying the quarterly volatility by the square root of the number of periods. In this case, we multiple the quarterly standard deviation by the square root of 4 to get an annualized number. Of course, the volatility could be positive or negative in any given year, so Michael uses the NORM.S.INV function. This function uses a cumulative distribution function and translates a random variable to go either positive or negative around a normal distribution. A random number of .5 will make the factor 0, while and cumulative probability of 16% (roughly at -1 standard deviation) will end up with a factor of -1 and so on. Calculating the standard deviation on the projected volatilities should return a number close to the historical 7%.

Inertia

For momentum, we use the gross rent data and take the first difference of it, which is the rate of change from year to year. A linear regression was performed with a lagged percentage change in rents to arrive at an inertia factor. Please refer to the linear regression performed in the AR tab of the “2 WTC assumptions” to see how the rate of 33.5% for inertia was retrieved. If the market volatility was negative last term, the current period will have some downward pressure from momentum.

Market Cyclicality

Peak-to-peak, real estate cycles have lasted between 15 and 20 years (Geltner, 2013). While not always synced, the real estate and asset and space markets appear to have similar durations. To model this cyclical effect, Michael uses a sine curve to create a factor for rents each year. The coefficient in front of a sine curve affects the amplitude, or the height of the waves, and the numbers inside the sine function affect the duration and position of the curve. The Sine curve on its own has a cycle duration of 2π and amplitude range of 1 to -1. So, Michael translates the Sine curve to work in the spreadsheet by using the formula:

$$\frac{a}{2} \times \sin\left((y + p) \times \frac{2\pi}{d}\right) + 1$$

Where: $a =$ Maximum amplitude in %

$y =$ Number of years since start year, with start year = 0

$p =$ Cycle Starting Position (years after upward mid-point)

$d =$ Duration of one full cycles in years

The +1 at the end of the function shifts the entire sine cover up to have a mid-point of 1 instead of 0.

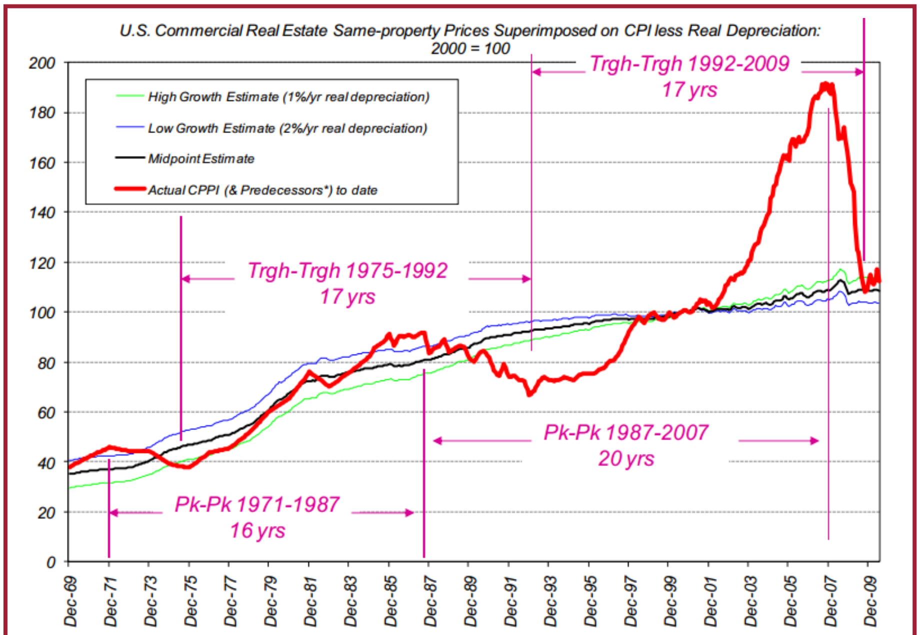


Figure 15: Real Estate Cycle Length

The peak-to-peak and trough-to-trough duration is between 15 and 20 years (Geltner, 2013).

For amplitude, Michael does a quick analysis of NYC office rents between 1988 and 2011. First, he converts the nominal rents to real rents, then he observes the difference between lowest and highest values with the total change from peak to trough observed to be close to 40%. Lastly, Michael looks at the Moody's/RCA CPPI TBI to see where the real estate environment is in the cycle. It seems like it should be around half way on to the peak in 2013, but Michael decides to retard the cycle in the analysis a bit because economic recovery has been slower than usual.

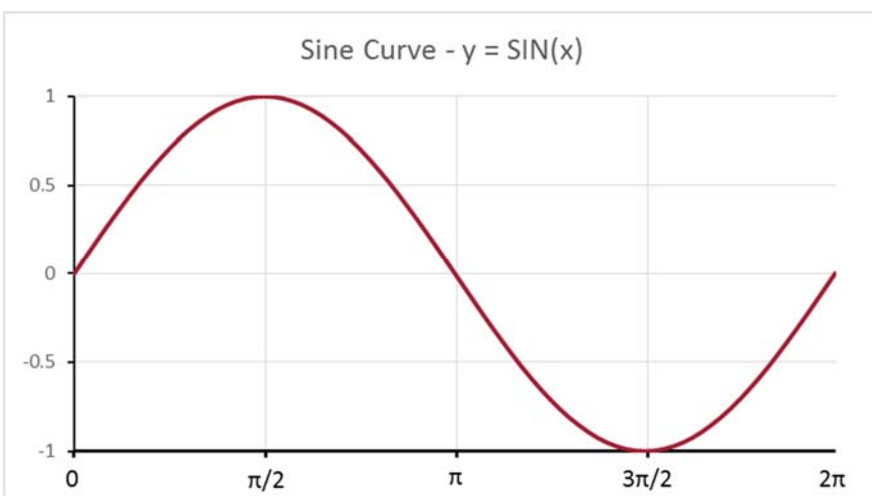


Figure 16: The Regular Sine Curve

When not transformed, the sine curve has a cycle duration of 2π and an amplitude of 1. The cycle's y-intercept is 0.

With all the parameters accounted for, the Sine wave is converted to a factor with the current year as a factor of 1.

Noise

Noise is taken in to account symmetrically with a 10% range normal distribution. 10% is a typical range used by appraisers when providing their opinion of value.

Black Swan

Black swans are by definition, impossible to predict but we can still simulate the effect. Michael gives each year a 5% chance to occur, which gives about a 40% chance of a black swan event occurring every 10 years. The magnitude of impact is set at -25%, more than half of the cycle effect occurring one year seems appropriate for a meaning event that will affect the investment.

Now that rents are modeled, Michael turns to modeling other items which need to be projected forward.

Cap Rate

Projecting future cap rates is important because it greatly affects our terminal value calculation and option trigger. Again, looking at RCA data, the cap rate seems to fluctuate between 8.5% and 5%, giving a midpoint of 6.75%. Since 2 WTC will be a modern blue chip office tower, Michael decides to set the mean cap rate a little lower at 6.5% with the same half range of 1.75%. The asset market cycle tends to lead the space market cycle but they can be out of sync at times. Going for the rule rather than the exception, Michael sets the position of the cap rate cycle one year in front of the space market cycle.

Operating Expenses and Construction Costs

Michael uses the same method to account for uncertainty in expenses and construction costs as the ModerateCo example in chapter 3 of this thesis. Operating expenses are set to grow at around the targeted rate of inflation by the US Federal Reserve Bank, 2%.

For Construction Costs, RSMeans data says that office building construction costs in New York have risen about 5% in the last year (Carrick, 2013). The hard cost quoted by RSMeans is $223 per square foot. Assuming that 2 WTC will be a high quality, expensive building, Michael uses $300 per square for hard costs. Adding a rule of thumb (30% of hard cost) for soft costs, brings the total construction cost number to $390 per square foot.

7.4 *Pro Formas*

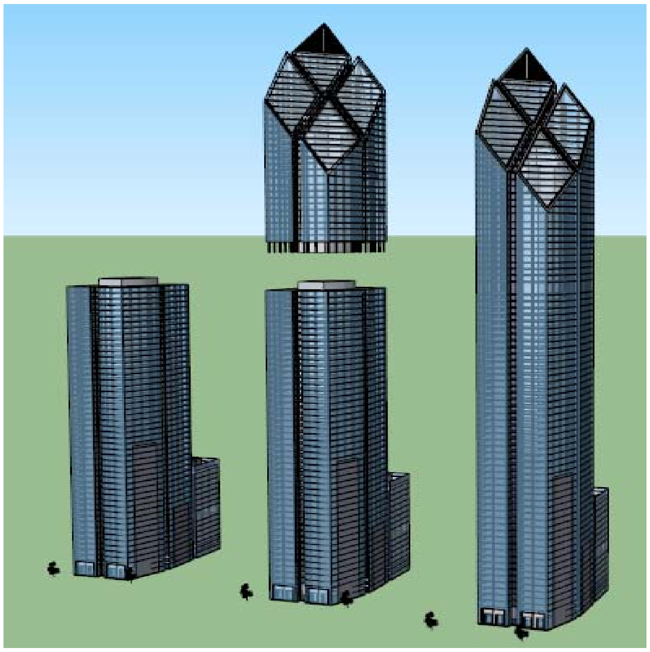


Figure 17: 2 WTC Sketch up of Alternatives
The building on the left is a model of a 30 floor inflexible design. The middle building represents a flexible design option while the building on the right is the inflexible 60 floor office tower design.

Now with our parameters projected in to the future, Michael is ready to set up *pro formas* to test out his real options hypothesis. He will create 3 separate *pro formas* and compare their financial performance under uncertainty. The first *pro forma* will be a 60 floor inflexible case. Another inflexible case will be modeled, but for 30 floors instead of 60 floors. The last one will have a real option imbedded into the design by starting with 30 floors and allowing for the flexibility to build another 30 floors on top in the future.

The construction time should be longer for the 60 floor tower, so Michael makes sure that the 60 floor tower takes 4 years to build versus the 3 that the 30 floor towers need. The construction costs are discounted at an OCC of 1.6%, with 50 basis points from the risk premium of construction cash flows and 110 basis points for the risk free rate. The risk premium for construction cash flows reflect the low systematic risk involved. Construction costs are usually locked in with a contact and do not move with capital markets.

The risk free rate is 1.1%, using the 10 year Treasury bill rate, minus 150 basis points to account for the yield curve effect (Bloomberg Markets, 2014). The cash flow from the tower during its fully leased, stabilized phase is discounted at 5.1%, reflecting the average risk premium of 4.5% for all NCREIF commercial real estate between 1970 and 2010 (Geltner, 2014). Michael haircut the risk premium by 50 basis points because this office tower will be a blue chip property in one of the most desirable locations in the world. Lastly, the stabilized NPV will be discounted in the development phase by 7.1% to account for the risk involved in leasing up a primarily speculative development. Geltner (2014) states that the risk premium for development phase cash flows typically have a 50-200 basis point premium over equivalent properties in the stabilized phase.

Many developers may be tempted to use a single discount rate for the entire project. However, it is important that each phase with a different level of risk have a different opportunity cost of capital. These discount rates should also be justifiable through market evidence because it is the capital markets that determine these rates.

The *pro forma* models 5 and 10 year leases with rent escalations using if statements. This is key to our analysis because lease rates will be locked in by their lease terms while the market can move either way during the lease. Since Michael doesn't know what length the leases will be, he has used the RAND function to determine if the leases will be 5 year or 10 year leases, the most common lease lengths in commercial real estate.

The inflexible 30 and 60 floor *pro formas* do not stray much from a standard *pro forma*. The flexible 30 floor *pro forma* will need to model the real option though.

7.5 Real Option Triggers

To model a real option, two things need to be defined. First off, the trigger conditions need to be modeled. For the flexible case, the trigger is pulled whenever the addition becomes profitable. To measure profitability, Michael uses the NPV investment decision rule:

1. Maximize the NPV across all mutually exclusive alternatives;
2. Never choose an alternative that has NPV < 0.

The acquisition cost of the land does not factor into this decision because it is what is called a "sunk cost". Sunk costs are costs incurred in the past that cannot be recovered regardless of future outcomes. So, they should not be considered when making future decisions.

The NPV for each year is easy to calculate using project cap rates and construction costs. Michael uses an IF statement to act as a switch to signify if construction begins on the addition or not. Since the addition can only take place once, another IF statement is used to ensure that no addition has started previously before beginning construction in that year.

The other piece that needs to be defined for a real option is the consequence. In this case, what happens is that an extra 1.3 million square feet is added two years (to account for construction) after the year of the construction trigger. New leases have to kick in, and a 10% construction cost premium is added to the construction cost. The NPV is then calculated the same way as the inflexible *pro formas*.

7.6 Results

in Millions		Inflex 30	Flex 30	Flex 40	Flex 50	Inflex 60
Expected NPV		($191)	$14	$9	$4	$14
Median		($251)	($145)	($137)	($128)	($95)
Mode		($300)	($500)	($500)	($300)	($300)
Std Deviation		$371	$710	$714	$719	$731
Percentiles						
Value	5%	($686)	($792)	($847)	($901)	($962)
At	10%	($604)	($708)	($742)	($770)	($808)
Risk	25%	($455)	($543)	($535)	($527)	($502)
Median	50%	($251)	($145)	($137)	($128)	($95)
Value	75%	$7	$400	$397	$388	$414
At	90%	$295	$979	$963	$948	$977
Gain	95%	$509	$1,383	$1,372	$1,349	$1,380

Figure 18: 2 WTC Financial Model Results
The flexible 30 floor design and the inflexible 60 floor design outperform the other buildings. Note that the flexible 30 floor design has the lowest potential losses, yet maintains good gains when the economy is good.

Michael compares the returns with distributions after performing a Monte Carlo Simulation and the results are intriguing. The first thing that pops out is how the inflexible 30 floor model's financial performance is terrible. Also unfortunate, the real option did not have as much of an effect on expected NPV as Michael hoped. The Flexible 30 floor option has the same expected NPV as building the entire 60 floors outright. However, not all is lost because the flexible design is doing its job, protecting Arden Forest Properties when the economy is doing poorly. The cumulative distribution shows how the flexible design behaves like the 30 floor inflexible project at the low end of possibilities, but then starts to behave like the 60 floor inflexible project at the higher end. The flexible 30 floor option is definitely the most preferable option.

If there was no construction cost premium, the flexible 30 floor option actually out performs even at the top end in the same environmental conditions as the inflexible 60 floor option. During these conditions, the real option is almost always exercised right away in the first year possible, 2017. Despite taking an extra year to construct 60 floors, the flexible option comes on out above the inflexible option because of the lease timings. The economy reaches the peak of the cycle in 2019, right when the leases from the new addition come online, while all of the leases of the inflexible schemes are stuck at lower rates signed 2 years previously. Those poor performing leases will also be renewed at another low point in the cycle 10 years afterwards in the lowest point of the real estate cycle.

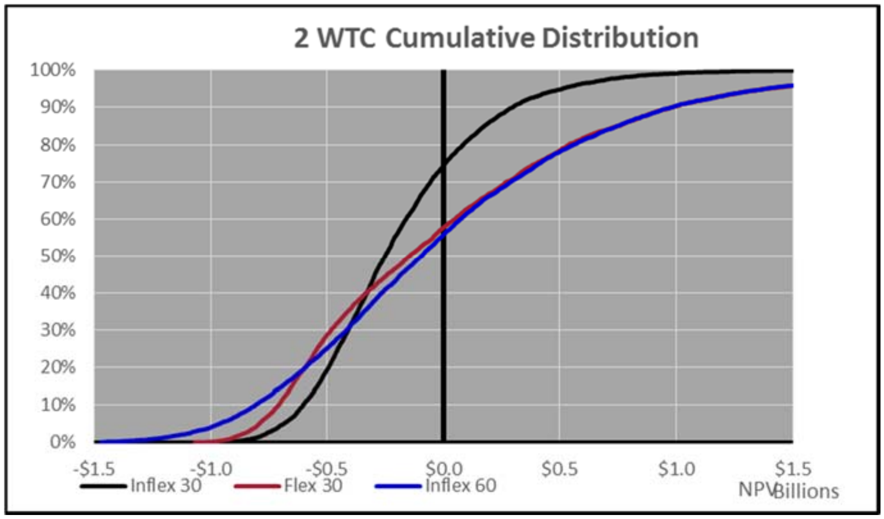


Figure 19: 2 WTC Distribution Function
The flexible 30 floor design's CDF performs like the inflexible 60 floor design, except at the low end of NPVs where the downside is minimized.

While the flexible 30 floor model outperforms the other schemes, it does so under specific conditions modeled by the Analyst. Regardless, Michael is convinced that real option analysis will help Arden Forest win the bid, arming the company with knowledge of distributions will help the company focus on managing uncertainty rather than relying on guesswork and gut feelings in deterministic financial modeling of Real Estate.

CHAPTER 8: Conclusion

"The more you know, the more you know that you do not know" is a commonly used maxim about Socratic Ignorance. It seems that uncertainty persists more today than ever before. Perhaps the human race is just learning more about uncertainty through humbling events such as the mega-recession in 2008. One thing that we can be assured of is that uncertainty will continue to play a role in real estate investing whether professionals acknowledge uncertainty or not.

There is little doubt that real estate firms and investors would like to incorporate the techniques presented in this thesis. The unfamiliarity with stochastic methods is the main stumbling block and is understandable when millions of dollars are on the line with each real estate project. At the same time, it is the fact that a lot of dollars are at stake which makes these techniques important. Engineers and scientists have relied on Monte Carlo Simulations to build atomic bombs, fly to the moon, and save lives during pandemics for almost a century now. Surely stochastic methods will add value in real estate if used properly.

In this thesis, it was shown how simple it can be to add uncertainty into otherwise deterministic *pro formas* which every real estate professional is familiar with. The unpleasant effect of Jensen's inequality on return measures in deterministic models is exposed. Monte Carlo Simulations were demystified in under 5 minutes using a few keystrokes in Excel. The value of distributions over point estimate was displayed, which gave an entirely new perspective on financial returns. Real Options was the tool presented that enables a real estate venture take advantage of uncertainty.

In Chapters 5 and 6, these concepts were further detailed with solid theory and empirical evidence. Uncertainty in real estate was broken down and explained, using new data tools and indices to quantify volatility and risk. The academic underpinning of real options was presented with theory borrowed from financial options. Then we revealed the mechanics of real options in the context of real estate.

To help translate theory in to practice, a modern example of 2 World Trade Center was presented. For simplicity, only one out of a variety of options was employed in the analysis, but the power of real options in real estate was clearly on display. While it is not a guarantee that real options in a real estate venture will increase value monetarily, the act of analyzing real options is a valuable option in its own right for real estate where irreversible investment decisions are made frequently.

Perhaps the greatest advantage of understanding these concepts is the change in mindset when it comes to approaching real estate problems. A new grand avenue is opened up when uncertainty becomes part of the analysis. There are endless possibilities with real option analyses and creative problem solvers will be the greatest benefactors.

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APPENDIX

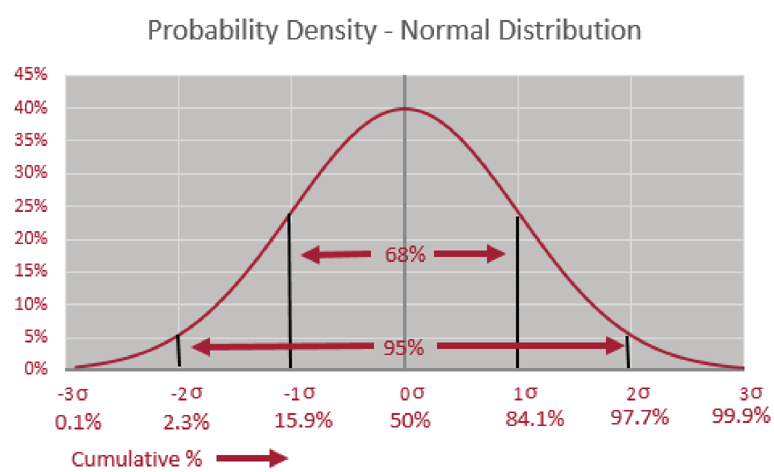
Appendix A   Incorporating Uncertainty into a Financial Model

The RAND function in Excel randomly generates a number between 0 and 1. Each time a recalculation occurs, the RAND function generates a new number. This function is the source of uncertainty in the ModerateCo model. Each number in the RAND function has the same probability of occurrence. For example, 0.1 has the same probability of occurring as 0.9.

To translate this random number to a working rent growth rate another function must be used in conjunction with the RAND function.

Chapter 5 is dedicated to modeling uncertainty in the real world. For now, a very simplified method is used to translate the random numbers generated using the RAND function in to rent growth rates. Instead of every number in the RAND function occurring with equal chance, extreme numbers or outliers should occur with less probability than numbers in the "center" or near a long-run mean.

The NORM.INV function takes in a random probability and translates the number using a normal distribution function. Recall that 68% of values occur within one standard deviation from the mean. 95% and 99% of value occur within two and three standard deviations of the mean respectively.



Nesting a RAND function as the probability input for the NORM.INV function allows the output value to be normally distributed instead of evenly distributed.

The probability accepted by the NORM.INV function is taken in as a cumulative probability of a normal distribution. If a 0.5 is given by the RAND function, the NORM.INV function will output the mean. If 0.023 is given by the RAND function, the NORM.INV function will output a number two standard deviations below the mean. Any probability between 0.16 and 0.84 will return a value within one standard deviation from the mean. The mean and standard deviation must be specified to make the NORM.INV function work.

ModerateCo Tower Assumptions		Mean	Std. Dev.
Purchase Price	$100 /gsf		
Gross Floor Area	170,000 sf		
Efficiency	90%		
Office Rent	$30 /sf		
Rent Growth Rate	=NORM.INV(RAND(),C9,D9)	3%	2%
Expense Growth Rate	3%		
Stabilized Vacancy	5%		
Expenses	$15 /sf		
Capital Expenditures	10% of NOI		
Terminal Cap Rate	11.00%		
OCC/Discount Rate	12.50%		

The deterministic SimpleCo example used a 3% rent growth rate. To maintain consistency, 3% is also used as the mean in the ModerateCo rent growth rate formula. As an assumption, 2% is used was the standard deviation for rent growth rate. Using 2% as our assumed standard deviation means that growth rate should be within ±2% of the mean (or between 1% and 5% in our example) 68% of the time and should be within ±4% of the mean 95% of the time.

(in 000's)	Year	1	2	3	4	5	6	7	8	9	10	11
Potential Gross Revenue		$4,590	$4,783	$4,985	$5,195	$5,414	$5,642	$5,880	$6,128	$6,386	$6,656	$6,936
Vacancy		$230	$239	$249	$260	$271	$282	$294	$306	$319	$333	$347
Effective Gross Revenue		$4,361	$4,544	$4,736	$4,935	$5,144	$5,360	$5,586	$5,822	$6,067	$6,323	$6,589
Operating Expenses		$2,550	$2,627	$2,705	$2,786	$2,870	$2,956	$3,045	$3,136	$3,230	$3,327	$3,427
Net Operating Income		**$1,811**	**$1,918**	**$2,031**	**$2,149**	**$2,273**	**$2,404**	**$2,541**	**$2,686**	**$2,837**	**$2,996**	**$3,162**
Capital Expenditures		$181	$192	$203	$215	$227	$240	$254	$269	$284	$300	
CF From Operations		$1,629	$1,726	$1,828	$1,934	$2,046	$2,164	$2,287	$2,417	$2,553	$2,696	
Reversion (Purchase and Sale)	-$17,000										$28,749	
PBTCF	**-$17,000**	**$1,629**	**$1,726**	**$1,828**	**$1,934**	**$2,046**	**$2,164**	**$2,287**	**$2,417**	**$2,553**	**$31,445**	
NPV	**$3,019**											

Appendix B Performing Monte Carlo Simulations

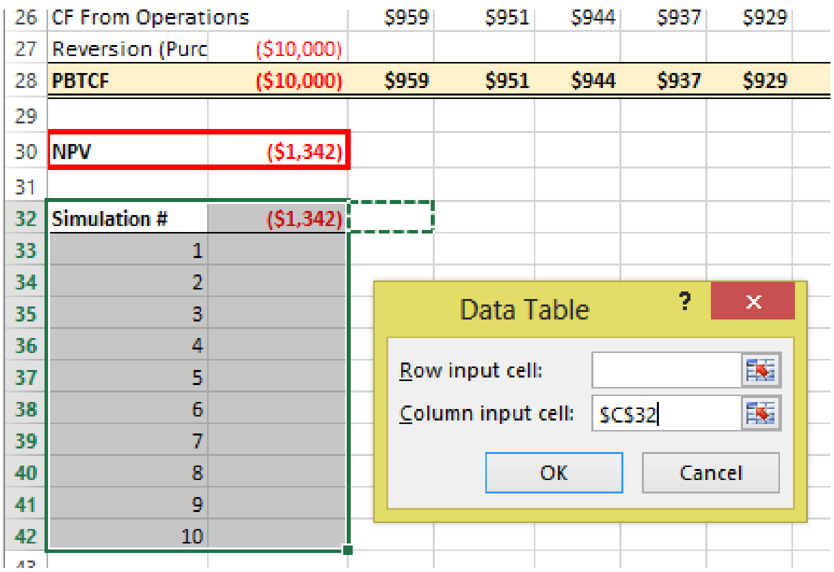
Our probabilistic ModerateCo *pro forma* looks exactly the same as the deterministic SimpleCo *pro forma*, except that our NPV now changes when we recalculate formulas by hitting F9. Each recalculation represents a different scenario under uncertainty. While it is interesting to see the NPV jump around, it would be useful if there was a way to record the NPV values for many iteration/simulation runs.

This *pro forma* is now set up for Monte Carlo simulations. Many iterations of the model are ran and the output values (in our case, NPV) are recorded into a table.

To set up the 2 column simulation table, reference the output value (NPV) in the top row of the right column.

	A	B
29		
30	NPV	($162)
31		
32	Simulation #	=B30
33	1	
34	2	
35	3	
36	4	
37	5	
38	6	
39	7	
40	8	
41	9	
42	10	
43		

The next step is to select the entire table and bring up the Data table window from Data -> What if Analysis -> Data Table. For the column input select, just select any blank cell in the spreadsheet and it should populate the rest of the simulations.



In this simplistic example, we ran 10 simulations. The number of simulations which can be ran is only limited by the processing power of computers.

29		
30	**NPV**	**$856**
31		
32	**Simulation #**	**$856**
33	1	($149)
34	2	$286
35	3	($932)
36	4	($488)
37	5	$777
38	6	$251
39	7	($942)
40	8	$528
41	9	$62
42	10	($1,960)

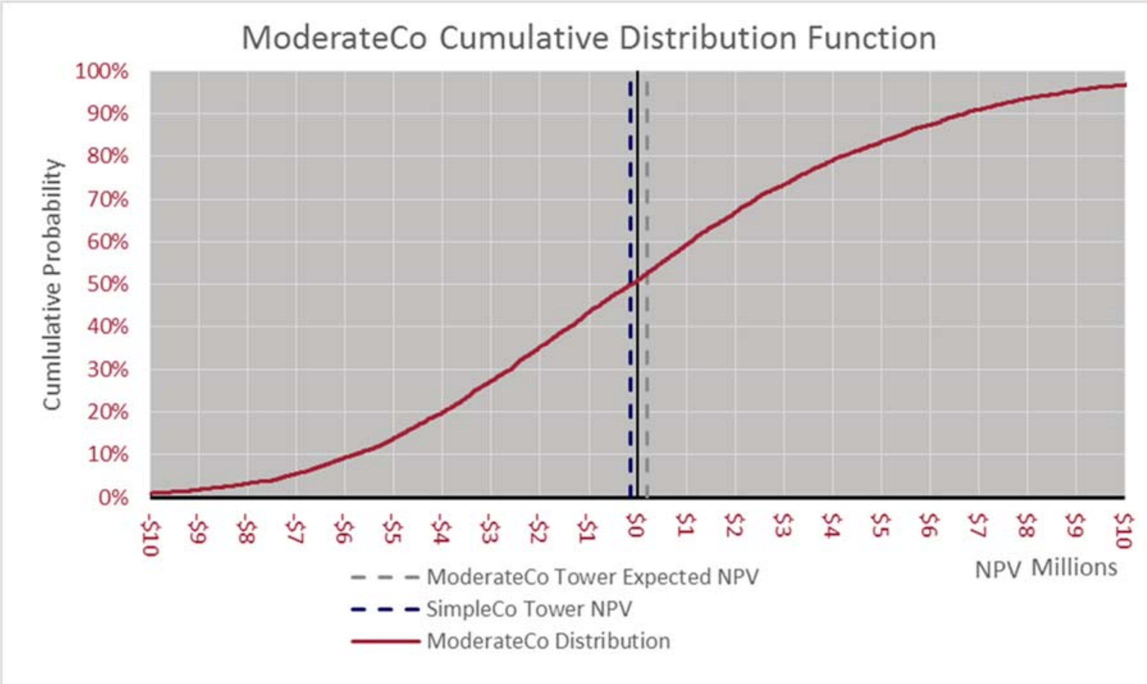
Appendix C Creating Cumulative Distribution Functions (CDFs) in Excel

Creating a cumulative distribution function of the results from the Monte Carlo Simulation is simple to do in Excel. The CDF is the chief output result from the *pro forma* and provides much more information than a single NPV value.

Once a data table is created to record the results from the simulations, a new table can be created, sorting the results from smallest to largest using the SMALL function.

	A	B	C	D	E
1	Sim#	NPV		NPV	ModerateCo Distribution
2	1	$ (23,724)		=SMALL($B$2:$B$5001,A2)	0.02%
3	2	$ 491,197		$ (13,086,915) SMALL(array, k)	0.04%
4	3	$ (2,547,986)		$ (12,920,797)	0.06%
5	4	$ 4,448,899		$ (12,499,669)	0.08%
6	5	$ 93,734		$ (12,152,808)	0.10%
7	6	$ 5,372,176		$ (12,093,495)	0.12%
[illegible]	7	$ [illegible]		$ [illegible]	[illegible]

The SMALL function selects a number from the result corresponding to the rank specified as "k". Thus, rank 1 would refer to the smallest number in the result set, and rank 2 would refer to the second smallest number in the result set. The sorted NPV values will be the x-axis values for the cumulative distribution function. For the y-axis values, 1/(number of iterations) is given to each result. Effectively, in 5,000 runs of the model, each result accounts for a 1/5000 or (0.02%) slice of the distribution. As a cumulative distribution function, 0.02% is added to each successive result. Graph the table as a scatter plot and format as necessary.



A few observations can be made about the CDF function of ModerateCo Tower. The greater the slope of the graph, the greater the likelihood of the corresponding NPVs. In ModerateCo, there is about a 50% chance of positive NPV and 50% chance of negative NPV, but the loss and returns are not symmetrical. The loss at the worst 10% of scenarios was at least -$6 million, which the gain at the best 10% of scenarios was at least $7 million. These numbers are called the Value-at-risk (@ 10%) and Value-at-gain (@ 90%) numbers. Another observation one could make is that there is about a 30% chance that the final NPV will be between -$2,000 and $2,000.

In ChallengeCo Tower, multiple CDFs are plotted on the same graph to compare and analyze the probabilistic outcomes across multiple alternatives.

Appendix D Using IF Statements to Model Real Options for Real Estate Ventures

As presented in section 5.3, The Binomial Lattice method is commonly used to value real options, but the focus of this appendix will be the Monte Carlo method used in conjunction with IF statements to model the behavior of Real Options because of the simulation method's ability to model several different sources of uncertainty, In the hands of a creative analyst, a plethora of different situations and circumstances can be modeled with the flexibility that the Monte Carlo Simulation method offers.

The IF statement is an important logic function in Excel that allows the program to make decisions automatically for you (because manually making 5,000 switches is madness). Basically, the IF statement can be used as an automatic switch - in the context of Real Options, the IF statement allows the spreadsheet to make its own decision on whether to exercise an option or not.

For ChallengeCo, the option to build 10 more floors sometime in the future needs to be modeled. When will construction begin for the 10 additional floors? Construction will only commence if the economy does well; few developers would want to exercise this option when rents are low and vacancy is high! The first step in modeling real options is to specify the 'trigger' conditions.

ChallengeCo Parameters	Year 0	1	2	3
Total Development Cost	$100 /gsf			
Efficiency	90%			
Gross Floor Area (Addition Incl)	170,000 sf			
Option Exercise Trigger: Rent	$35 /sf			
Flex Development Cost	$105 /gsf	$ 108.15	$ 110.30	$ 111.37
Office Rent	$30 /sf	$ 30.90	$ 31.51	$ 31.82
Rent Growth Rate	3%	1.99%	0.97%	0.79%
Expenses	$15 /sf	$ 15.45	$ 16.00	$ 16.59
Expense Growth Rate	3%	3.56%	3.70%	7.11%
Vacancy	10%	19.38%	15.20%	26.38%
Capital Expenditures	10% of NOI	10.16%	10.84%	10.97%
Terminal Cap Rate	11.00%	11.18%	11.84%	11.57%
OCC/Discount Rate	12.50%			

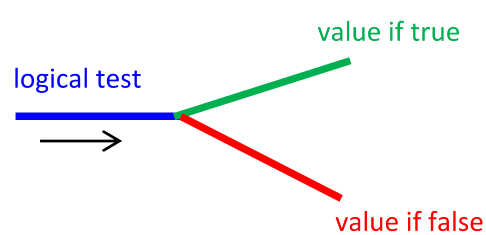
In ChallengeCo, the input assumptions are given a healthy dose of random walk, plus two new assumptions appear: total development cost and flex development costs. Total development cost will be the initial hard and soft costs of constructing the office building. The flex development cost represents the cost to construct an addition, inflated by the same rate as the rent growth rate. The option exercise trigger cannot be missed with a thick red border.

The goal here is to model the construction of an additional 10 floors sometime in the future when the economy improves. In this case, the rent is initially $30 per square foot, so the option should be exercised when the rent rises up. Let us see what happens if we trigger construction of the additional 10 floors when the rents reach $35 per square foot.

ChallengeCo Parameters	Year 0	1	2	3	4	5	6
Total Development Cost	$100 /gsf						
Efficiency	90%						
Gross Floor Area (Addition Incl)	170,000 sf						170,000 sf
Option Exercise Trigger: Rent	$35 /sf						
Flex Development Cost	$105 /gsf	$ 108.15	$ 108.46	$ 110.78	$ 113.79	$ 116.91	$ 123.14
Office Rent	$30 /sf	$ 30.90	$ 30.99	$ 31.65	$ 32.51	$ 33.40	$ 35.18
Rent Growth Rate	3%	0.29%	2.13%	2.73%	2.74%	5.33%	0.43%
Expenses	$15 /sf	$ 15.45	$ 16.05	$ 16.62	$ 17.12	$ 17.73	$ 18.59
Expense Growth Rate	3%	3.89%	3.55%	2.97%	3.61%	4.84%	0.86%
Vacancy	10%	4.79%	12.95%	10.16%	9.99%	4.88%	0.00%
Capital Expenditures	10% of NOI	9.85%	9.76%	9.44%	10.04%	9.72%	9.51%
Terminal Cap Rate	11.00%	10.83%	11.31%	11.08%	11.60%	11.92%	11.49%
OCC/Discount Rate	12.50%						

In the 6th year of the *pro forma*, the rent climbs above $35 per square and immediately, an additional 170,000 square feet (10 floors) is added through by using IF statements. The IF statement is used as a switch between adding 170,000 square feet and *not* adding more floor area.

At its core, the IF Statement has three parts.



1) Logical test

2) Value if true

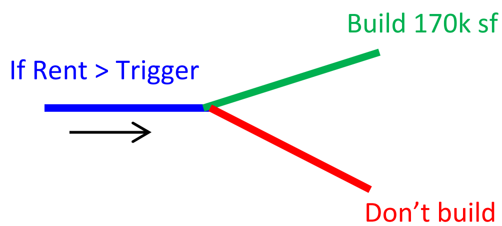
3) Value if false

Organized in this format: =IF (logical test , value if true , value if false)

Think of the IF statement as a "fork in the road" with the logical test as the input. Value if true and value if false are outputs.

The logical test is an equation that tells the computer what to do when it encounters the fork in the road. Verbally, the logical test will read: "if the rent is greater than $35…" In excel it would be: "E8>B5" with B8 being the rent in the current year and B5 as the trigger rent.

'Value if true' is the outcome which will occur when the logical test is true, with the opposite being true for the "value if false".



IF (Rent > Trigger, 170000 , 0)

For each year that the possibility exists to construct another 170,000 sf, an IF statement is required.

New Problem: In current form, the IF statements we created will automatically construct an additional 170,000 sf without memory of what happened in the previous years. Thus, there could be 170,000 sf of construction every year even though the real option that is being modeled can only occur one time.

To solve this, an IF statement is nested inside another one to create a switch with more than two outcomes. Here are the components:

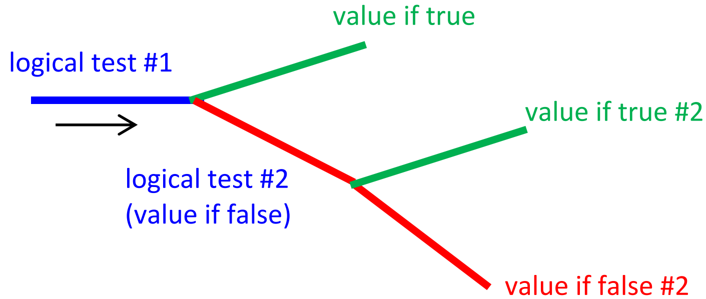
1) Logical test

2) Value if true

3) Value if false...another IF statement

3a) Value if true

3b) Value if false

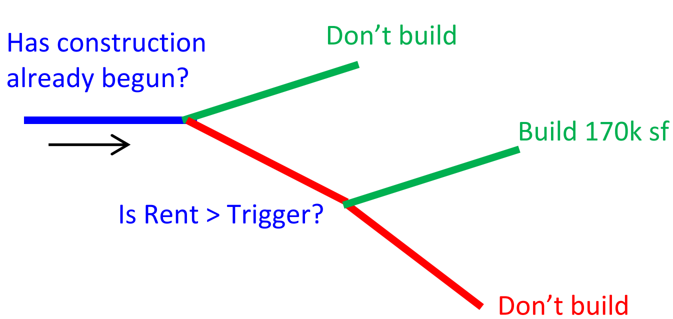


In the nested IF statement, the first logical test is set up to see if 170,000 square feet was constructed beforehand. If there has been another 170,000 square feet built beforehand, then no construction takes place (effectively, ignoring anything rent does in that year). If the addition has not been constructed yet, then proceed to the second IF Statement level.

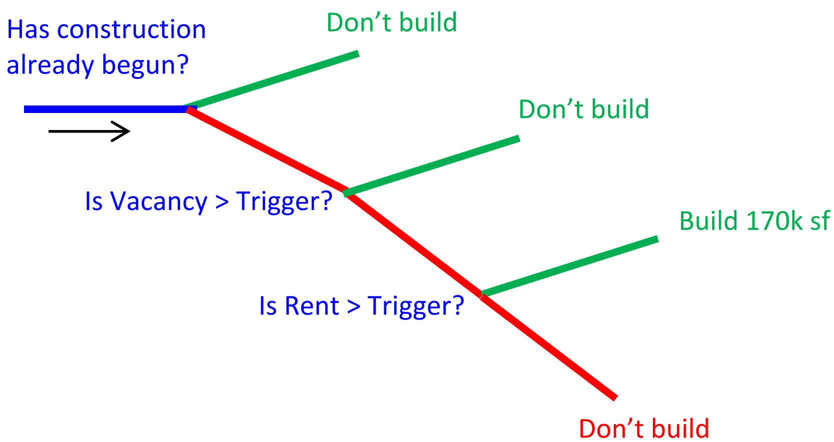
On the second level, the previous logical test and outcomes are the same.

IF (SUM(previous years sf)>0 , 0, IF (Rent > Trigger, 170000 , 0))

Now, the Real Option of building an additional 10 floors sometime in the future is modeled and is ready for the Monte Carlo Simulation.



Rarely does rent make up a trigger on its own for real estate, vacancy is important factor also. Adding vacancy as another trigger is simple with a third nested IF statement. Notice that the decision is to only build if the vacancy rate falls below the trigger. The logic can be followed by the tree below:

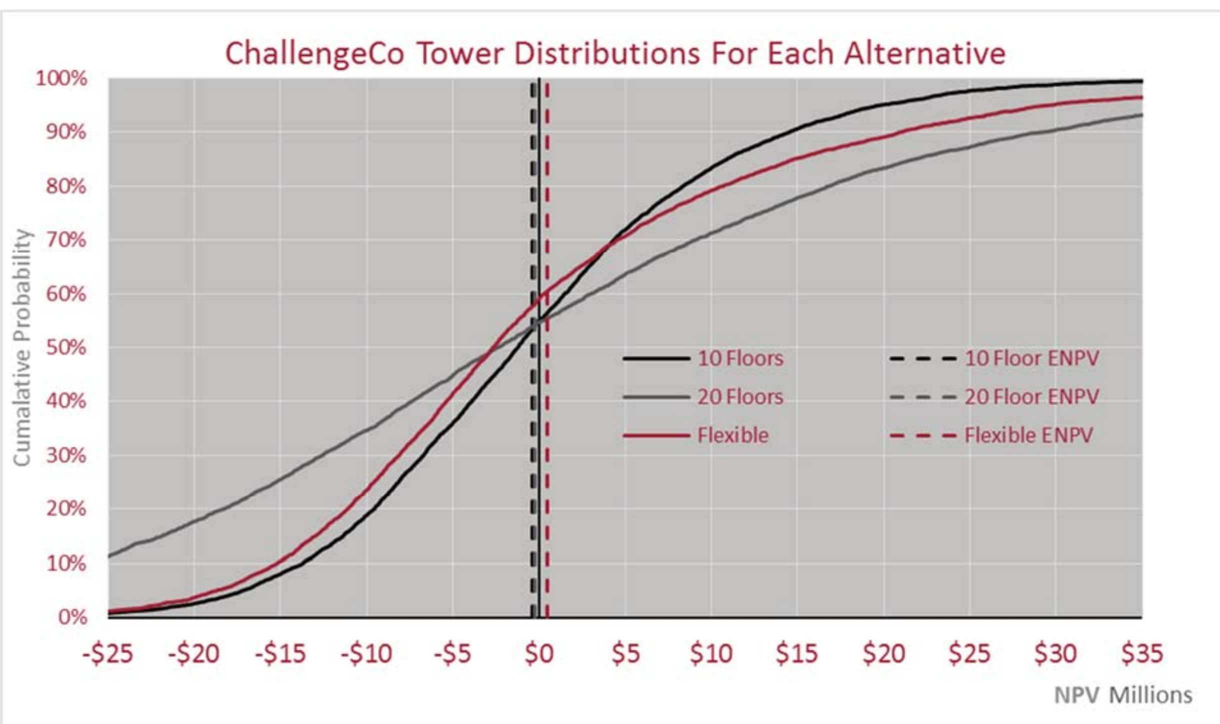


IF (SUM(previous years sf)>0 , 0, IF (Vac > Trigger, 0 , IF (Rent > Trigger, 170000 , 0))

IF statements become messy very quickly, but with good organization and patience, even the most the complex decision rules in Real Options can be modeled. Once there is confidence that the additional 170,000 square feet is constructed when the decision rules are satisfied, the parameters can be tied in to the rest of the *pro forma* to eventually calculate down to the NPV.

ChallengeCo Parameters	Year 0	1	2	3	4	5	6
Total Development Cost	$100 /gsf						
Efficiency	90%						
Gross Floor Area (Addition Incl)	170,000 sf					170,000 sf	
Option Exercise Trigger: Rent	$35 /sf						
Option Exercise Trigger: Vacancy	5.00%						
Flex Development Cost	$105 /gsf	$ 108.15	$ 111.67	$ 117.06	$ 119.86	$ 128.57	$ 139.67
Office Rent	$30 /sf	$ 30.90	$ 31.91	$ 33.45	$ 34.24	$ 36.73	$ 39.91
Rent Growth Rate	3%	3.26%	4.82%	2.39%	7.27%	8.64%	6.85%
Expenses	$15 /sf	$ 15.45	$ 15.71	$ 15.41	$ 15.29	$ 14.74	$ 14.91
Expense Growth Rate	3%	1.69%	-1.92%	-0.76%	-3.59%	1.12%	0.74%
Vacancy	10%	1.61%	4.01%	4.74%	0.00%	0.00%	1.13%
Capital Expenditures	10% of NOI	10.18%	10.23%	10.32%	10.40%	9.53%	8.20%
Terminal Cap Rate	11.00%	10.96%	11.27%	11.85%	12.06%	12.12%	11.56%
OCC/Discount Rate	12.50%						
ChallengeCo Flexibility							
(in 000's)	**Year**	**1**	**2**	**3**	**4**	**5**	**6**
Potential Gross Income		$4,728	$4,882	$5,117	$5,239	$5,620	$12,212
Vacancy		$76	$196	$243	$	$	$138
Effective Gross Income		$4,652	$4,686	$4,874	$5,239	$5,620	$12,073
Operating Expenses		$2,364	$2,404	$2,358	$2,340	$2,256	$4,562
Net Operating Income		**$2,288**	**$2,282**	**$2,517**	**$2,900**	**$3,364**	**$7,511**

Applying the same Monte Carlo Simulation as in ModerateCo Tower, the impact of the Real Option is evident the CDF is compared to the CDFs of models without flexibility built in.



When compared to an otherwise identical 10 floor officer tower, ChallengeCo Tower with a Real Option of building an additional 10 floors in the future is slightly worse off when economic conditions turn out to be poor. The flexible office tower's CDF is slightly shifted to the left of the 10 floor office tower's CDF because of the slightly higher construction costs we modeled in for flexibility ($105/sf vs $100/sf). On the other hand, if economic conditions turn out excellent, the flexible ChallengeCo Tower dominates the 10 floor office tower because the real option to expand is exercised and allows the flexible building to take advantage of favorable conditions.

Now contrast the flexible ChallengeCo Tower with the 20 floor non-flexible building. The 20 floor building is exposed to much more risk as the upside is good, but the downside is absolutely disastrous. This high level of risk in the 20 floor office building occurs because of the high operation leverage created from the large construction cost.

There is no standard way of modeling Real Options into your financial model. The level of complexity is only limited by the creativity and patience of the analyst creating the *pro forma*. Armed with knowledge of a handful of functions in Excel, any real estate professional can easily model uncertainty and real options in to *pro formas* to provide valuable insights for their multi-million dollar projects.

As illustrated in this example, incorporating design and/or financial flexibility into real estate investments can have a significant impact in not only Expected Net Present Value, but risk profiles as well. Understanding the effects of under certainty on real estate ventures can lead a tremendous competitive advantage.