

`$SPAD/src/input pdecomp0.as`

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**Abstract**

# Contents

1 License

3

# 1 License

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— \* —

```
#pile
#include "axiom.as"

--% Polynomial composition and decomposition functions
-- If  $f = g \circ h$  then  $g = \text{leftFactor}(f, h)$  &  $h = \text{rightFactor}(f, g)$ 
-- SMW Dec 86

--% PolynomialComposition
--)abbrev package PCOMP PolynomialComposition
--)abbrev package PDECOMP PolynomialDecomposition

PolynomialComposition(UP: UnivariatePolynomialCategory(R), R: Ring): with
  compose: (UP, UP) -> UP
```

```

== add
  compose(g:UP, h:UP):UP ==
    r: UP := 0
    while g ~= 0 repeat
      r := leadingCoefficient(g)*h**degree(g) + r
      g := reductum g
    r

-- Ref: Kozen and Landau, Cornell University TR 86-773

--% PolynomialDecomposition

PolynomialDecomposition(UP:UPC F, F:Field): PDcat == PDdef where
  UPC ==> UnivariatePolynomialCategory
  NNI ==> NonNegativeInteger
  LR ==> Record(left: UP, right: UP)

PDcat ==> with
  decompose: UP -> List UP
  decompose: (UP, NNI, NNI) -> Union(value1:LR, failed:'failed')
  leftFactor: (UP, UP) -> Union(value1:UP, failed:'failed')
  rightFactorCandidate: (UP, NNI) -> UP
PDdef ==> add

  import from F
  import from LR
  import from Union(value1:UP, failed:'failed')
  import from Float
  import from NNI
  import from UniversalSegment NNI
  import from Record(quotient:UP, remainder:UP);

  leftFactor(f:UP, h:UP):Union(value1:UP, failed:'failed') ==
    g: UP := 0
    for i in 0.. while f ~= 0 repeat
      fr := divide(f, h)
      f := fr.quotient
      r := fr.remainder
      degree r > 0 => return [failed]
      g := g + r * monomial(1, i)
    [g]

  decompose(f:UP, dg:NNI, dh:NNI):Union(value1:LR, failed:'failed') ==
    df := degree f
    dg*dh ~= df => [failed]
    h := rightFactorCandidate(f, dh)
    g:Union(value1:UP, failed:'failed') := leftFactor(f, h)

```

```

g case failed => [failed]
[[g.value1, h]]

decompose(f:UP):List UP ==
df := degree f
for dh in 2..df-1 | df rem dh = 0 repeat
  h := rightFactorCandidate(f, dh)
  g := leftFactor(f, h)
  g case value1 => return
  append(decompose(g.value1), decompose h)
[f]

rightFactorCandidate(f:UP, dh:NNI):UP ==
f := f / leadingCoefficient f
df := degree f
dg := df quo dh
h := monomial(1, dh)
for k in 1..dh repeat
  hdg:= h**dg
  c := (coefficient(f,df-k)-coefficient(hdg,df-k))/
      (dg::Integer::F)
  h := h + monomial(c, dh-k)
h - monomial(coefficient(h, 0), 0) -- drop constant term

```

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## References

- [1] nothing