Lesson Two: Student Team Challenges:

A. Suppose, it is the first day of January, and there is an infectious disease going on, called X2019. The reproductive number is way lower than that often assumed for Coronavirus, it is about 1.1. A friend argues that 1.1 is a very small number and we shouldn't be worried about X2019! In the morning, you read on the newspaper that approximately 100 people in your city of 1,000,000 people are infected by X2019. The disease lasts for 1 day. You are tempted to check if you friend was right about the risks! How many patients do you think your town will have at the end of January, if no intervention is done? Your friend says it should grow by 30% or so, what do you think? First make a guess, then use your calculator to estimate it. [Answer: On day 31, you have 1,744 cases of infection (~17 times more than day 1), and cumulative cases of 18,194 (~182 times more than day 1)]. Lesson: Exponential growth is powerful, even if the exponential growth factor is just slightly greater than 1.0.

B. Suppose there is a disease Y2019. We know that it takes 10 days for people to recover and be non-infectious. But during the ten-day infection period they can infect others who are susceptible with a probability of only 0.011 per interaction. Let's **assume** they **come in contact with**, on average, 10 healthy people per day. Your same friend argues that, while on X2019 he made a mistake and it was dangerously infectious, this one is not as bad, given such a low probability of transmission. You say we should see how many people each person is infecting during the entire period of sickness. Do you recognize this as the basic productive number, R_0? And then you estimate it. What is the number?

C. **Herd immunity** occurs when the number of new infections is equal to the number of new recoveries. That means that any newly infected individual is, on average, infecting only one additional person. Thus, at the system's point of achieving herd immunity, the total number of active infections ceases to grow – it is held constant (at least momentarily). We have already read about Herd Immunity on the paper from *OR/MS Today*.
But now we go beyond that paper and write an equation for Herd Immunity, *H*, where *H* is the fraction of the population that is immune when "the system" first exhibits zero growth in total number of infections. The equation includes the parameter R_0. The result is stated simply:

$$H = 1 - 1/R_0$$

1) Does this check with your intuition for $R_0 = 2$?, $R_0 = 3$?, $R_0 = 1$?, $R_0 = 10$?
2) Can you derive the equation? *HINT*: Go back to the original equation for R_0, try to include H as a new parameter and recall that the average number of new infections at Herd Immunity is 1.0.
3) Does the development of Herd Immunity suggest a feedback loop in some system diagram? Explain.
4) Over time, what happens to the total number of active infections after Herd Immunity is achieved? Can you plot what you think will happen?

D. We like to help our community to be prepared for the flu. R_0 of the seasonal flu is around 1.4. Your teacher says you should help your community to decrease R_0. How do you think that can be done? Think about the equation between $R_0 = \lambda p T$. The value of R_0 is very important and we like to help our community to decrease. We can't do anything for T. But you might be able to do something for λ and p.
First think about potential actions, and then write a one-page report to your family on how your community can overcome the flu this year, based on the concept of reproductive number.