

Teacher Notes for Lesson One: Stocks and Flows

Teacher: Please study this material and then present it to the class in a way that they can interact with you and with each other as the development progresses. At the end of the week, the students should be excited to apply this knowledge to data from different countries. The concept of accumulation is explained in the first BLOSSOMS Video. Here students should find the main stock variables. This will lead them to build the core of an SIR model. We start simply:

Suppose from January 1st, every day 100 new people get infected with coronavirus, and this continues until the end of the month. Suppose from the 11th, every day 50 people recover. If someone asks you at day t , how many people in total have got infected, and how many people have recovered, how would you answer that?

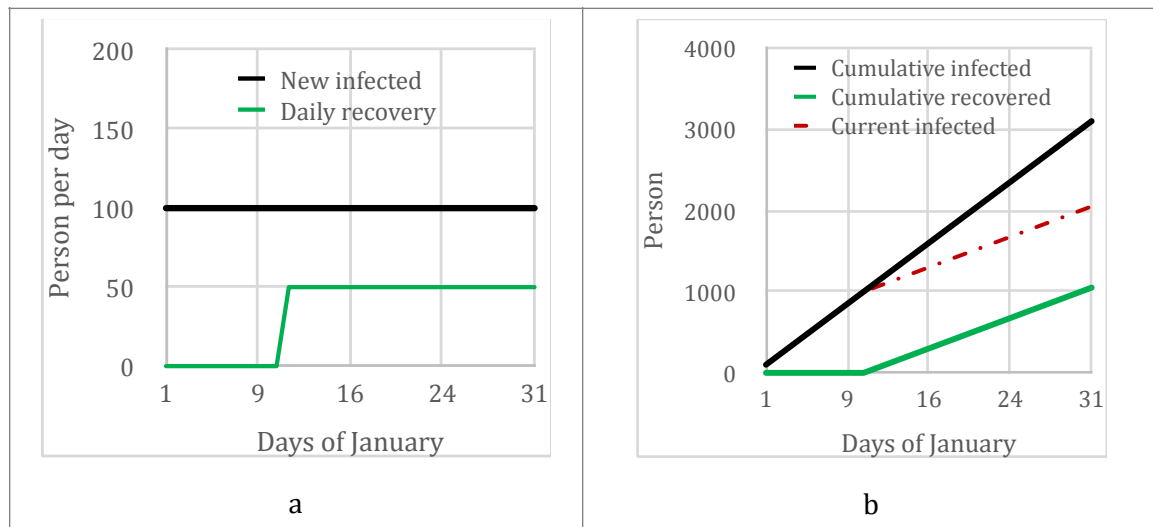


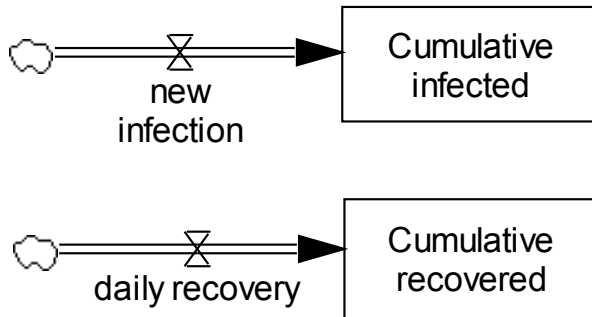
Figure 1: A simple example of stock-flow concepts: (a) flows, (b) stocks

This is simple: It is about adding of all daily incidents until day t . So we have

- Cumulative infected at time t equals $100*t$, and
- Cumulative recovered at time t equals: 0 if $t \leq 10$, and $50*(t-10)$ if $t > 10$.

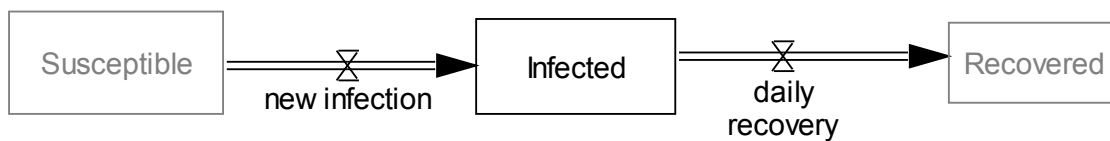
For example, after 20 days we will have cumulative infected of 2,000, and 500 recovered. And both numbers increase over time. Figure 1b shows the graphs for cumulative infected (black line) and recovered (green line) over time.

These 2 variables of cumulative infected and recovered are “stocks,” accumulating over time. (remember the BLOSSOMS video on ***The Surprising World of Complex Systems***).



“New infection” - the daily number of new infections, is the inflow to “Cumulative infected”; “Daily Recovery” is the inflow to “Cumulative recovered”. Neither has an outflow. It’s like water accumulation in a bathtub: if you keep the inflow of water to the bathtub constant, the bathtub water will continue to rise until it overflows! That’s why Daily Infected and Daily Recovered (after $t=10$) are positive constant numbers, their cumulative numbers keep increasing.

The cumulative number of infected counts everyone who ever got infected, even those who have recovered. But there is one more important question: Given the trends, at time t , how many people are infected and not recovered yet? That’s the difference between cumulated number of infected and cumulated number of recovered. To make a better sense of how the population moves from susceptible (people who have not yet been sick) to currently infected, and then recovered, we can offer a stock-flow diagram. See the following figure: it has three boxes (stocks) of *Susceptible*, [current] *Infected*, and [cumulative] *Recovered* (we intentionally use the word recovered instead of cumulative recovered as that’s how it is mentioned in the literature and students can see why this diagram is called S.I.R.). The diagram shows how people change their status, from left (*Susceptible*) to right (*Infected* and then *Recovered*). For now, let’s just focus on the box at the middle (*Infected*). Current infected is also a stock variable. It increases with each new infection (inflow) and decreases by daily recovery (outflow).



Student Team Challenge #1

Given this information, **can you use Figure 1a to estimate the number of current infected from new infection and daily recovery?** Hint: From January 1st until 10th, every day we are adding 100 new cases, and after January 10th, we are still adding 100 new cases each day while 50 are recovering each day (net increase of 50). Figure 1b, the dotted line shows the trend of current infected. As you see, the number of infected people is still increasing. And that continues as long as inflow > outflow (which was the case in this simple example). The bathtub analogy helps here too: water is flowing in to the bathtub much more than draining out, thus water is accumulating in the bathtub. This is physics ruling!

Student Team Challenge #2

The previous problem was a simple case. Consider the following. Figure 2a shows number of new infections, and daily recoveries. Daily infected (black line) starts with 50 for 5 days, then becomes 100 per day for 10 days, then back to 50 and finally to zero. Recovery (green line) follows the same pattern with a 10-day delay. **Can you estimate “cumulative infected”, “cumulative recovered” and “current infected?”**

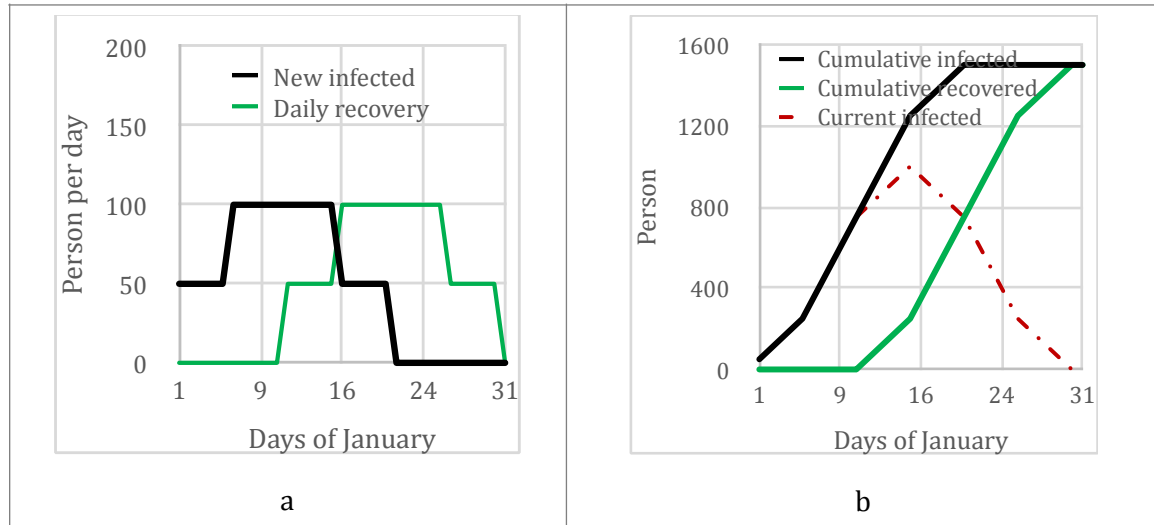
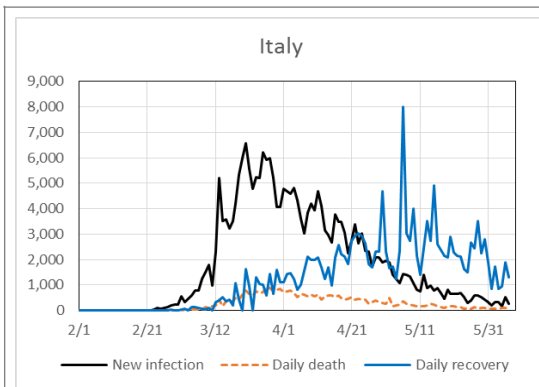


Figure 2: A more realistic example of stock-flow concepts in infectious diseases

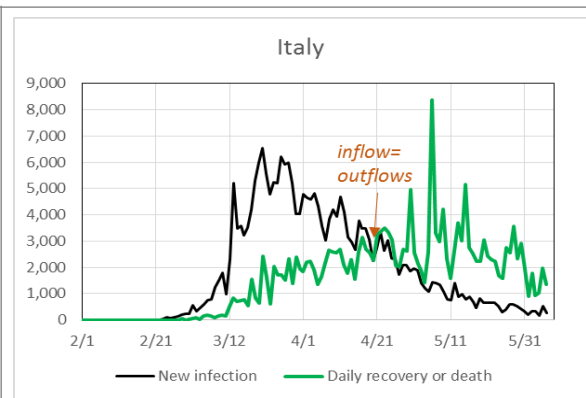
Figure 2b shows Cumulative Infected, and Cumulative Recovered which are totals of the cases that we see in the daily trends of Figure 2a. If you just focus on “New infected” in Figure 2a, we have to count all new cases over time. The first 5 days, every day we are adding 50 new cases, so by the end of the first 5 days we will have 250 cases (note the corresponding point in the black graph on Figure 2b). Then infections speed up, and every day for ten days we have 100 new cases. So by day 15 we will have 250+1000 total cases (note the corresponding point in the black graph on Figure 2b). And finally, we have 5 more days with 50 new infections and after day 20 we stop seeing new cases, and that’s when the cumulative cases levels off at 1,500. The same pattern exists for *Cumulative recovered*. In the absence of death, *Cumulative recovered* will eventually be equal to *Cumulative infected*.

In Figure 2b we also see current infected (red dashed line). How is that estimated? Think this way: every day we are adding the difference between New Infected and Daily Recovery, or subtracting if Recovery is more than New Infection. As long as New Infected is more than daily recovery (inflow > outflow) we see growth in Current Infected. When do we pass the peak? **The peak is when inflow=outflow!** See Figure 2a, in the day that the black line and green line cross (right at the middle of day 15 and 16), on Figure 2b, the dashed red line reaches its maximum. Quickly after that day, the dashed line starts decreasing, since on a daily base we have more Recovery than New Infection.

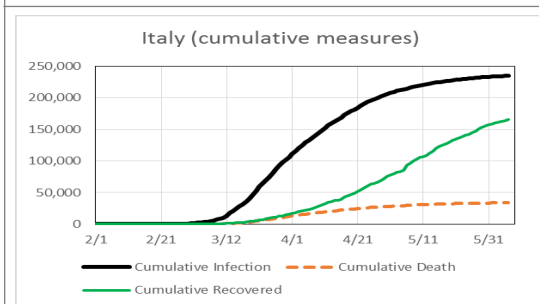
Now, let's consider a more realistic case, using data from Italy:



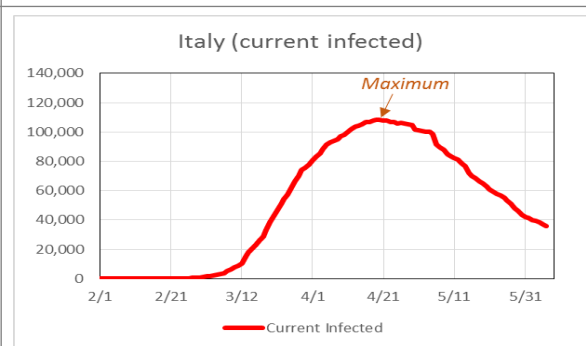
a: flows: daily new infection (inflow), daily death and daily recovery (outflows)



b: same as a, with one graph for total of daily death and recovery (outflow) pointing to the first date at which inflow=outflow



c: cumulative infection, death, and recovered



d: current infected showing the maximum point

This is the daily trend of infections from the 2020 coronavirus in Italy. At the time of designing this homework the disease was still spreading; you may search for more up-to-date numbers. **The questions are: What is the cumulative number of Infected and what is the cumulative number of Recovered. We unfortunately had death too. What is the cumulative number of deaths? Plot these variables over time.** When did we pass the peak of infection (the maximum point in current infected)? You can use the graph and draw current infected. Note that since there is death too, outflow is daily recovery plus daily death. Can you tell that without even drawing the current infected graph? Yes! Hint: maximum of the stock should be on one of the points that inflow=outflow!