<u>Teacher Notes for Lesson Two</u>: R_0, the Basic Reproductive Number

Teacher: *Please study this material and then present it to the class in a way that they can interact with you and with each other as the development progresses.* Emphasis is on math informality and intuition. Probably you have noticed that, early on in any pandemic, the infected population grows rapidly. That's because as the number of people in the infected population increases, a susceptible person is more likely to have contact with an infected person – as their numbers are growing! This is a reinforcing feedback loop (also known as a positive feedback loop). Let's think about this simple example:

> Let's say at the beginning, there is one infected person. If that person recovers in 1 day, but infects 2 people, then the day after we will have two infected individuals. If infection continues with the same trend, those 2 infect 4 new people, and those 4 infect 8, and so on. The daily number of infected people over time will be: 1, 2, 4, 8, 16, 32, 64, … Yes, in six days we went from 1 to 64.

This is an exponential growth that comes from the reinforcing loop of infection. As you note, we are doubling the daily infected population each day. Epidemiologists define a parameter, the basic reproductive number R_0, which is the number of susceptible people that on average an infected person infects before death or recovery. In our simple example R_0=2. But it can be higher. What if it were 3? Then the growth trend was much more rapid: 1, 3, 9, 27, 81, 243, 729, … yes, in six days it would be 729 rather than 64.

<u>Formula for R_0:</u> We teach and demonstrate that R_0 is directly proportional to (1) the mean number of other people one interacts with on a given day; (2) given a person-to-person interaction, the likelihood of infecting that given person; (3) the mean number of days that one is infected and still circulating in society. In equation form, we can write

$$R_0 = \lambda p T,$$

where λ is the mean number of other people one interacts with on a given day; p is the likelihood of infecting a given person, given contact; and (3) T is the mean number of days that one is infected and still circulating in society.

Think of this example (quoted from reading list, **COVID-19 Math: You, Me, R_0 and Rolling Re-entry): "*Here is the key:*** Past values of R_0 are provided by historical data -- depicting human behavior and disease characteristics; *future values of R_0 are determined by You!* (and me and all of us). Imagine that you are infectious and still going about your life, with a "personal R_0" for a given day of 4. You interact with 20 people each day. You could infect each of those 20, but on average you infect 4. The likelihood of infecting any one of them depends on the intensity and type of interaction. A hug or a handshake has high likelihood of transmission. The more careful you are, the less the chance of transmission. Now suppose you have just decided to interact with ten of these 20 people by Internet; for such virtual interactions, there is zero chance of passing on the infection. By reducing the number of face-to-face contacts by 50 per cent, you cut your personal daily R_0 in half, to 2.0! {This is the lambda term, λ}. Now, if your remaining ten face-to-face interactions use social distancing and good hygiene, you could easily achieve another 50% reduction, from 2.0 to 1.0, now a 75 percent total reduction! {This is the intensity-of-interaction term, *p*.} While any given disease may be more or less infectious and thus affect the value of *p*, it is ultimately human behavior that determines the numerical value of R_0. R_0 is not like the constant *pi* = 3.14159… "

Now ask the students if they feel that this formula is reasonable: $R_0 = \lambda p T$. Welcome their comments.

What if $R_0 = 1$? Then the number of infected people stays constant! For anyone who becomes recovered, one new person gets infected, so people are just replacing each other. What if R_0=0.5? The number of infected people should decline over time since every two people are replaced by 1. It appears that R_0=1 is an important value which will determine if the contagion should increase or not! In systems language, scientists say R_0=1 is a ***tipping point***.

Questions for Task 2:

A. Suppose, it is the first day of January, and there is an infectious disease going on, called X2019. The reproductive number is way lower than that often assumed for Coronavirus, it is about 1.1. A friend argues that 1.1 is a very small number and we shouldn't be worried about X2019! In the morning, you read on the newspaper that approximately 100 people in your city of 1,000,000 people are infected by X2019. The disease lasts for 1 day. You are tempted to check if you friend was right about the risks! How many patients do you think your town will have at the end of January, if no intervention is done? Your friend says it should grow by 30% or so, what do you think? First make a guess, then use your calculator to estimate it. [Answer: On day 31, you have 1,744 cases of infection (~17 times more than day 1), and cumulative cases of 18,194 (~182 times more than day 1)]. Lesson: Exponential growth is powerful, even if the exponential growth factor is just slightly greater than 1.0. {Note for teachers: During the first days of a pandemic, the change in population of susceptible is so small, so it is OK to assume constant R_0 (or constant probability of infection). In longer term as number of healthy people decline, probability of infection declines, and exponential growth stops. So for this example, it is important to focus only on the first few weeks; longer term will be more complicated.}

B. Suppose there is a disease Y2019. We know that it takes 10 days for people to recover and be non-infectious. But during the ten-day infection period they can infect others who are susceptible with a probability of only 0.011 per interaction. Let's assumed they see, on average, 10 healthy people per day. Your same friend argues that, while on X2019 he made a mistake and it was dangerously infectious, this one is not as bad, given such a low probability of transmission. You say we should see how many people each person is infecting during the entire period of sickness. Do you recognize this as the basic productive number, R_0? And then you estimate it. What is the number? {For teacher: R_0 = 0.011*(10 healthy people per day)*(10 days of passing infection)=1.1, same as before!}

C. **Herd immunity** occurs when the number of new infections is equal to the number of new recoveries. That means that any newly infected individual is, on average, infecting only one additional person. Thus, at the system's point of achieving herd immunity, the total number of active infections ceases to grow – it is held constant (at least momentarily). We have already read about Herd Immunity on the paper from *OR/MS Today*.

But now we go beyond that paper and write an equation for Herd Immunity, H, where H is the fraction of the population that is immune when "the system" first exhibits zero growth in total number of Infections. The equation includes the parameter R_0. The result is stated simply:

$$H = 1 - 1/R_0$$

1. Does this check with your intuition for $R_0 = 2$?, $R_0 = 3$?, $R_0 = 1$?, $R_0 = 10$?
2. Can you derive the equation? *HINT*: Go back to the original equation for R_0, try to include H as a new parameter and recall that the average number of new infections at Herd Immunity is 1.0.
3. Does the development of Herd Immunity suggest a feedback loop in some system diagram? Explain.
4. Over time, what happens to the total number of active infections after Herd Immunity is achieved? Can you plot what you think will happen?

D. We like to help our community to be prepared for the flu. R_0 of the seasonal flu is around 1.4. Your teacher says you should help your community to decrease R_0. How do you think that can be done? Think about the equation between $R_0 = lpT$. The value of R_0 is very important and we like to help our community to decrease. We can't do anything for T. But you might be able to do something for l and p. First think about potential actions, and then write a one-page report to your family on how your community can overcome the flu this year, based on the concept of reproductive number.