Activity 1: Raindrops (extra for teacher guide).

- Question 1a. What forces dictate the size of raindrops?

- Question 1b. Write the size *a* of a raindrop as a function of gravitational acceleration *g*, surface tension of water σ, and the densities of air and water, ρ_a and ρ_w, respectively.

- Question 1c. Calculate the numerical values for *a* and *U* using known values: $\varrho_a = 10^{-3}$ g/cm^3, $\varrho_w = 1$ g/cm^3, gravity g ≈ 1000 cm/s^2, surface tension σ = 70 dynes/cm

Useful units for calculations

Remember: dyne = 1 g cm/s^2 (force in cm-g-s units) = 10^{-5} Newtons. Using cgs units is easiest for calculations with small insects, as the values are often order-one.

- Density of air $\varrho_a = 10^{-3}$ g/cm^3
- Density of water $\varrho_w = 1$ g/cm^3
- Gravity $g \approx 1000$ cm/s^2
- Surface tension $\sigma = 70$ dynes/cm

Hints for Activity 1

This problem is most easily done using scaling, or the art of approximation. In scaling, you use ~ symbols rather than equality up to a constant. Thus one can ignore constant numerical factors such as 2 and

To solve this problem, first note that you have two unknowns so you need two equations.

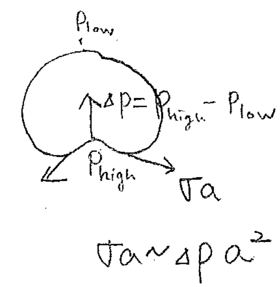
Background: pressure drag force $F_d \sim \Delta p\, a^2$ where I have approximated the cross sectional area of a sphere as $\sim a^2$. The difference in pressure between the front and back of the sphere is $\Delta p \sim \varrho_a U^2$, also known as the stagnation pressure.

First equation is simple equality of forces for terminal velocity, the gravitational force $F_g = mg \sim \varrho_w a^3$, and the pressure drag force $F_d \sim \varrho_a U^2 a^2$

Second equation is the criteria for drop breakage: this occurs at a vertical force balance at a point in front of the drop, right before it is cleaved in two. The drop will break if the local dynamic pressure, $\Delta p \sim \varrho_a U^2$ exceeds the curvature pressure at point, $\Delta p \sim \sigma / a$.

Activity 1 Solution: Raindrops

In order not to be breakable, the surface tension of a drop should be greater than the pressure difference between top and bottom of the drop.



It implies that $\sigma a \sim \Delta p a^2$.

And Δp is equal to $\rho_{air} v^2$.

$\therefore \sigma a \sim \rho_{air} v^2 a^2 \qquad \therefore \rho_{air} v^2 \sim \frac{\sigma}{a}$

At the terminal velocity, drag force is equal to gravity

$\therefore D = \rho_{air} v^2 a^2 \sim mg \sim \rho_w a^3 g$

$\therefore \frac{\sigma}{a} \cdot a^2 = \sigma a \sim \rho_w a^3 g \qquad \therefore a \sim \sqrt{\frac{\sigma}{\rho_w g}}$

$a \sim 2.3$ mm , $U \sim 500$ cm/s

Activity 1 Solutions (teacher's guide)

- Question 1a. What forces dictate the size and speed of raindrops? *Gravity, pressure drag, surface tension*

- Question 1b. Write the size *a* and speed *U* of a raindrop as a function of gravitational acceleration *g*, surface tension of water σ, and the densities of air and water, ρ_a and ρ_w, respectively. $a \sim (\sigma/\rho_w g)^{1/2}$, $U \sim (\sigma/\rho_a a)^{1/2}$ where ~ means equality up to a constant.

- Question 1c. Calculate the numerical values for *a* and *U* using known values: $\varrho_a = 10^{-3}$ g/cm^3, $\varrho_w = 1$ g/cm^3, gravity g ≈ 1000 cm/s^2, surface tension σ = 70 dynes/c:

  Solution: $a \sim 2.3$ mm, $U \sim 500$ cm/s

Answer 1: Raindrops are much heavier and faster than mosquitoes

Features of a raindrop

• raindrop radius ~ 2 - 5 mm

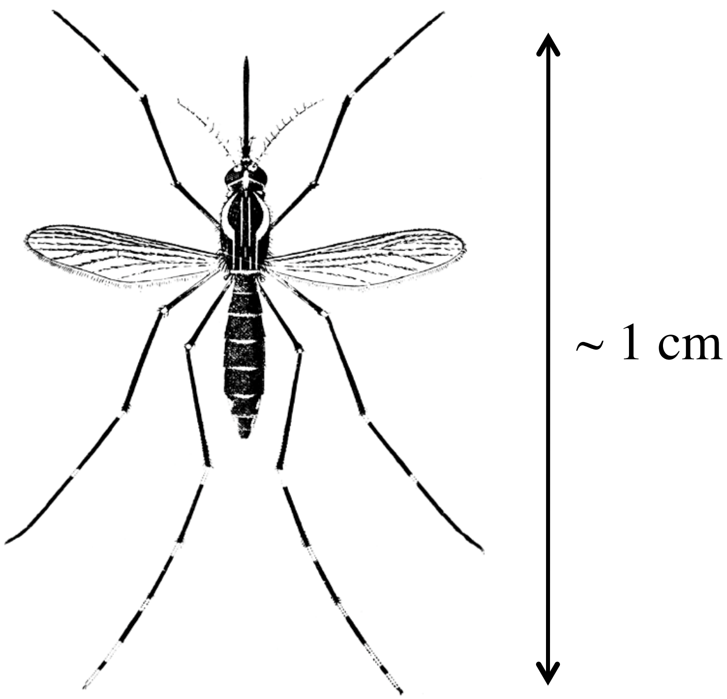
• raindrop weight ~ 2 - 50 mosquito weights

• raindrop speed ~ 5 - 9 m/s, much greater than mosquito speed (1 m/s)

*Clements, A. The sources of energy for flight in mosquitoes.

**Savile, D. & Hayhoe, H. The potential effect of drop size on

Activity 2: What is frequency f (impacts per second) of raindrop on a flying mosquito?



You are given:

Mosquito body area :

$A_m \sim 0.3 \text{ cm}^2$

Rain intensity:

$I \sim 50$ mm/hr ~ 0.0014 cm/s

Density of water:

$\varrho_w \sim 1 \text{ g/cm}^3$

Mass of drop:

$m \sim 10$ mg

Hint for Activity 2: What is frequency of impacts for a flying mosquito?

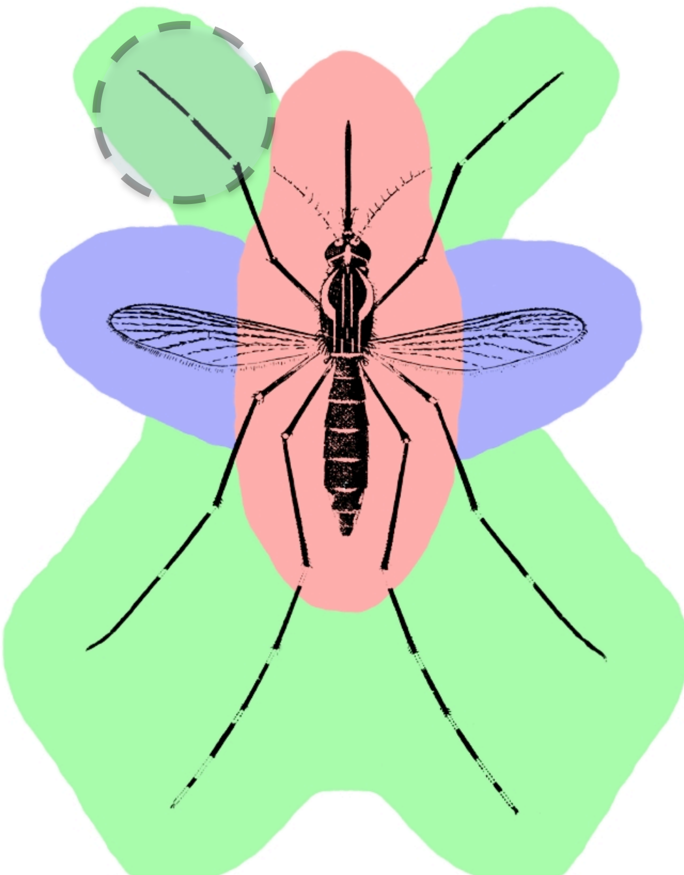
This is a problem of mass conservation. Rain intensity *I* is given in the units of cm/hour. We must convert this unit into a number of drops that falls on top of the mosquito per second.

This problem is related to the the first problem in "flying circus of physics", which asks is wetter to run or walk through the rain. Here we recognize that mosquitoes fly so slowly that impacts on the mosquito's frontal area are negligible compared to that on top.

So consider only drops falling atop the mosquito where plan-view area of wings and legs is A_m. This area is given by considering all drops that impact or even graze the legs (See diagram on next page where an area is sketched out that is one drop radius wider than legs and body). Students should estimate this area A_m to 1 significant digit.

First convert *I* to cm per second. Then, every second, we consider the volume of drops that fall to fill a volume that is A_m wide and *I* tall. We can convert this into a mass of fluid falling per second using the density of water ϱ used in activity 1. Lastlly, we can find frequency of drops *f* by remembering each drop has a fixed volume *m* calculated from activity 1.

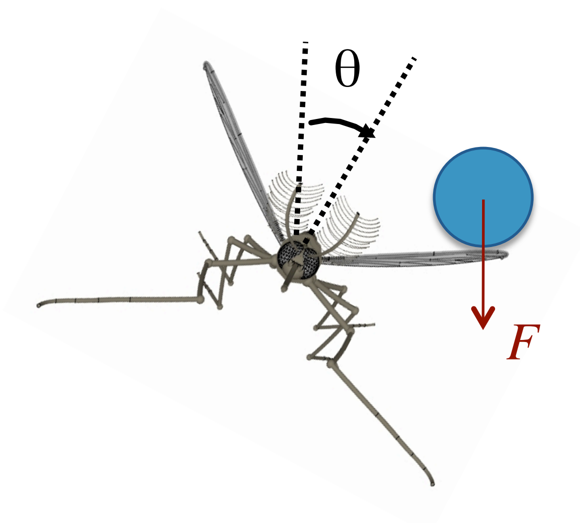
Activity 2 solution: Impact frequency f

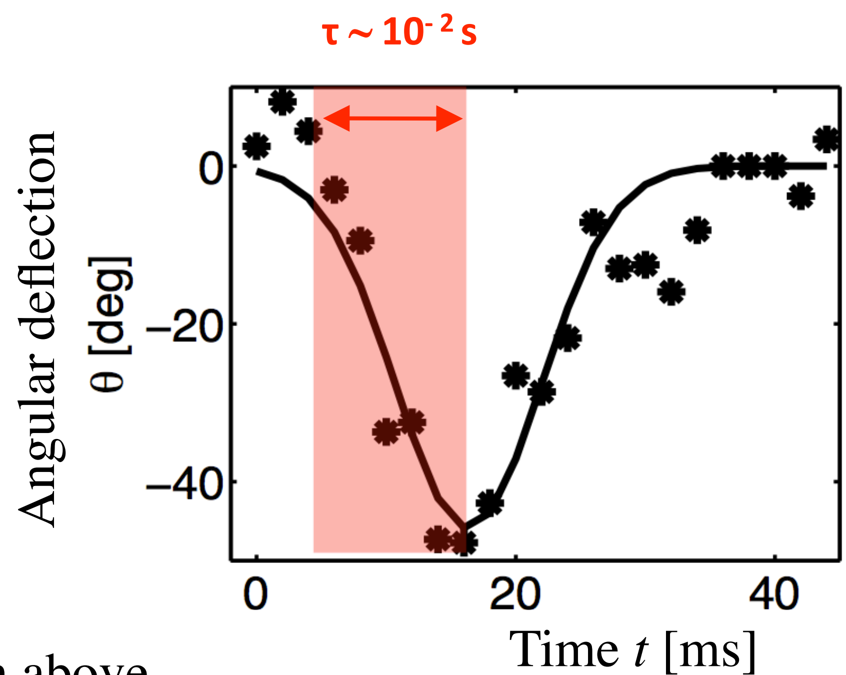


Frequency of impacts: once every 25 sec

$$f = \frac{\rho_w I A_m}{m_{drop}} \sim \frac{1}{25}\frac{\text{hits}}{\text{s}}$$

Activity 3: What is the raindrop's force F from a glancing blow on the wing?



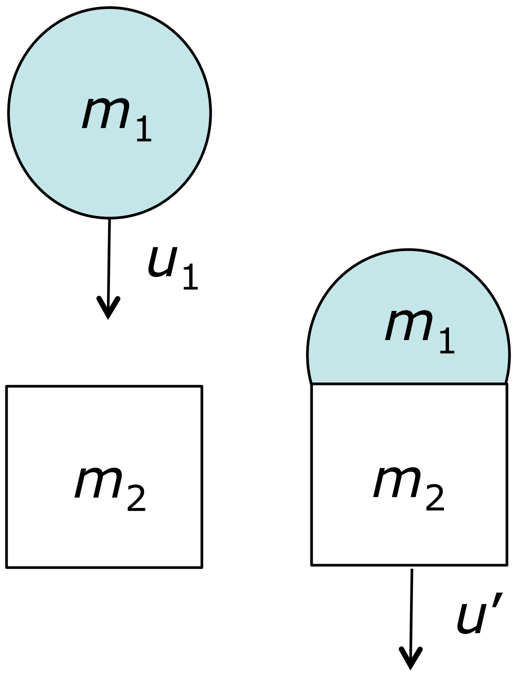


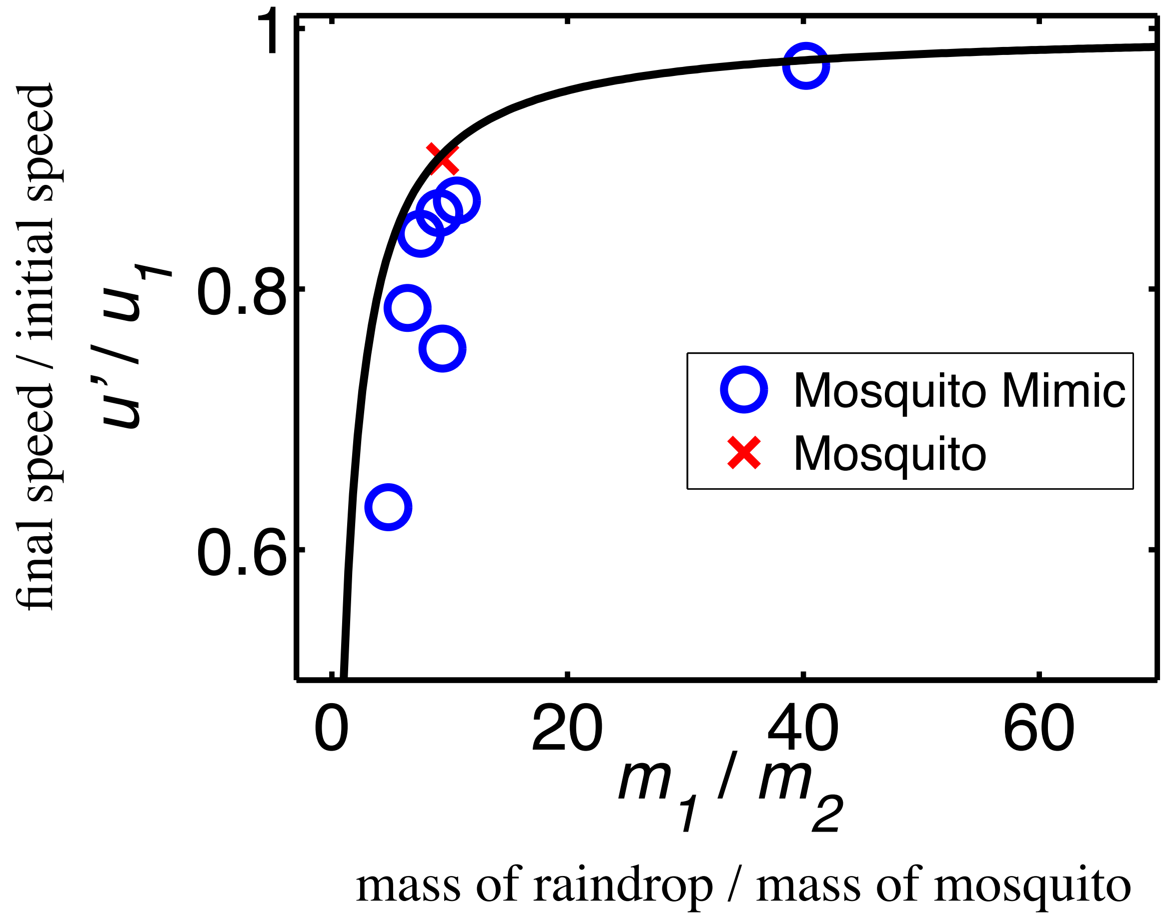
Given:
- -angular acceleration as shown in graph above
- -impact radius r = 1 mm
- -radius of mosquito, R ~ 1 mm, and mass m ~ 1 mg

Activity 4: What is the final speed of raindrop-cum-mosquito after impact?

Consider conservation of linear momentum. In particular consider the momentum before and after impact.

Inelastic Impact





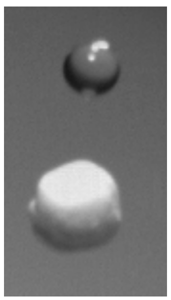
Activity 4: What would happen with other kinds of insects?

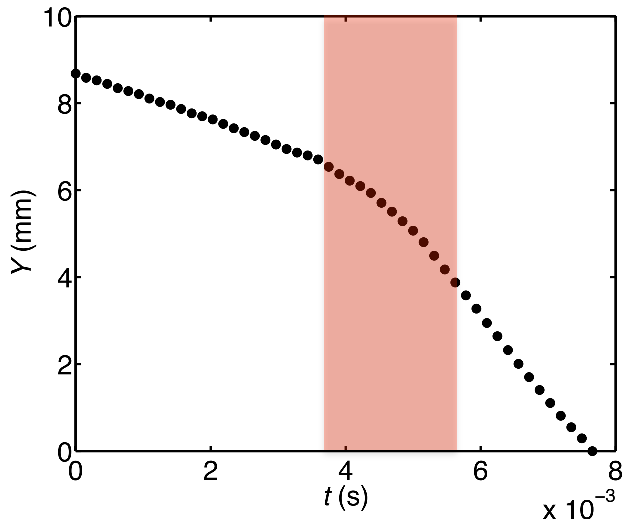
Insects	Mass (mg)
Parasitic wasp, *Encarsia formosa*	0.025
Black fly, *Simulium* Latreille	0.8
Fruit fly, *Drosophila melanogaster*	1
Woolly aphid, *Eriosomatina*	1.2
Mosquito, *Aedes aegypti*	3.5
Plume moth, *Emmelina monodactylus*	8.4
Crane-fly, Tipula obsoleta	11.4
Hoverfly, *Episyrphus balteatus*	21.8
March fly, *Bibio marci* male	26.6
Conopid fly, *Conops strigatus*	27.1
Ladybird, *Coccinella 7-punctata*	34.4
Crane-fly, Tipula paludosa	49.8
Bluebottle fly, *Calliphora vicina*	62
Orchid bee, *Euglossa dissimula*	91
Honeybee, *Apis mellifera*	101.9
Orchid bee, *Euglossa imperialis*	151.7
Dronefly, *Eristaltis tenax*	165.9
Bumblebee, *Bombus hortorum*	226
Bumblebee, *Bombus lucorum*	231
Bumblebee, *Bombus terrestris*	595
Orchid bee, *Eulaema meriana*	819.6
Hawkmoth, *Manduca sexta* male	1199
Black-chinned hummingbird, *Archilochus alexandri*	3000
Magnificent hummingbird, *Eugenes fulgens*	7400

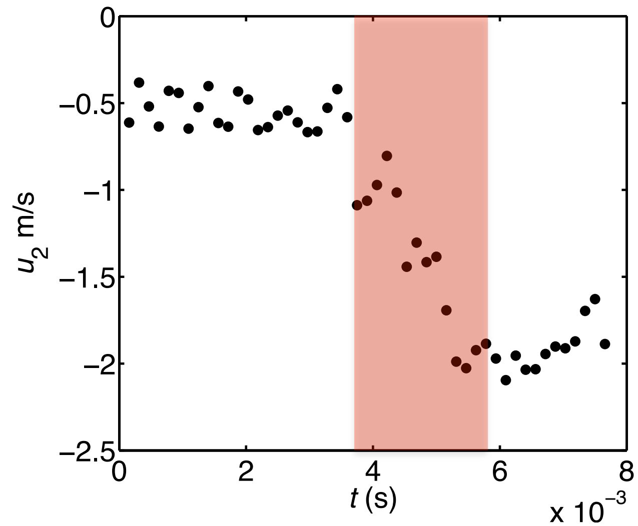
Handout 1: Position and velocity of mimics

Position Y | Velocity U_2

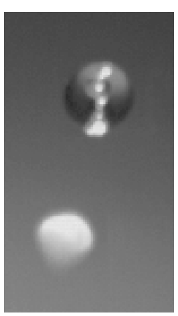
Small drop

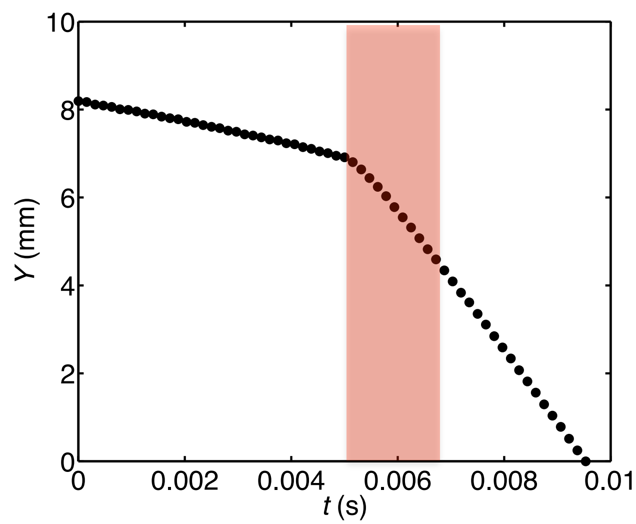


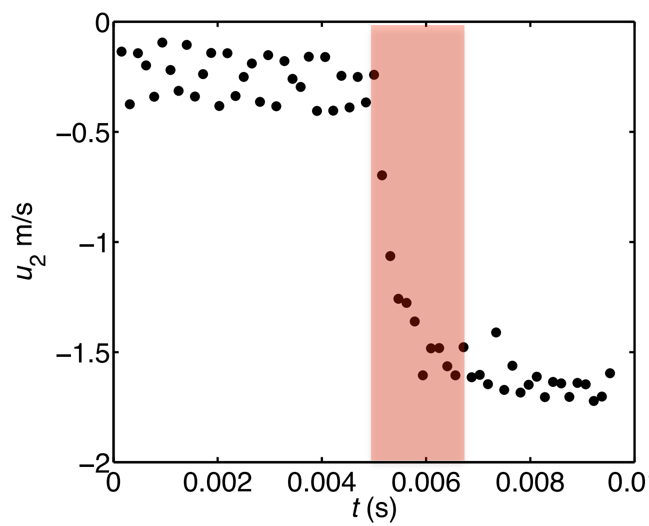




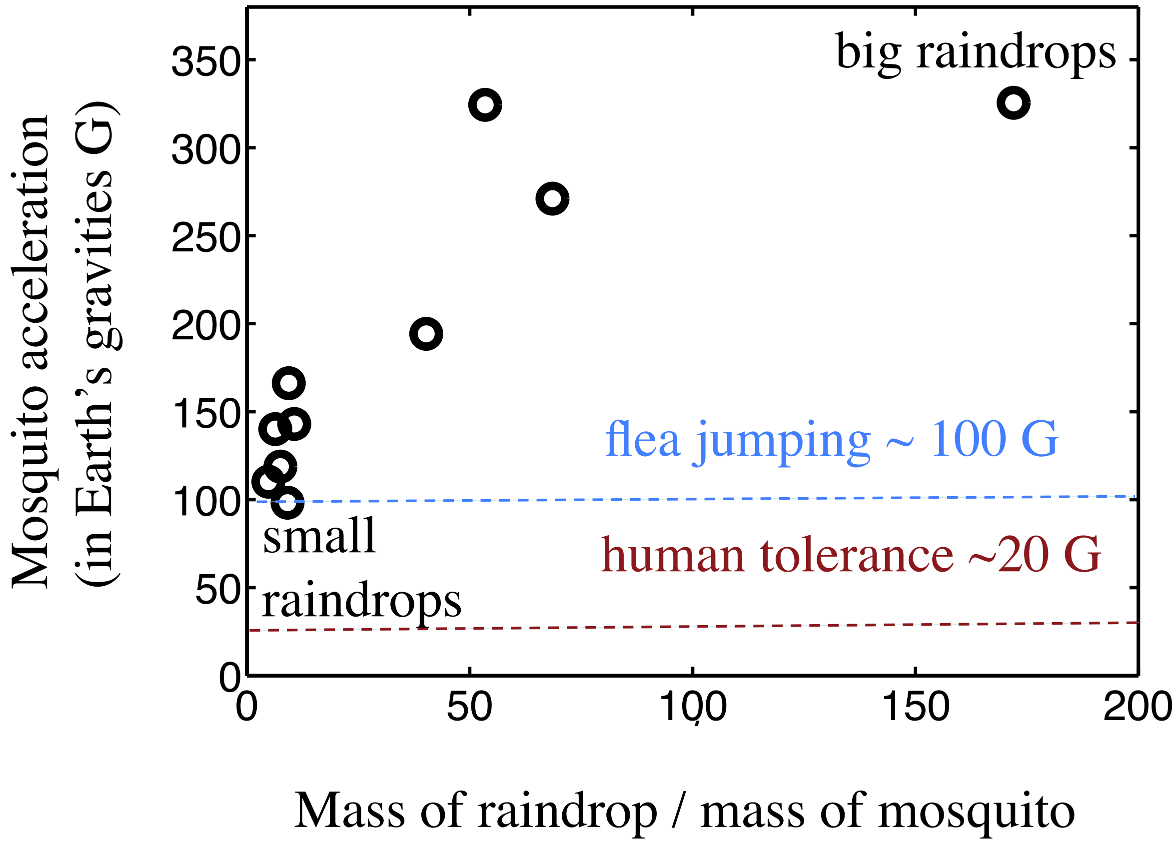
Large drop







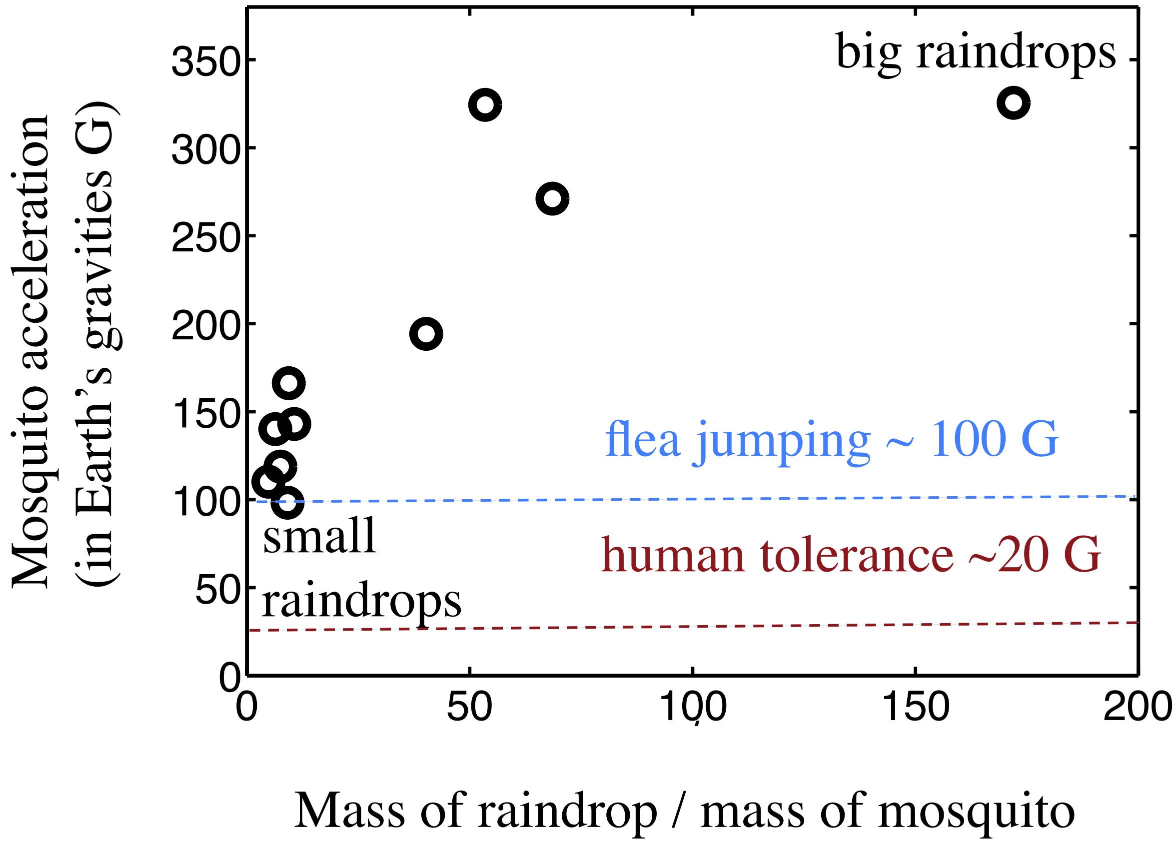
Mosquitoes Accelerate 50-300 G



$$F = 2 \text{ dynes} \times 300 \text{ G} = 600 \text{ dynes} \approx 0.61 \text{ gf}$$

Max force: 4000 dynes (flight still possible) – 10,000 dynes (death)

Mosquitoes Accelerate 50-300 G



$$F = 2 \text{ dynes } \times \text{ } 300 \text{ G} = 600 \text{ dynes} \approx 0.61 \text{ gf}$$

Max force: 4000 dynes (flight still possible) – 10,000 dynes (death)

Dimensionless Acceleration