MIT Blossoms lesson
on
“Elasticity: studying how Solids change shape and size”
Handouts for students

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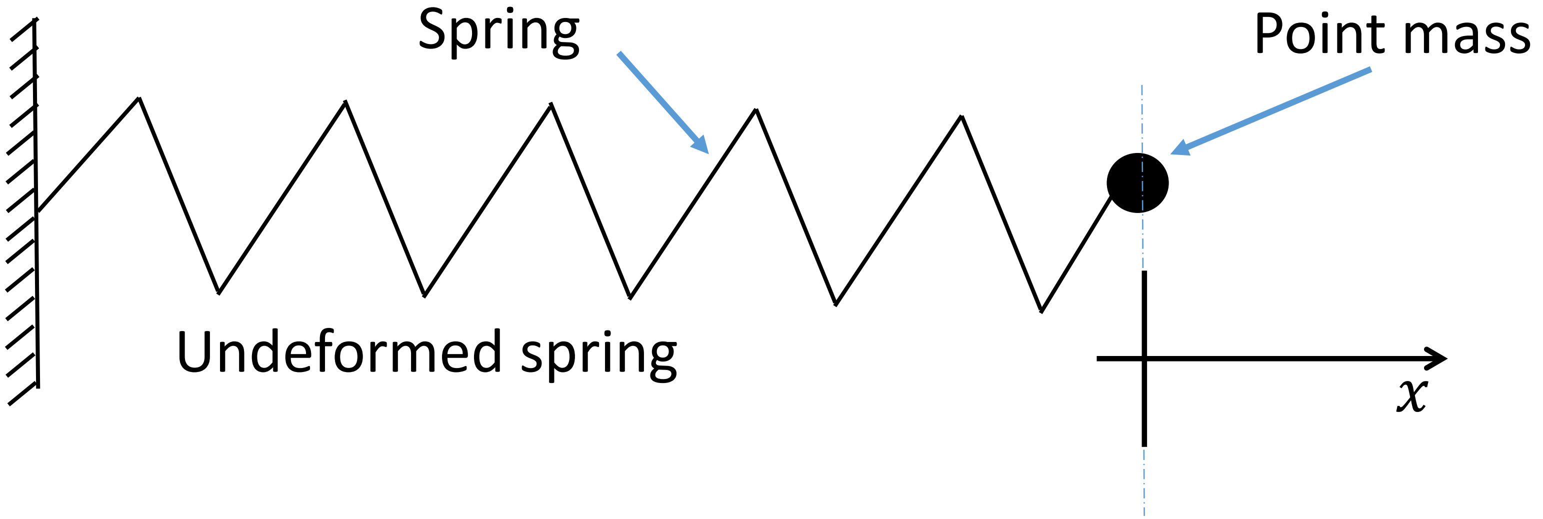
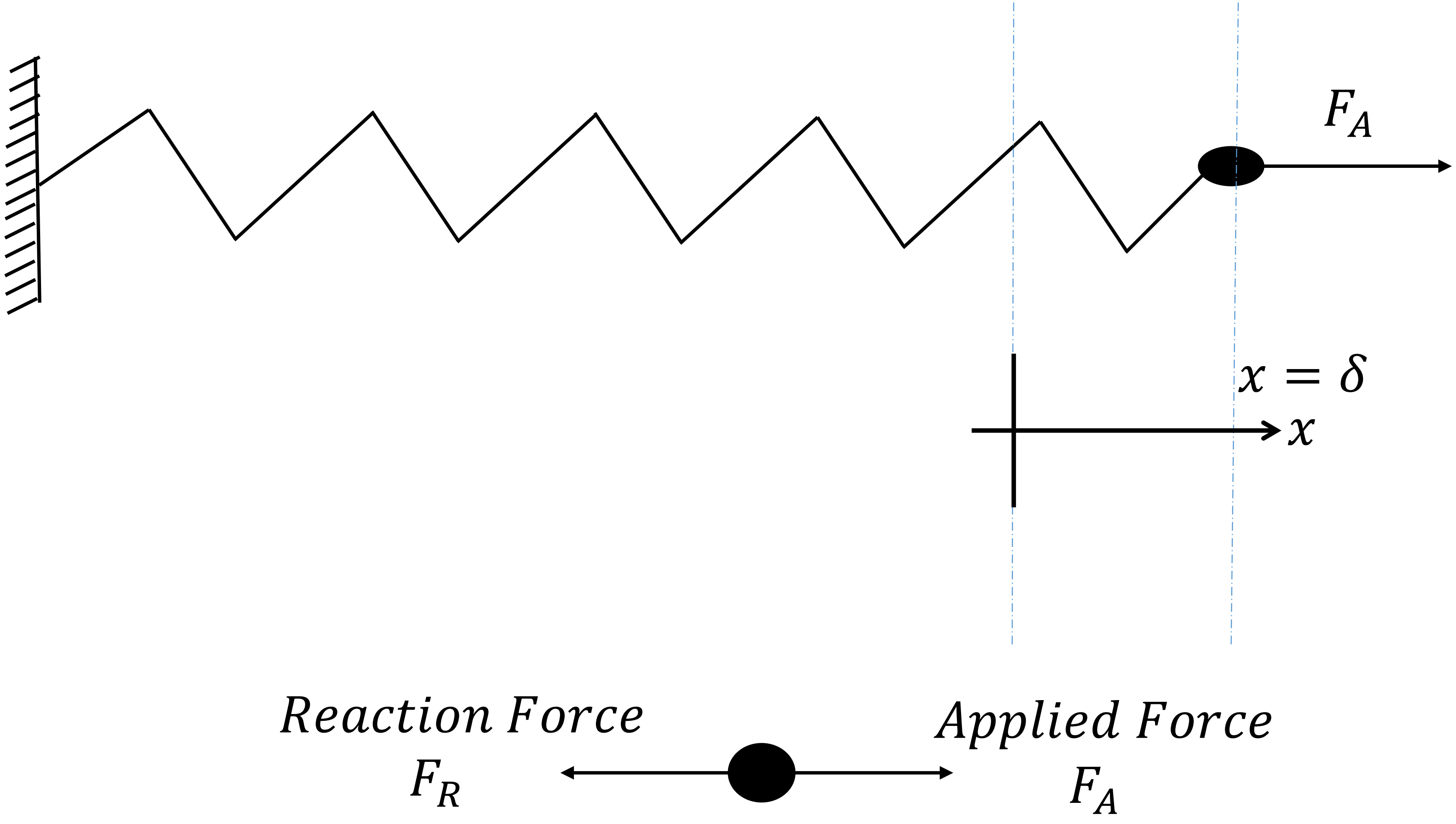
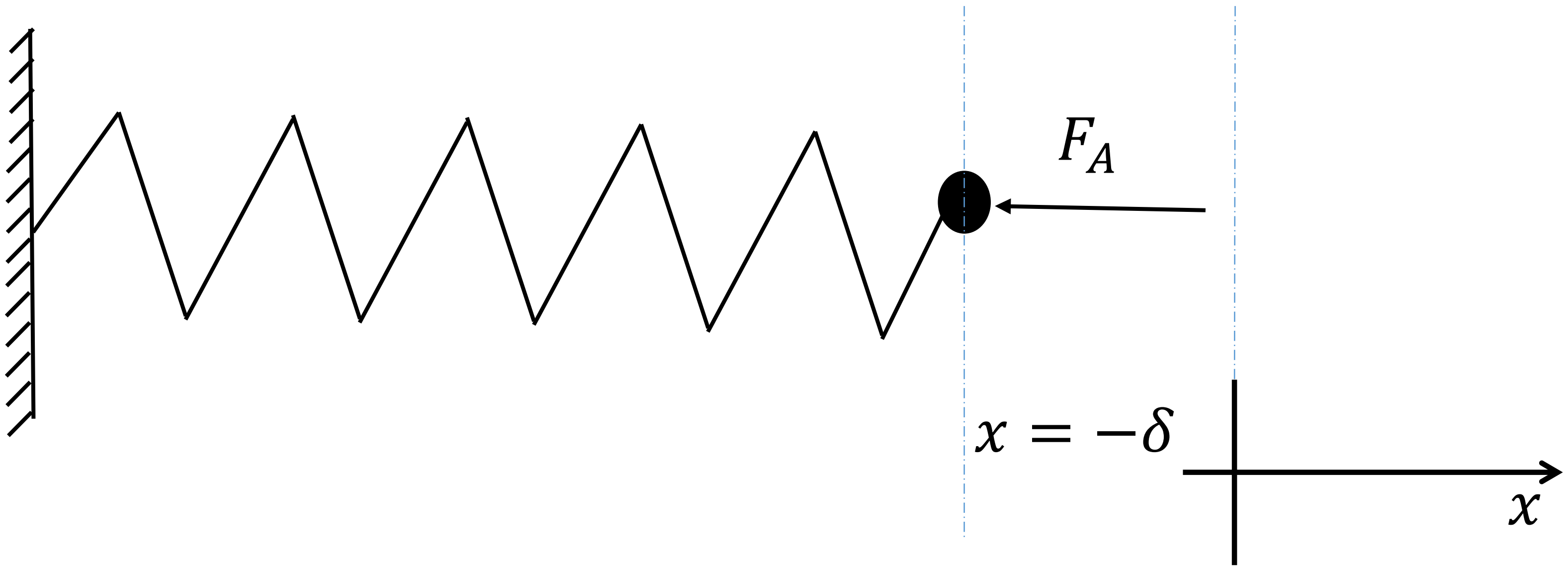
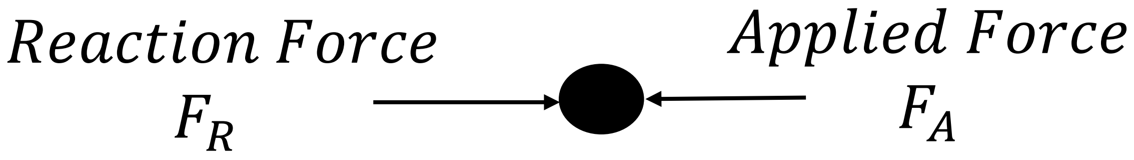

Spring
Point mass
Undeformed spring
x

Deformation under tensile load



Deformation under compressive load





General case: $F_R \propto \delta^n$

$$\Rightarrow F_R = k\delta^n$$

k : Spring Constant (a measure of stiffness of the spring)

Special case: $n = 1$

$\Rightarrow F_R = k\delta$ → Hooke's law (linear spring)

From Newton's 2nd Law of motion,

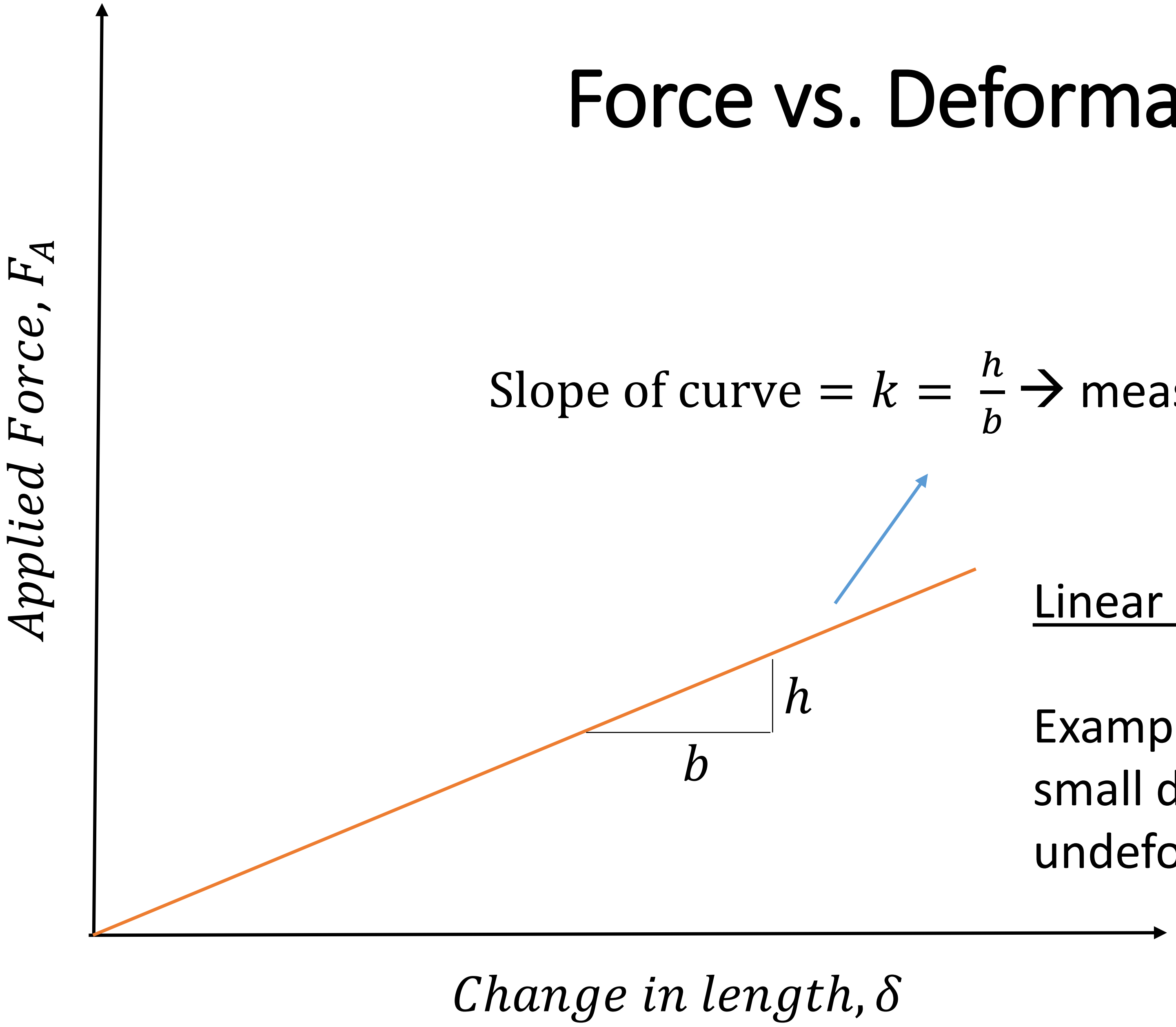
$F_R - F_A$= (mass)(acceleration)

When acceleration is absent and/or mass is negligible,

$k\delta^n - F_A$ = 0

Or, $F_A = k\delta^n$

Force vs. Deformation curve

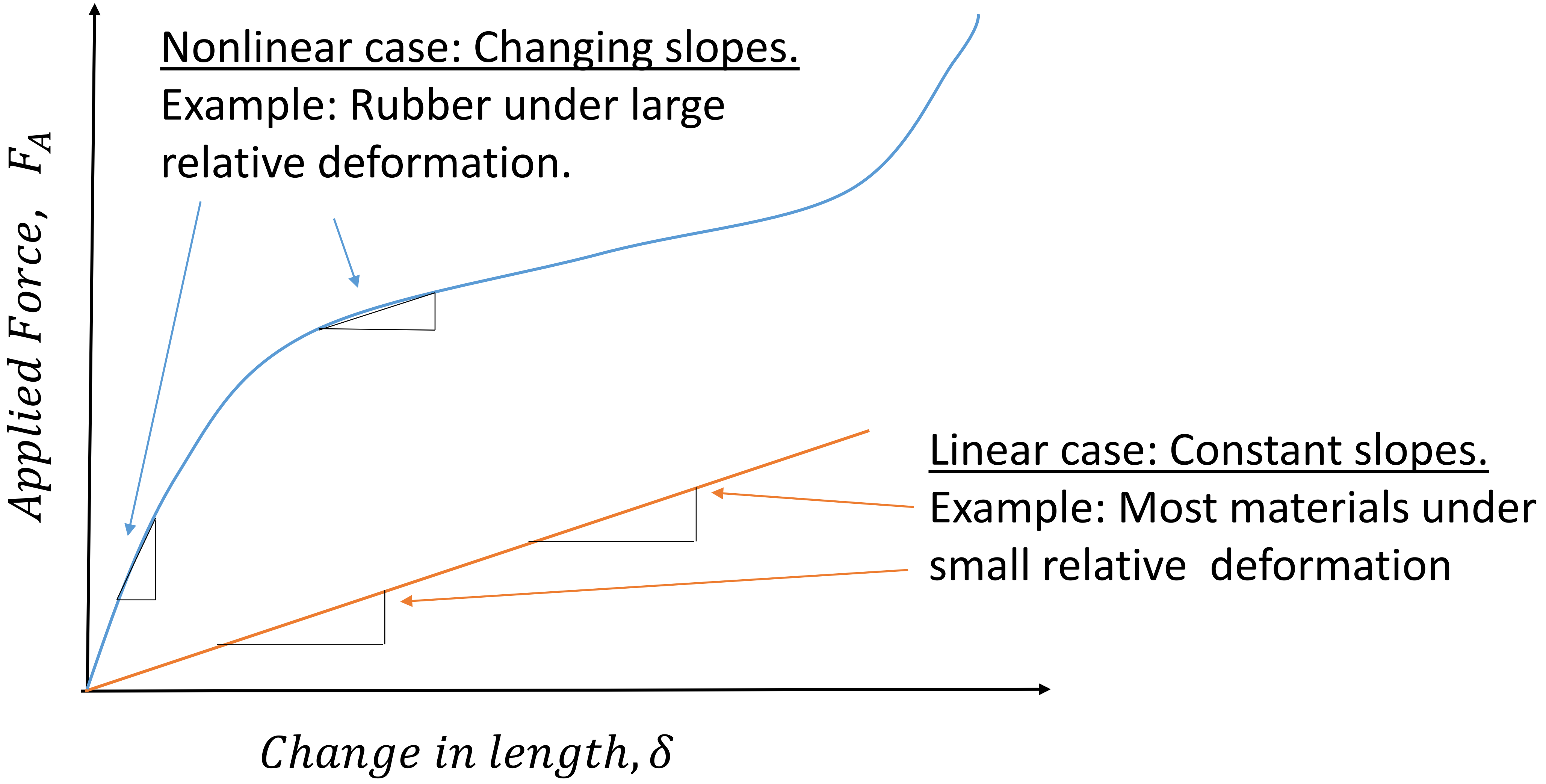


Slope of curve $= k = \frac{h}{b}$ → measure of stiffness of the material

<u>Linear elasticity ⇒ Constant slopes</u>

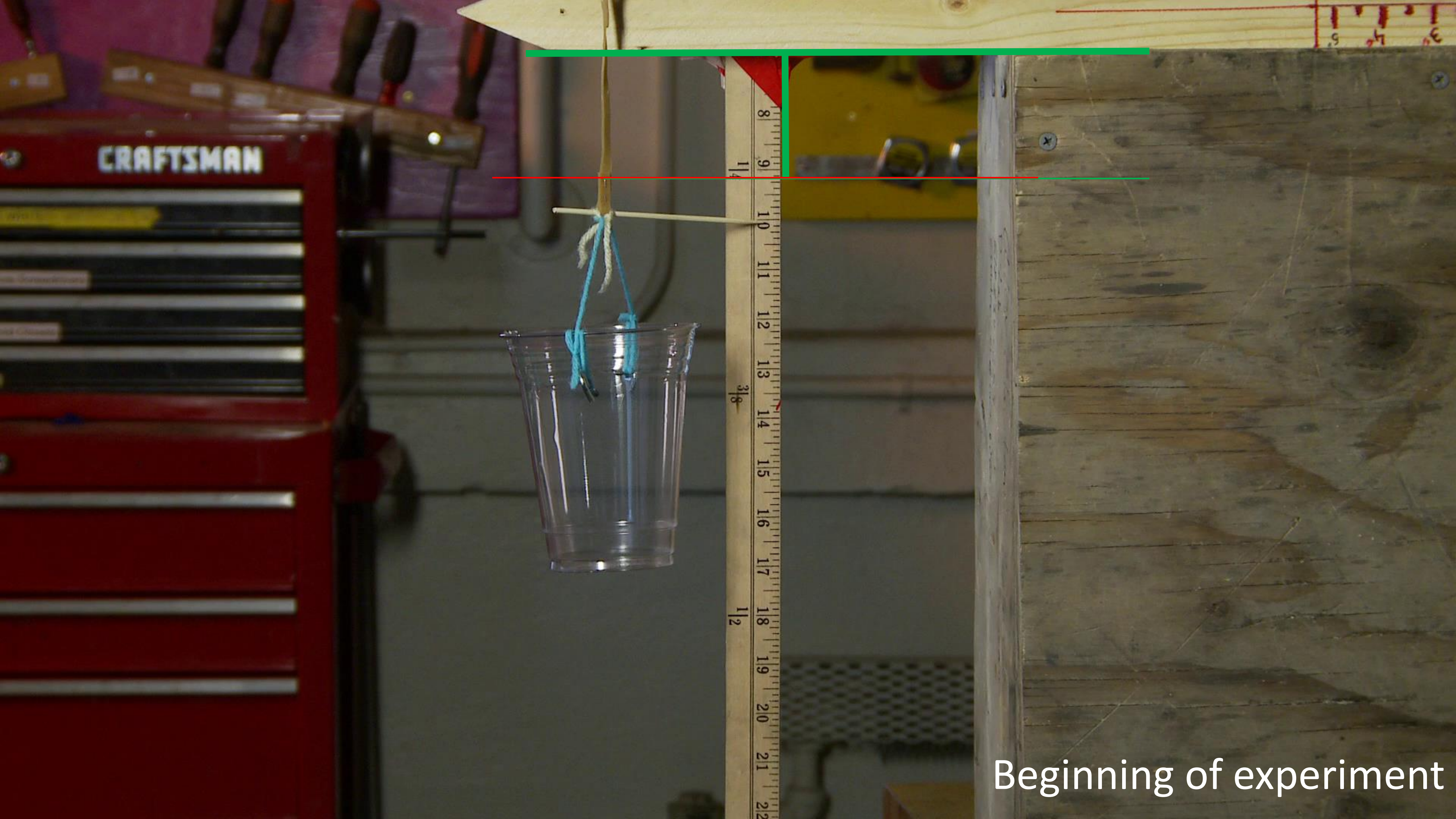
Example: Most materials under small deformation relative to undeformed configuration

Force vs. Deformation curves for Linear and Nonlinear Elasticity

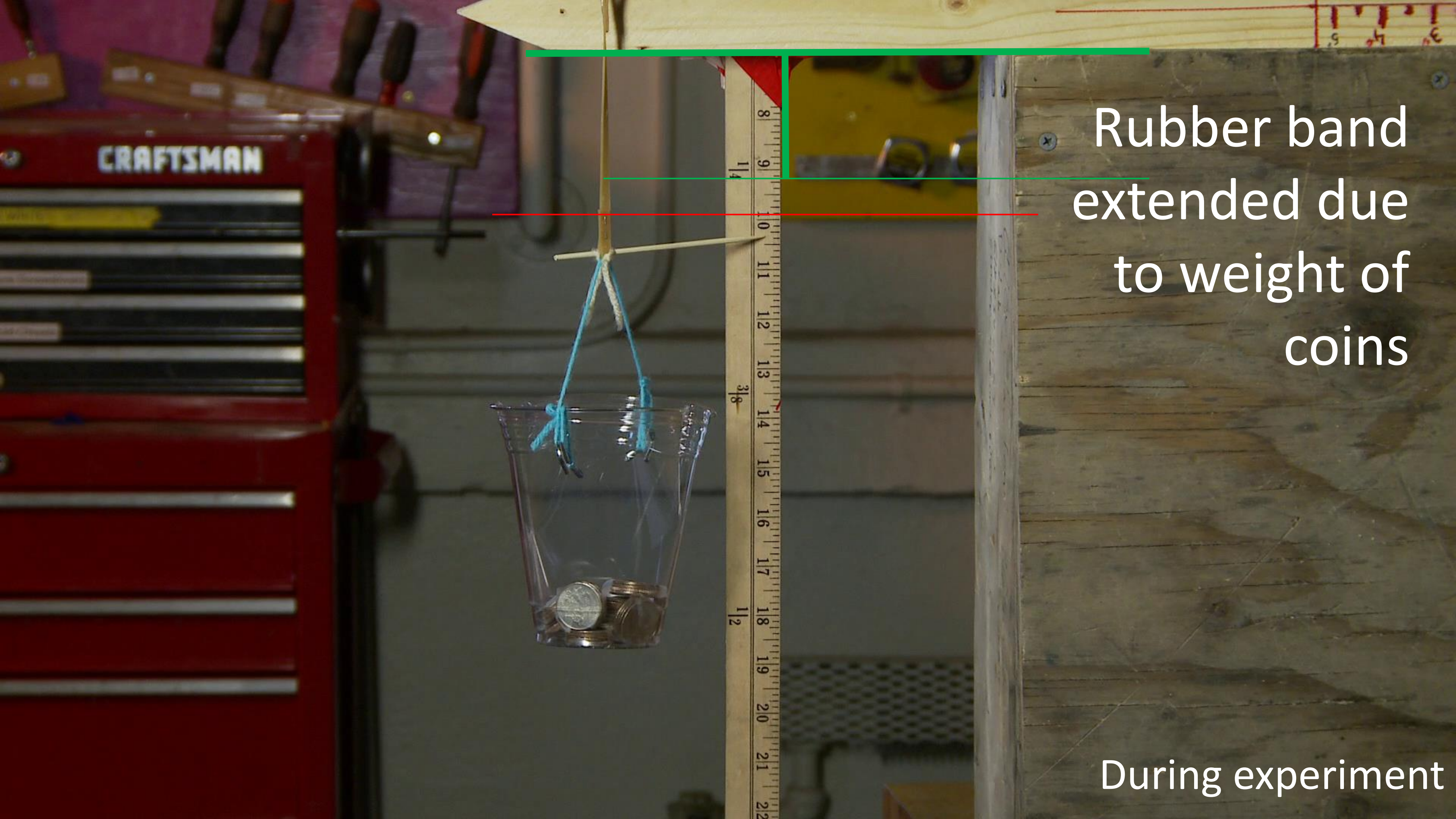


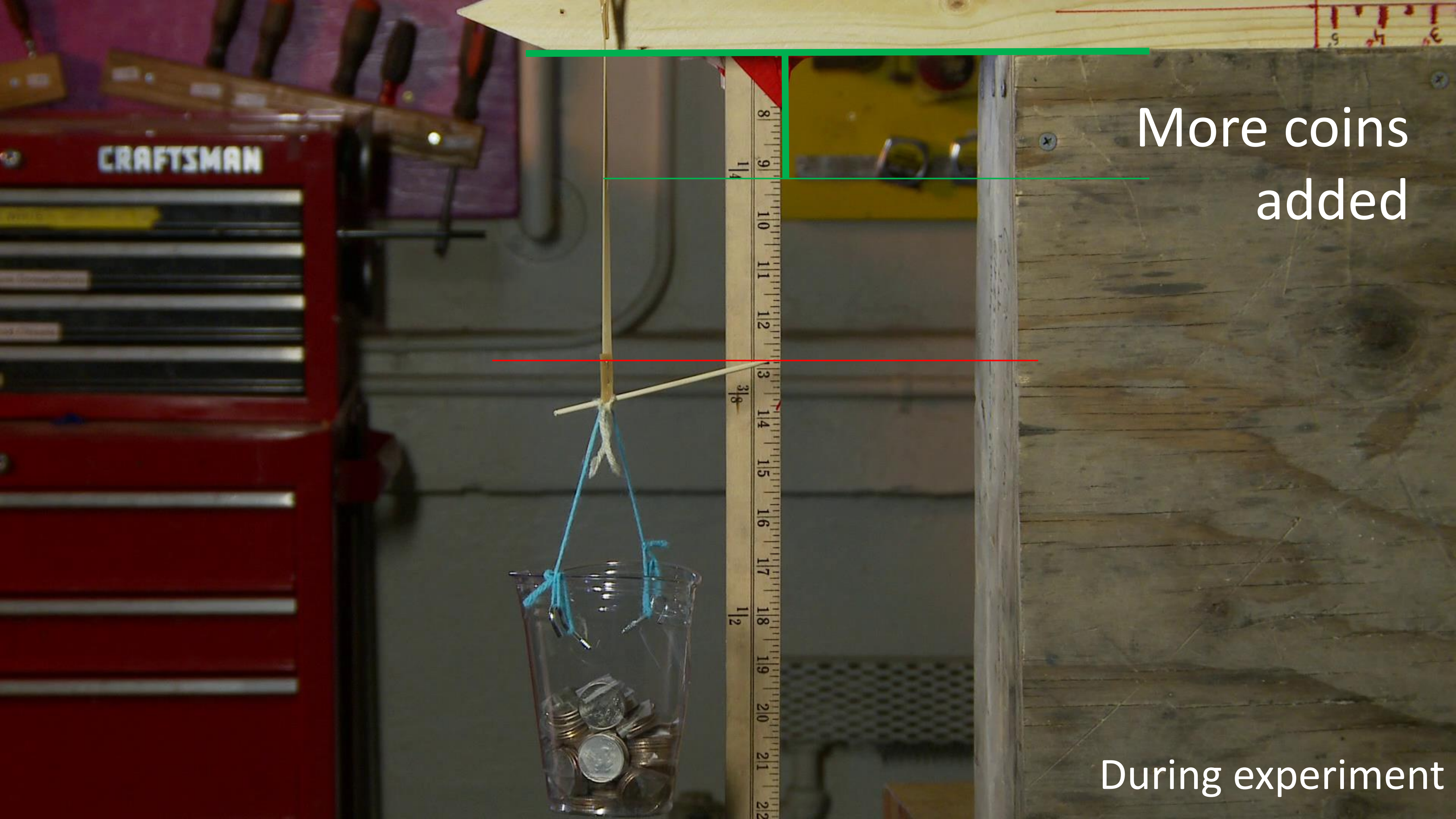
Example of Activity 2

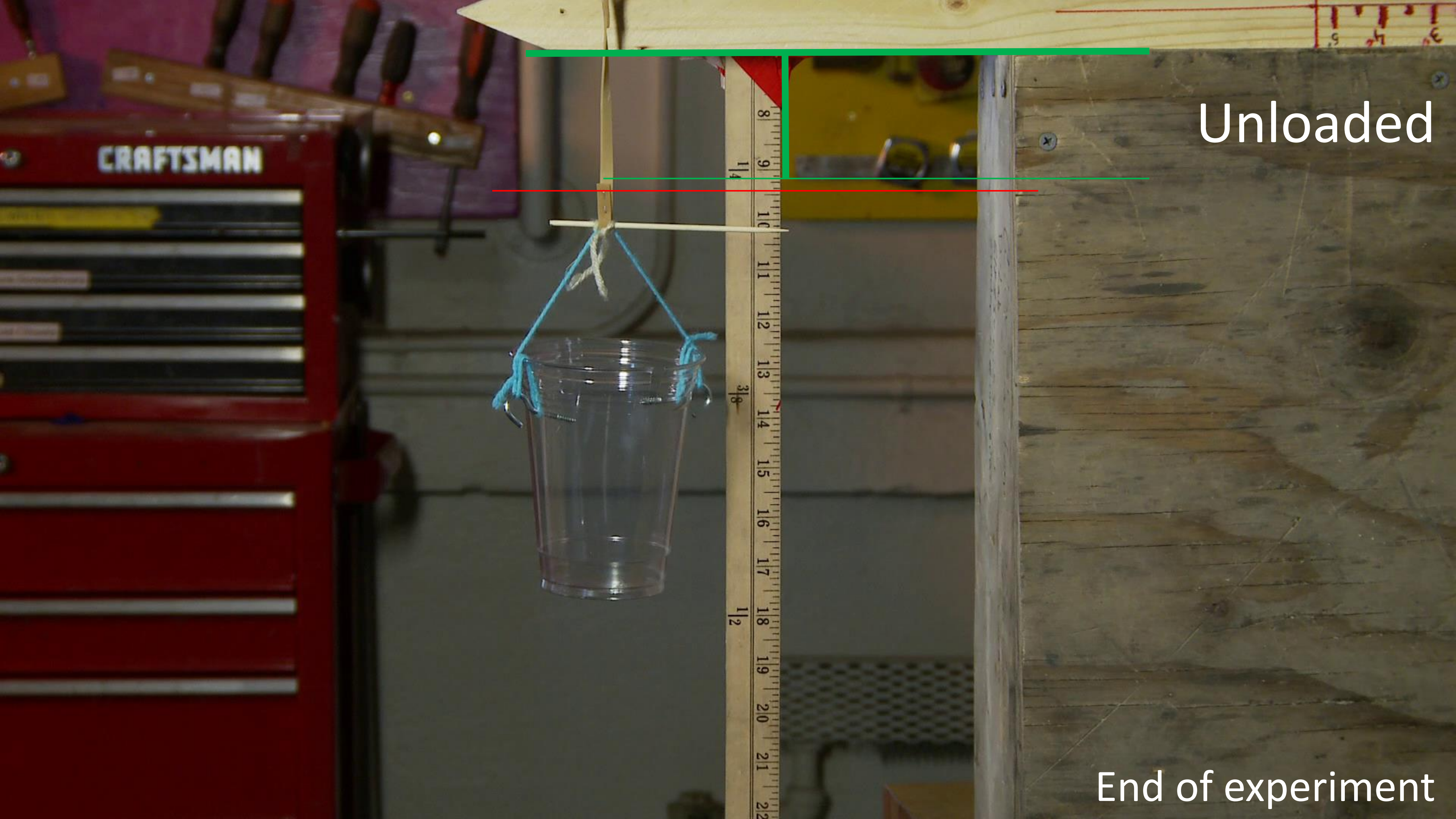
(Controlled extension of rubber band)



Beginning of experiment


CRAFTSMAN
Rubber band extended due to weight of coins
During experiment

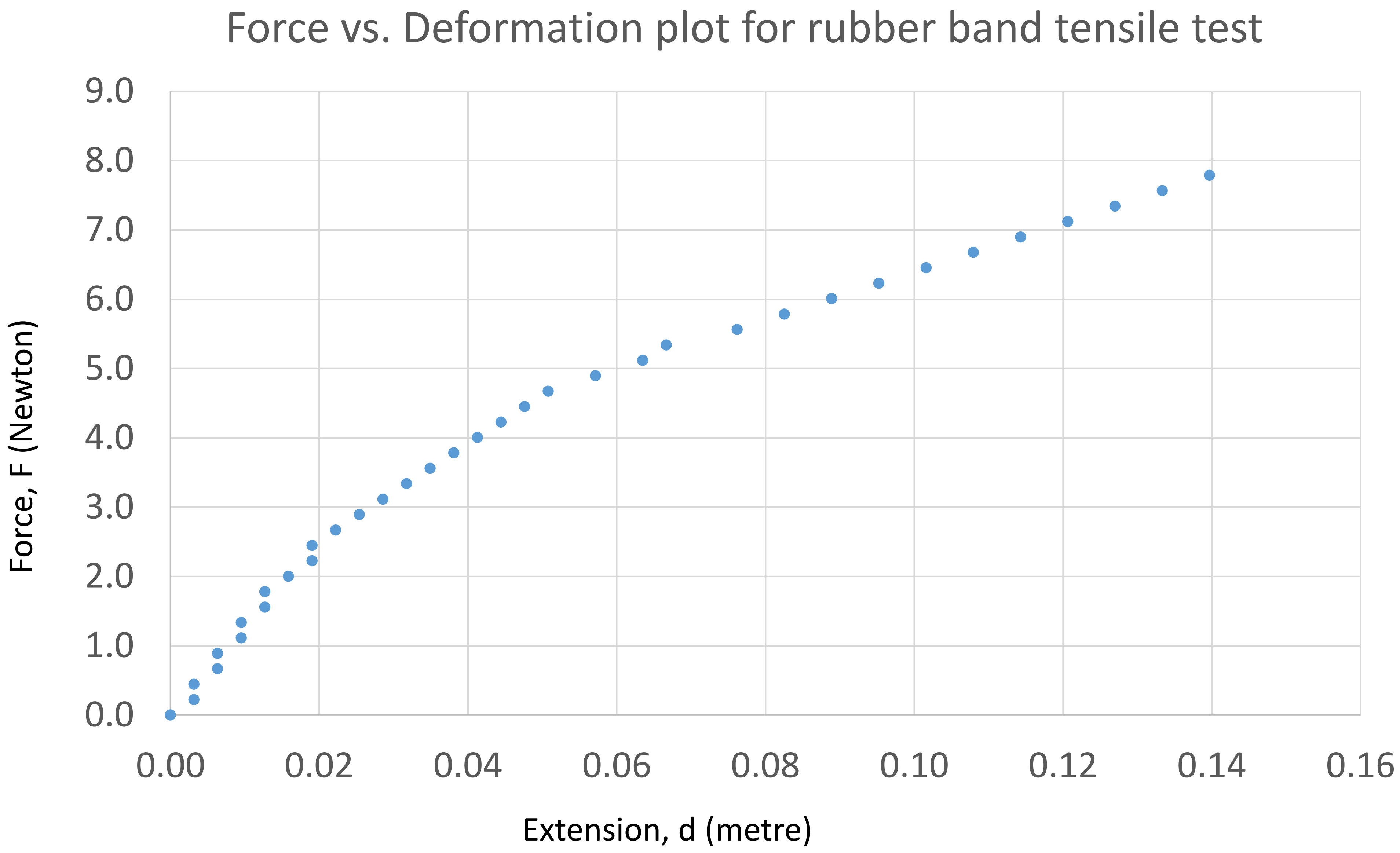

More coins added
CRAFTSMAN
During experiment


Unloaded
End of experiment
CRAFTSMAN

Loading sequence number, n	Mass (kg)	Reading from ruler, Y (metre)	Extension, d= $Y - Y_0$ (metre)	Force = Mass*9.81 (newton)
0	0	Y_0	0	0
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				

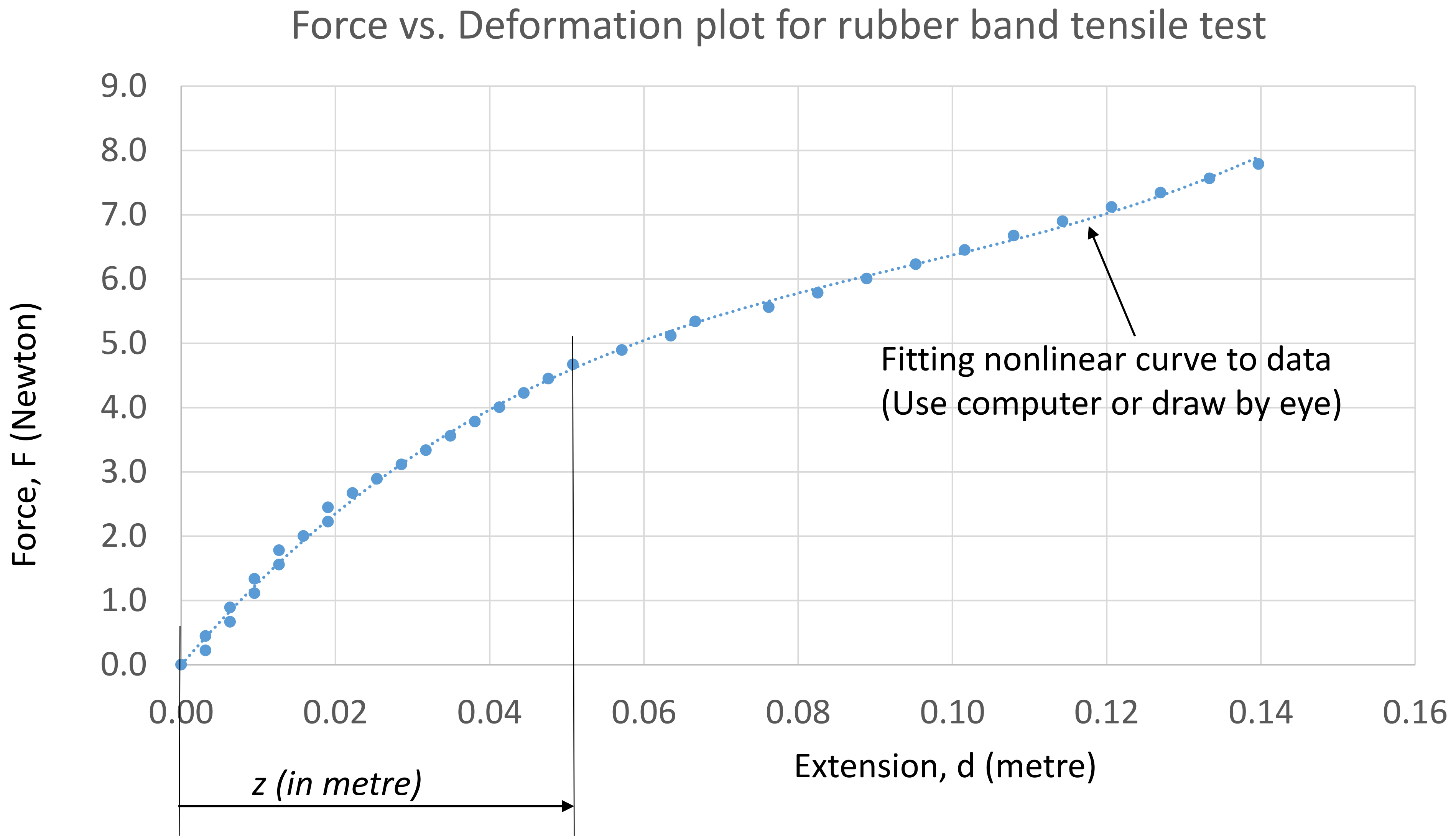
Loading sequence number, n	Mass (kg)	Reading from ruler, Y (metre)	Extension, d= $Y - Y_0$ (metre)	Force = Mass*9.81 (newton)

Example of Activity 2: Controlled extension of rubber band



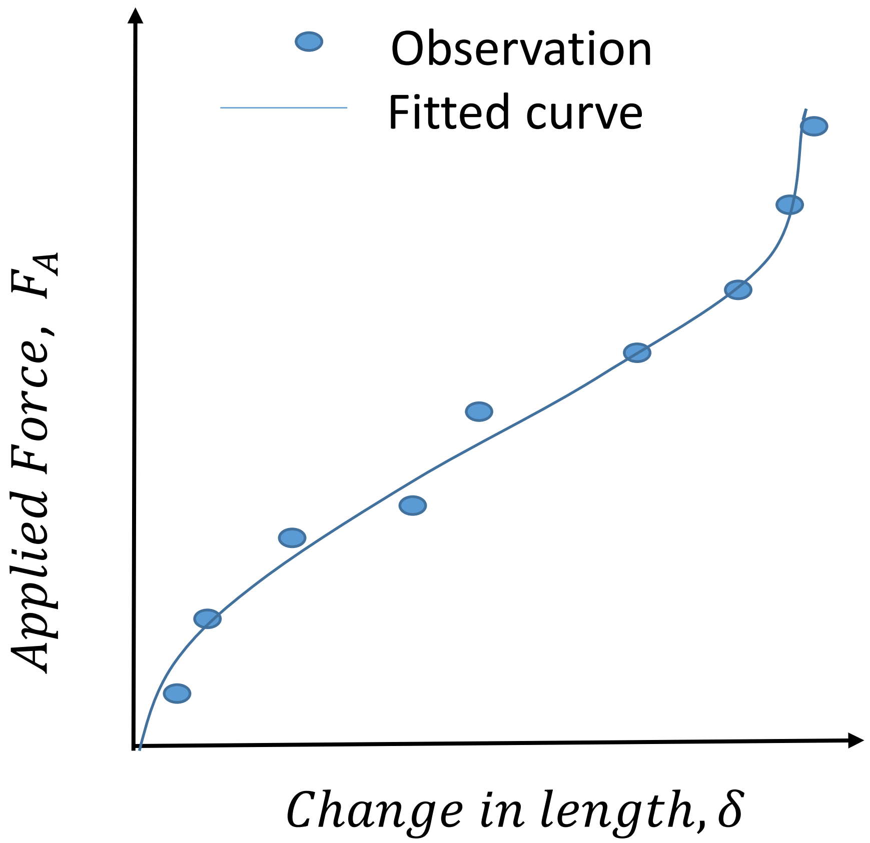
Example of Activity 3

(Launching match-stick using rubber-band as a projectile)

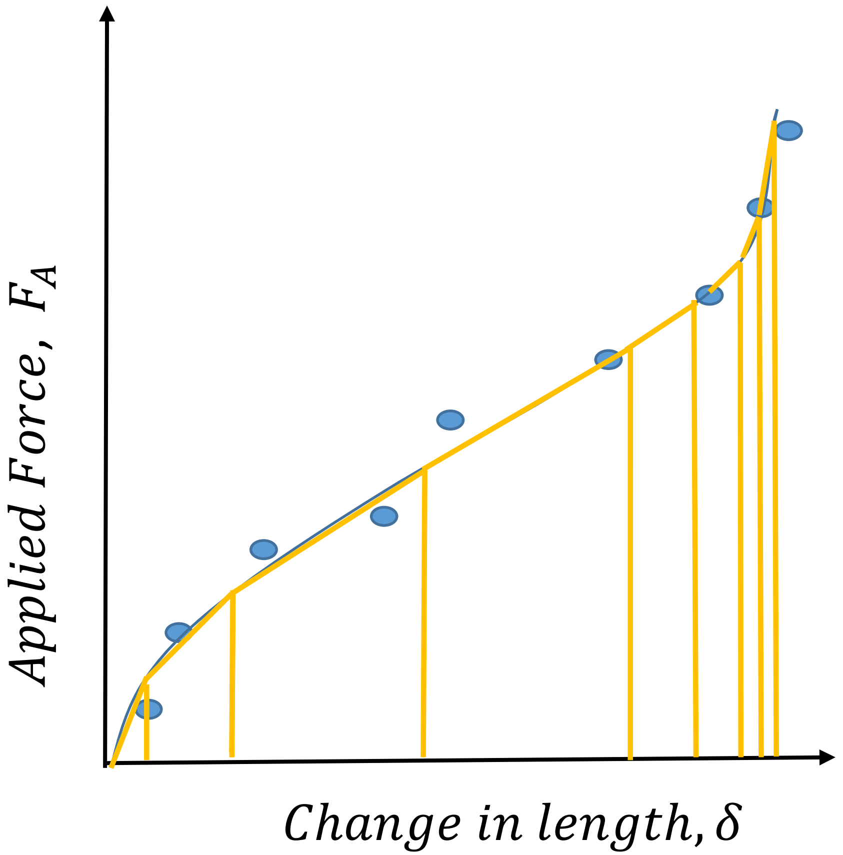


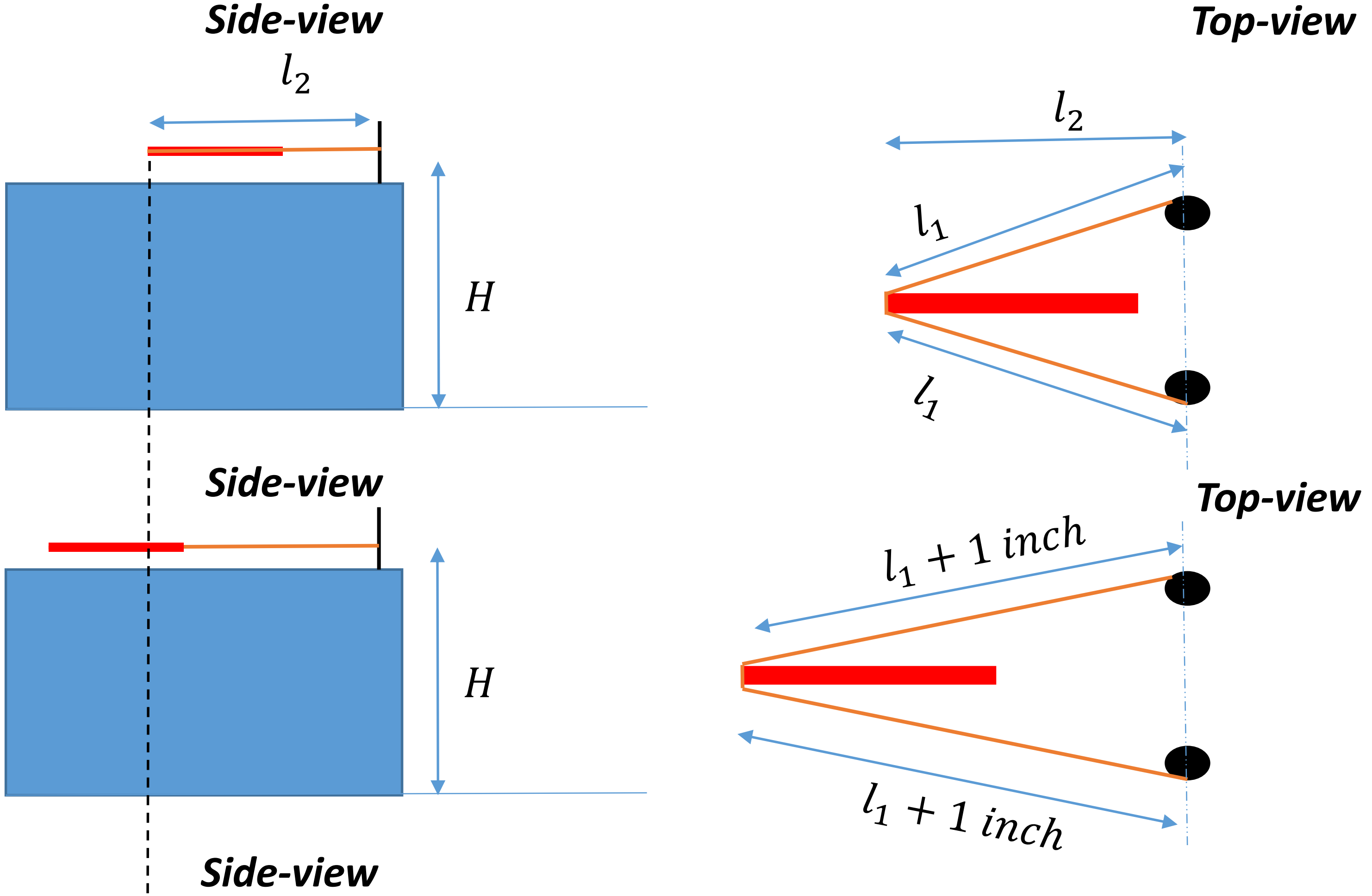
Potential energy stored in the rubber band for extension of *z* metre (area under the Force vs. Deformation curve from d= 0 up to *z* metre)

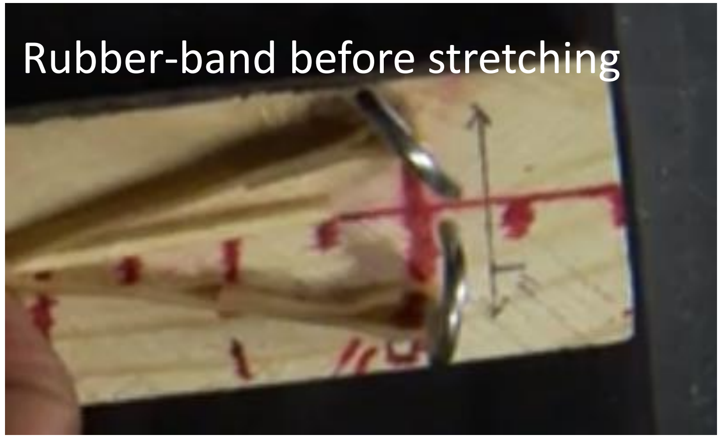
Total elastic potential energy stored = area under Force vs. Deformation curve



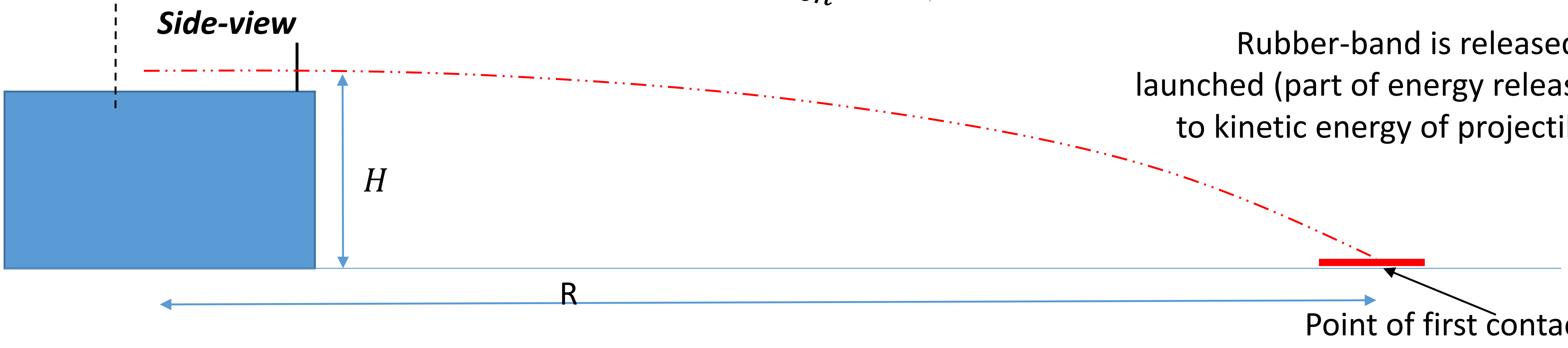
Approximating the area under the Force vs. Deformation curve as a sum of areas of multiple non-overlapping trapezia/trapezoids







After rubber-band is stretched by 2 inch → Potential energy stored in rubber band



Rubber-band is released → Projectile is launched (part of energy released is converted to kinetic energy of projectile) → Projectile hits ground.

#	Range, R (metre)
1	
2	
3	
4	
5	
Median	

Kinetic energy of projectile at launch when rubber band is released

Assuming: (1) Projectile launched horizontally, and (2) No air resistance,

$$\text{Kinetic energy of projectile} = (\text{Mass of projectile})(\text{Horizontal velocity of projectile at launch})^2/2$$

$$(\text{Horizontal velocity of projectile at launch})^2 = \frac{(\boldsymbol{Range\ of\ projectile})^2(\boldsymbol{Acceleration\ due\ to\ gravity})}{\mathbf{2}(\boldsymbol{Elevation\ of\ launch\ point})}$$

$$\frac{\boldsymbol{Kinetic\ Energy\ of\ Projectile\ at\ launch}}{\boldsymbol{Potential\ Energy\ stored\ in\ Rubber\ Band}} < \mathbf{1}$$

Activity 3: Launching a match-stick using the rubber-band as a projectile

- Range, R (median of range values from 5 repetitions of projectile launch)
- Elevation of launch, H
- Acceleration due to gravity, g = 9.81 m/sec^2
- Mass of projectile, M
- Velocity of launch of projectile, $\mathrm{v} = \mathrm{R}\sqrt{g/2H}$
- Kinetic energy of projectile at launch, $\mathrm{KE}^{(\mathrm{Pr})} = \mathrm{Mv}^2/2$
- Potential energy stored in rubber-band for z metre of extension, $\mathrm{PE}^{(\mathrm{Rb})}$ in Joule
- **Ratio = $\mathbf{KE}^{(\mathbf{Pr})}/\mathbf{PE}^{(\mathbf{Rb})}$ = ?**

DataAnalysisForActivities2and3.xlsx (sheet 2)