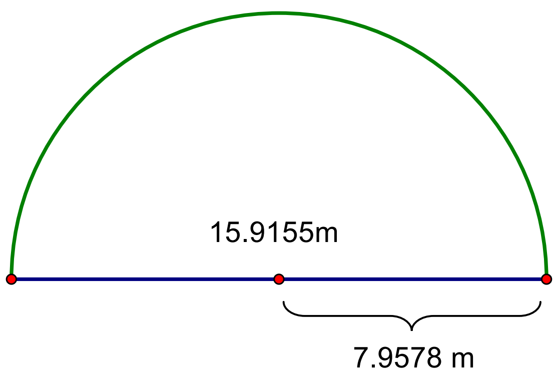
Homework for Fence Your Equation

For your reference, this is the answer to the homework related to semi-circular shape:

Since there are 25 meters of fencing, one half of the circumference of the corresponding circle would be 25 meters in length. Thus, if the pen were a full circle, the circumference would be 50 meters. And recall that the constant *p* is the ratio of the circle circumference *c* to its diameter *d*. So, we can write

$$\frac{\text{circle circumference}}{\text{circle diameter}} = \frac{c}{d} = \pi.$$

Thus we have $d = c/\pi = 50/\pi \approx 15.9155$. This means that the wall part of the semi-circular pen would be $d \approx 15.9155$ meters in length. Now the radius *r* of the circle (or semi-circle) is equal to *d*/2, or $r = d/2 = 25/\pi \approx 7.9578$. But we all recall that the area of a circle is equal to πr^2. So the area (A_{sc}) of the semi-circular pen would be



$$A_{sc} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(\frac{25}{\pi})^2 = \frac{(25)^2}{2\pi} = \frac{625}{2\pi} \approx 99.472$$

Appendix A

(i) If x represents the longer side of the pen,

$$Y = -\frac{1}{2}x^2 + \frac{25}{2}x$$

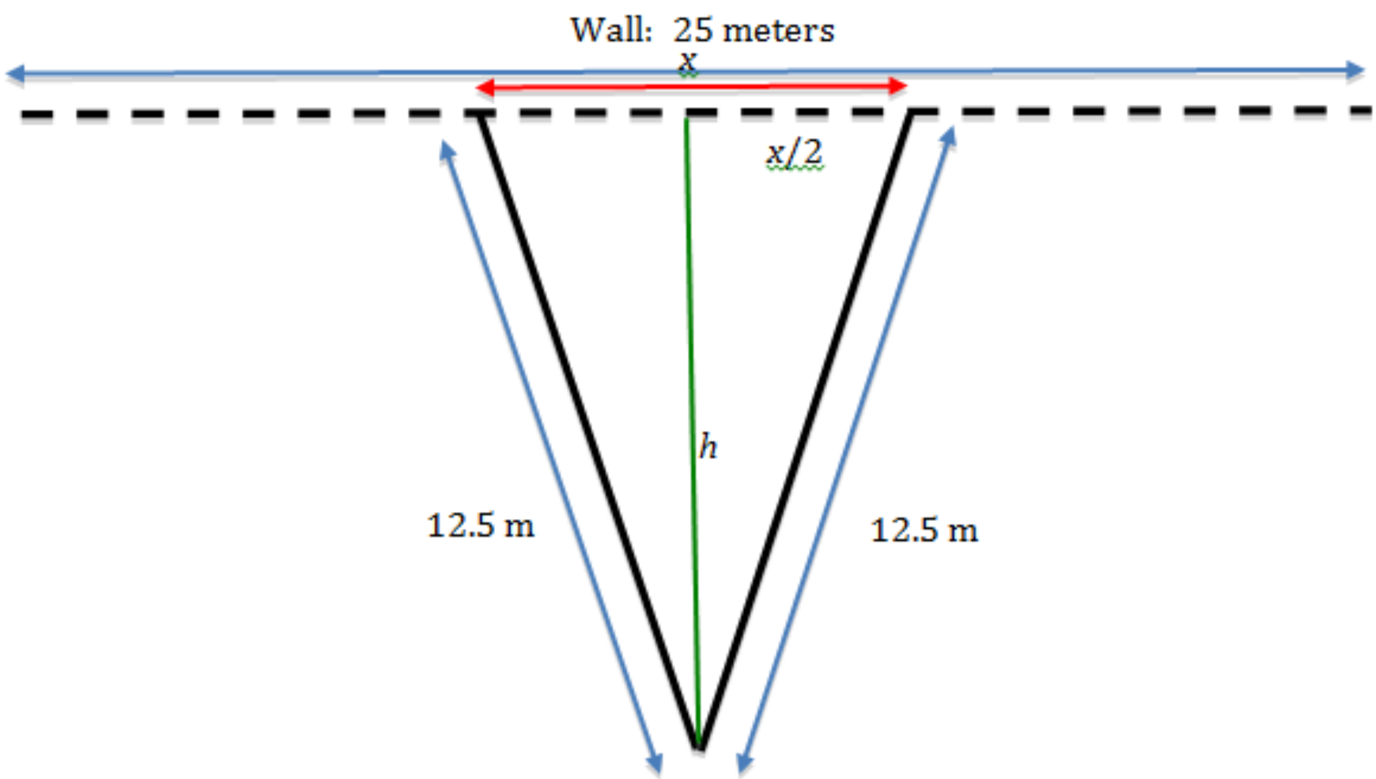
X	0										
Y	0										

(ii) If x refers to two <u>equal</u> sides,

$$Y = -2x^2 + 25x$$

X	0										
Y	0										

Appendix B



Appendix C

Solution for Isosceles Triangle

$$(x/2)^2 + h^2 = (12.5)^2$$

$$h = \sqrt{(12.5)^2 - (x/2)^2}$$

$$A(x) = \tfrac{1}{2} x \sqrt{(12.5)^2 - (x/2)^2}$$

$$\frac{dA(x)}{dx} = 0 = \frac{1}{2}\sqrt{(12.5)^2 - (x/2)^2} + \frac{1}{2} x \frac{-(1/2)2x/4}{\sqrt{(12.5)^2 - (x/2)^2}}$$

$$-\frac{1}{2}\sqrt{(12.5)^2 - (x/2)^2} = \frac{1}{2} x \frac{-(1/2)2x/4}{\sqrt{(12.5)^2 - (x/2)^2}}$$

$$-\sqrt{(12.5)^2 - (x/2)^2} = x \frac{-(1/2)2x/4}{\sqrt{(12.5)^2 - (x/2)^2}}$$

$$-[(12.5)^2 - (x/2)^2] = x[-(1/2)2x/4]$$

$$-(12.5)^2 + (x/2)^2 = -x^2/4$$

$$x^2/4 + (x/2)^2 = (12.5)^2$$

$$x^2/4 + x^2/4 = (12.5)^2$$

$$2(x^2/4) = (12.5)^2$$

$$x^2/2 = (12.5)^2$$

$$x^2 = 2(12.5)^2, \text{ or } \quad x = x^* = 12.5\sqrt{2}. \approx 17.68$$

$$A(x^*) = \tfrac{1}{2} x^* \sqrt{(12.5)^2 - (x^*/2)^2} = \tfrac{1}{2} 12.5\sqrt{2}\sqrt{(12.5)^2 - (12.5\sqrt{2}/2)^2}$$

$$A(x^*) = \tfrac{1}{2}(12.5)^2\sqrt{2}\sqrt{1-(\sqrt{2}/2)^2} = \tfrac{1}{2}(12.5)^2\sqrt{2}\sqrt{1-(1/2)} = \tfrac{1}{2}(12.5)^2 \approx 78.125 \text{ m}^2.$$

Solution for Equilateral Triangle

$$(12.5)^2 + (12.5)^2 = 2(12.5)^2 = (x^*)^2$$

$$h'^2 + (12.5/2)^2 = (12.5)^2$$

$$h' = \sqrt{(12.5)^2 - (12.5/2)^2} = 12.5\sqrt{1-1/4} = (12.5/2)\sqrt{3}.$$

$$A(12.5) = (1/2)b'h' = (1/2)12.5(12.5/2)\sqrt{3} = (12.5)^2\frac{\sqrt{3}}{4} \approx 67.66 \text{ m}^2$$

$$A(12.5) = (12.5)^2\frac{\sqrt{3}}{4} < A(x^*) = A(12.5\sqrt{2}) = \tfrac{1}{2}(12.5)^2$$

The ratio of Area of Isosceles Triangle to Area of Equilateral Triangle

$$\frac{A(x^*)}{A(12.5)} = \frac{A(12.5\sqrt{2})}{A(12.5)} = \frac{\frac{1}{2}(12.5)^2}{(12.5)^2\sqrt{3}/4} = \frac{2}{\sqrt{3}} \approx \frac{2}{1.732} \approx 1.155$$