

Activity 1: solve for size and speed of a drop.

5. In order not to be breakable, the surface tension of a drop should be greater than the pressure difference between top and bottom of the drop.

It implies that $\sigma a \sim \Delta p a^2$.

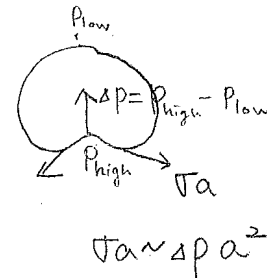
And Δp is equal to $\rho_{\text{air}} v^2$.

$$\therefore \sigma a \sim \rho_{\text{air}} v^2 a^2 \quad \therefore \rho_{\text{air}} v^2 \sim \frac{\sigma}{a}$$

At the terminal velocity, drag force is equal to gravity

$$\therefore D = \rho_{\text{air}} v^2 a^2 \sim mg \sim \rho_w a^3 g$$

$$\therefore \rho_{\text{air}} \frac{\sigma}{a} \cdot a^2 = \sigma a \sim \rho_w a^3 g \quad \therefore a \sim \sqrt{\frac{\sigma}{\rho_w g}}$$



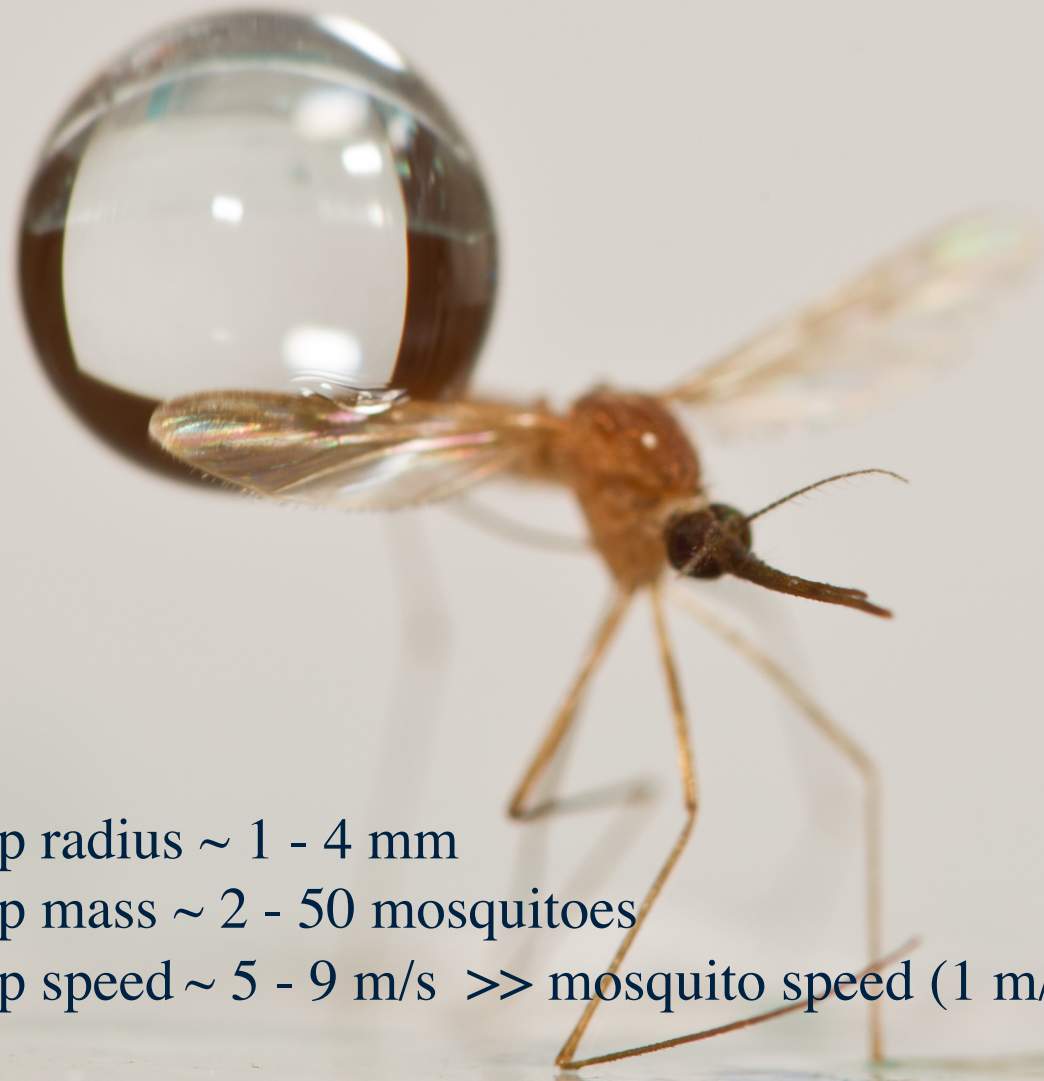
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Useful units for calculations

Remember: dyne = 1 g cm/s² (force in cm-g-s units) = 10⁻⁵ Newtons. Using cgs units is easiest for calculations with small insects, as the values are often order-one.

- Density of air $\rho = 10^{-3}$ g/cm³
- Density of water $\rho_w = 1$ g/cm³
- Gravity $g \approx 1000$ cm/s²
- Surface tension $\sigma = 70$ dynes/cm

Answer 1: Raindrops heavier and faster than mosquitoes



- raindrop radius $\sim 1 - 4$ mm
- raindrop mass $\sim 2 - 50$ mosquitoes
- raindrop speed $\sim 5 - 9$ m/s \gg mosquito speed (1 m/s)

Activity 2: What is frequency of impacts for a flying mosquito?

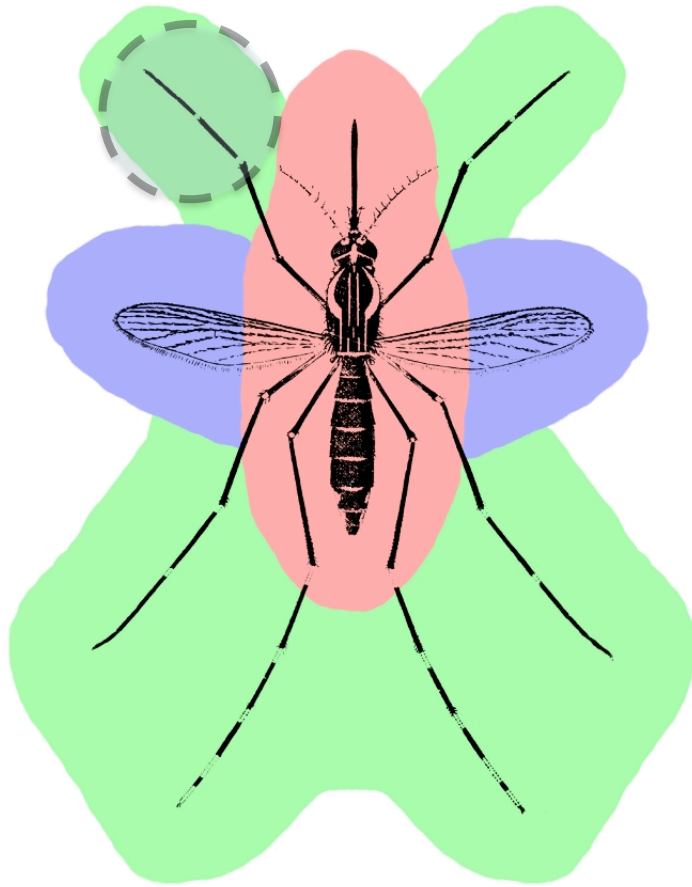
This is a problem of mass conservation. Rain intensity I is given in the units of cm/hour. We must convert this unit into a number of drops that falls on top of the mosquito per second.

This problem is related to the the first problem in “flying circus of physics”, which asks is wetter to run or walk through the rain. Here we recognize that mosquitoes fly so slowly that impacts on the mosquito’s frontal area are negligible compared to that on top.

So consider only drops falling atop the mosquito where plan-view area of wings and legs is A_m . This area is given by considering all drops that impact or even graze the legs (See diagram on next page where an area is sketched out that is one drop radius wider than legs and body). Students should estimate this area A_m to 1 significant digit.

First convert I to cm per second. Then, every second, we consider the volume of drops that fall to fill a volume that is A_m wide and I tall. We can convert this into a mass of fluid falling per second using the density of water ρ used in activity 1. Lastly, we can find frequency of drops f by remembering each drop has a fixed volume m calculated from activity 1.

Activity 2 solution: Frequency of impacts



impact area $A_m \sim 30\text{-}40 \text{ mm}^2$.

rain intensity $I \sim 50 \text{ mm/hr}$

frequency of impacts: once every 25 sec

$$f = \frac{\rho I A_m}{m_{drop}} \sim \frac{1 \text{ hits}}{25 \text{ s}}$$

Direct impact (red) zone
composes 1/4 total area.

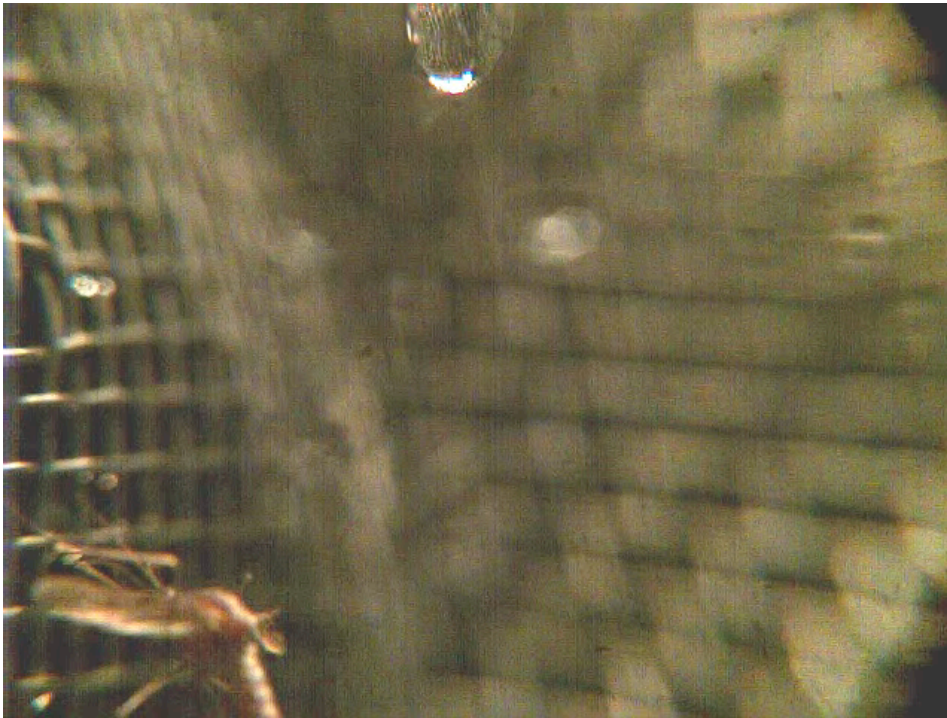
Glancing Impacts are 3X more
likely.

Activity 3: What is force of raindrop after glancing blow

This is a problem of conservation of angular momentum.

The

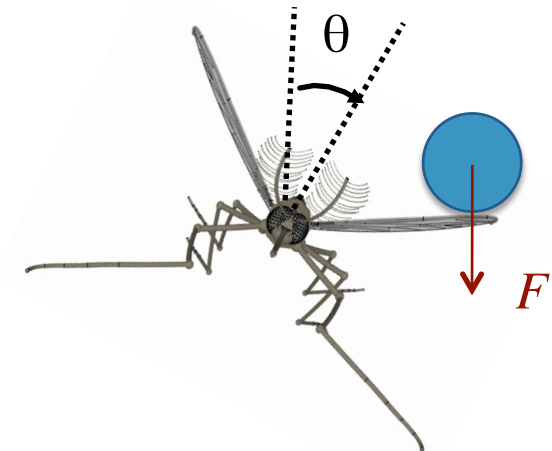
Activity 3: Glancing Blow



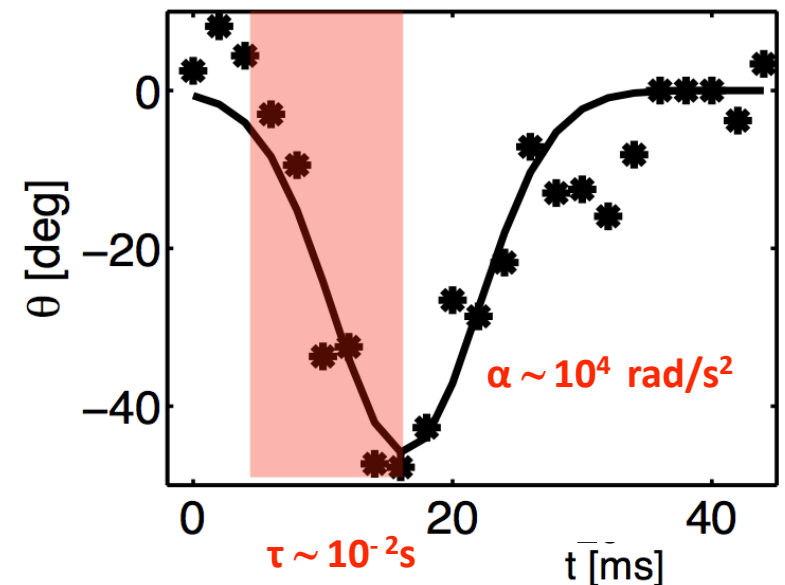
Torque applied by the drop: $r \times F = I \alpha$

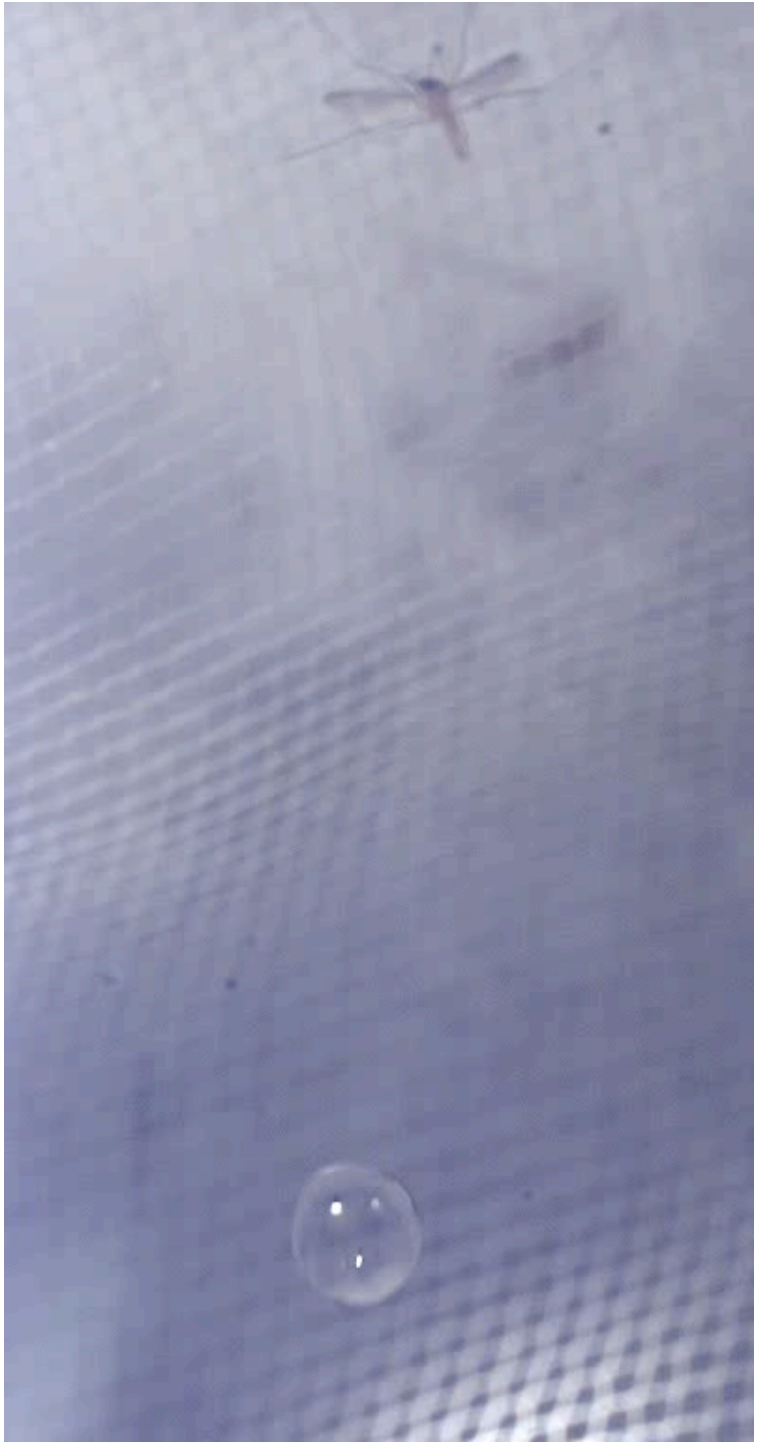
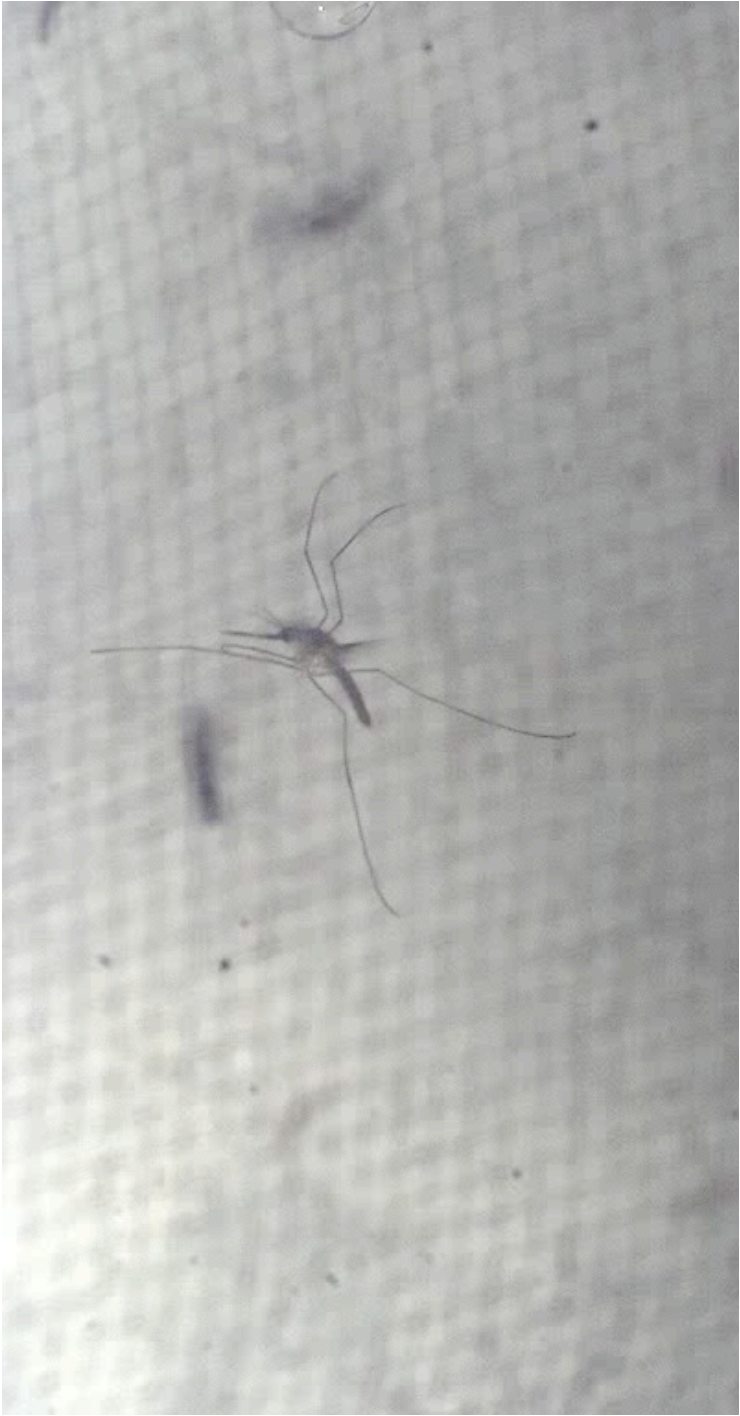
Geometry: $I = m_2 R^2 / 2 \approx 4 \times 10^{-5} \text{ g cm}^2$
 $R = 1 \text{ mm}$

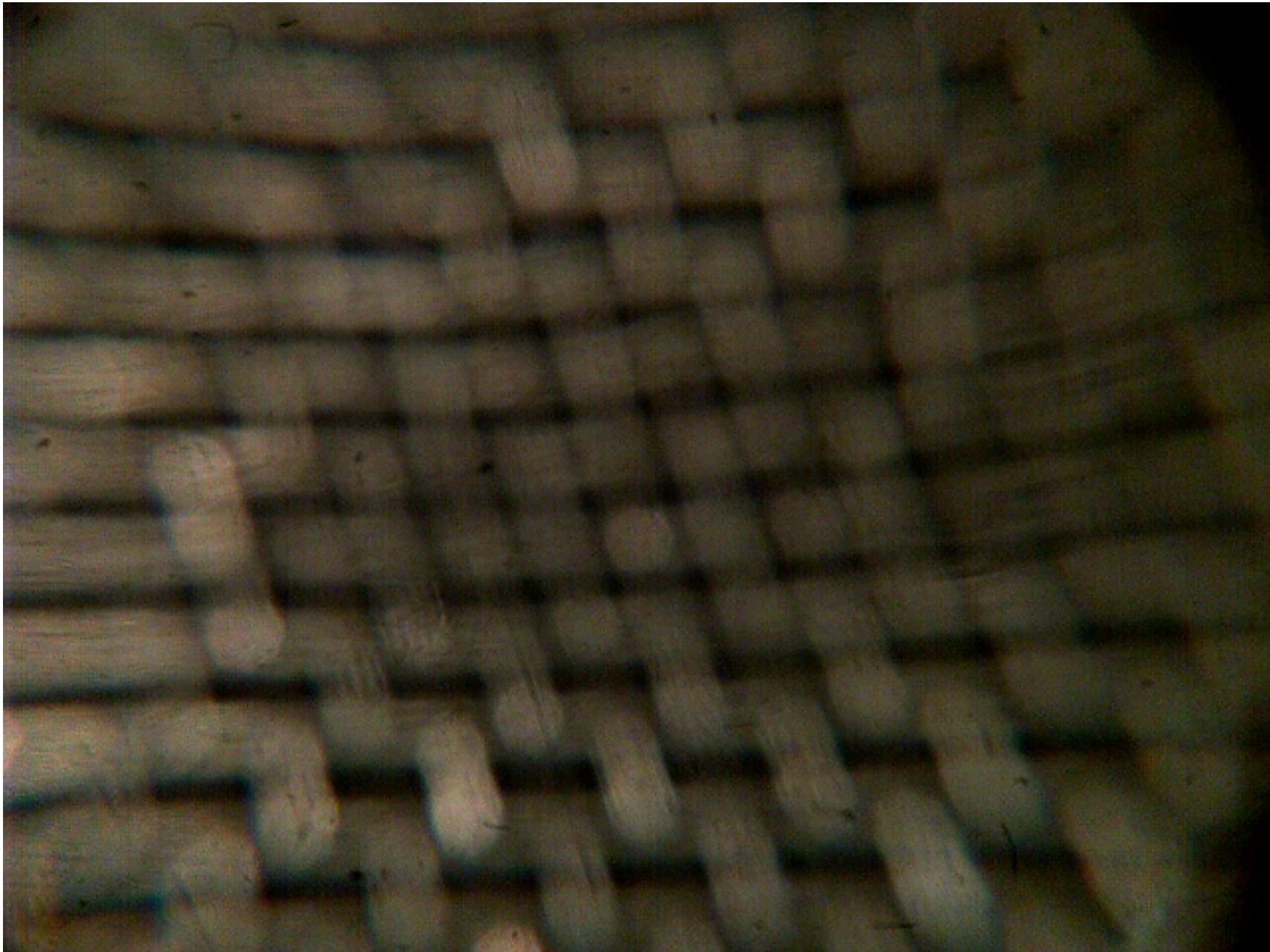
→ $F \sim 3.5 \text{ dynes} \sim 2 \text{ mosquito weights}$



Kinematics



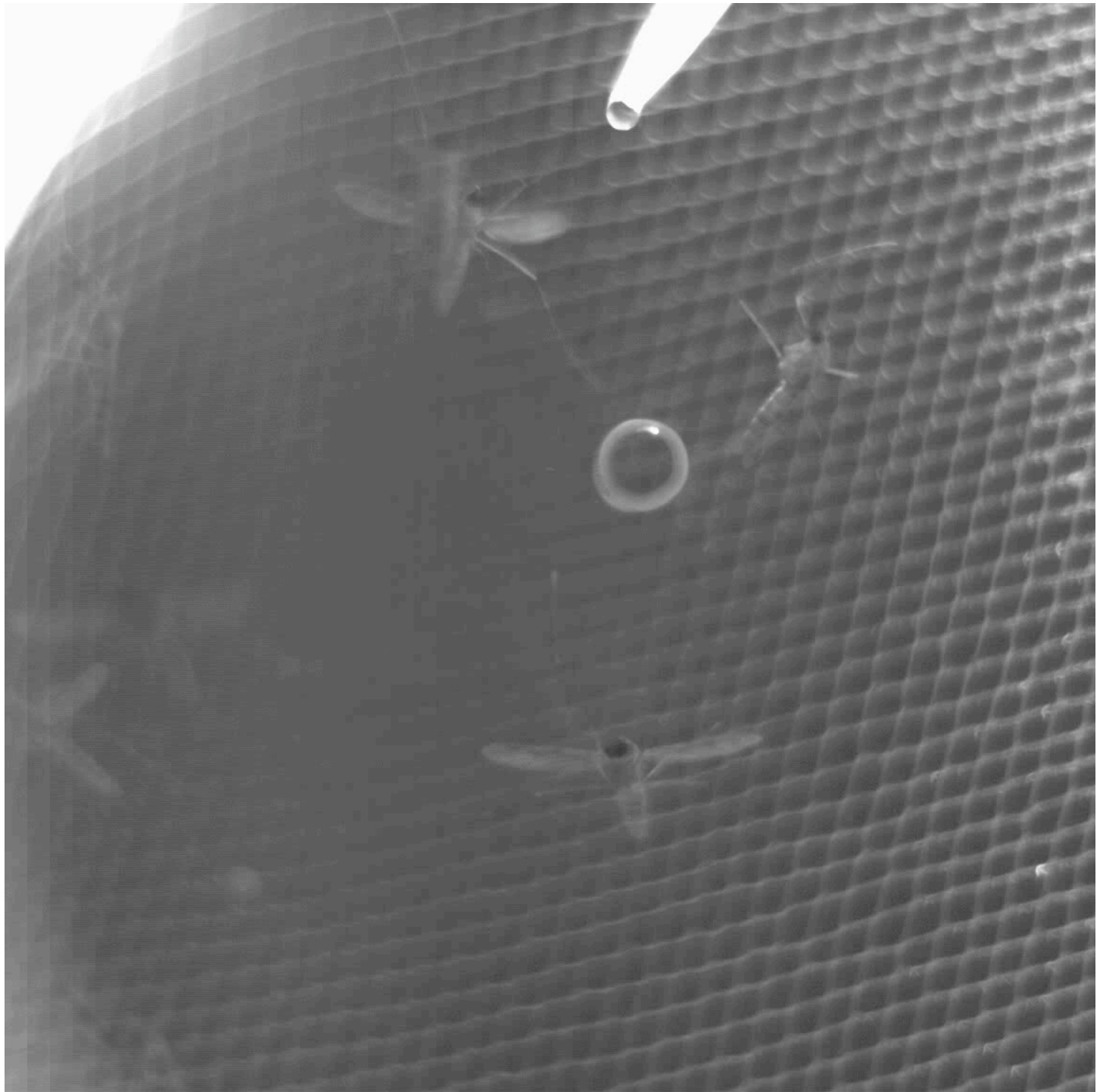




Direct Hit

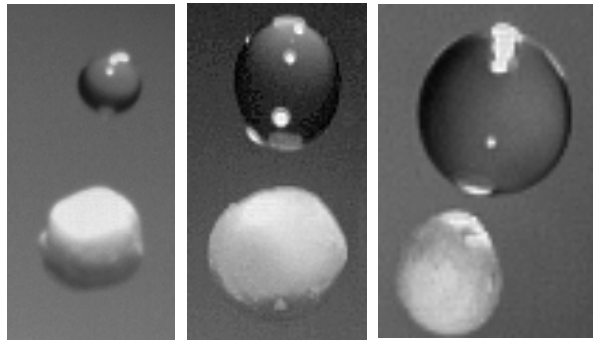


Inelastic collision—mosquito and drop move as one mass.
Mosquito pushed many body lengths before released.



Insect Mimics

Styrofoam “Mosquitoes”

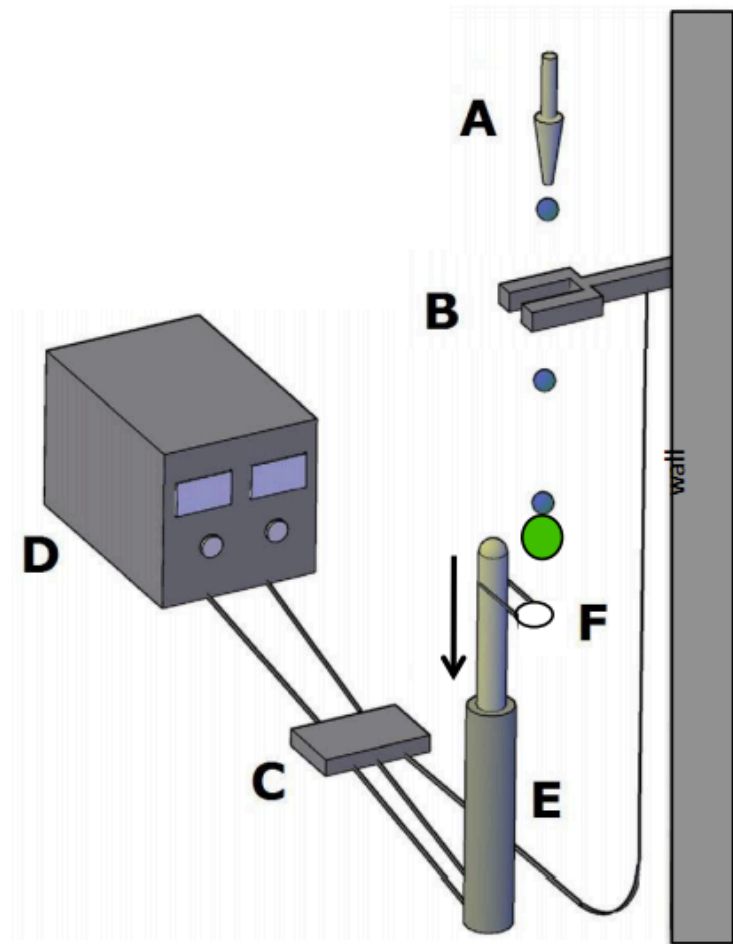


$m_2 \sim 0.4 - 4 \text{ mg}$



$m \sim 2 \text{ mg}$

Free body impacts

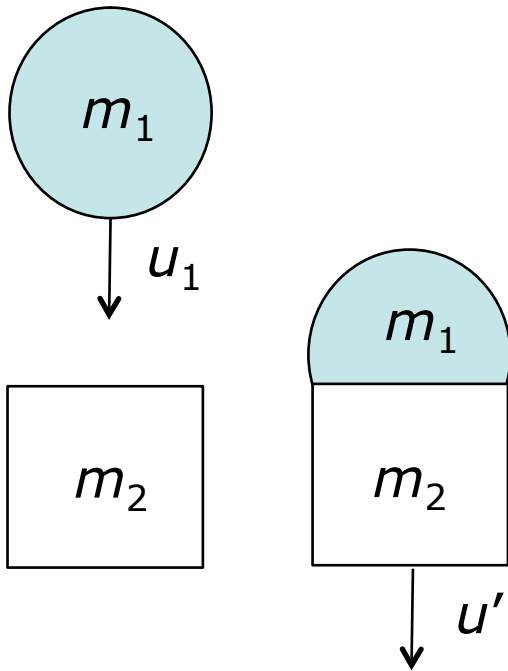


Activity 4: What is the final speed of raindrop-cum-mosquito after impact?

Consider conservation of linear momentum.

In particular consider the momentum before and after impact.

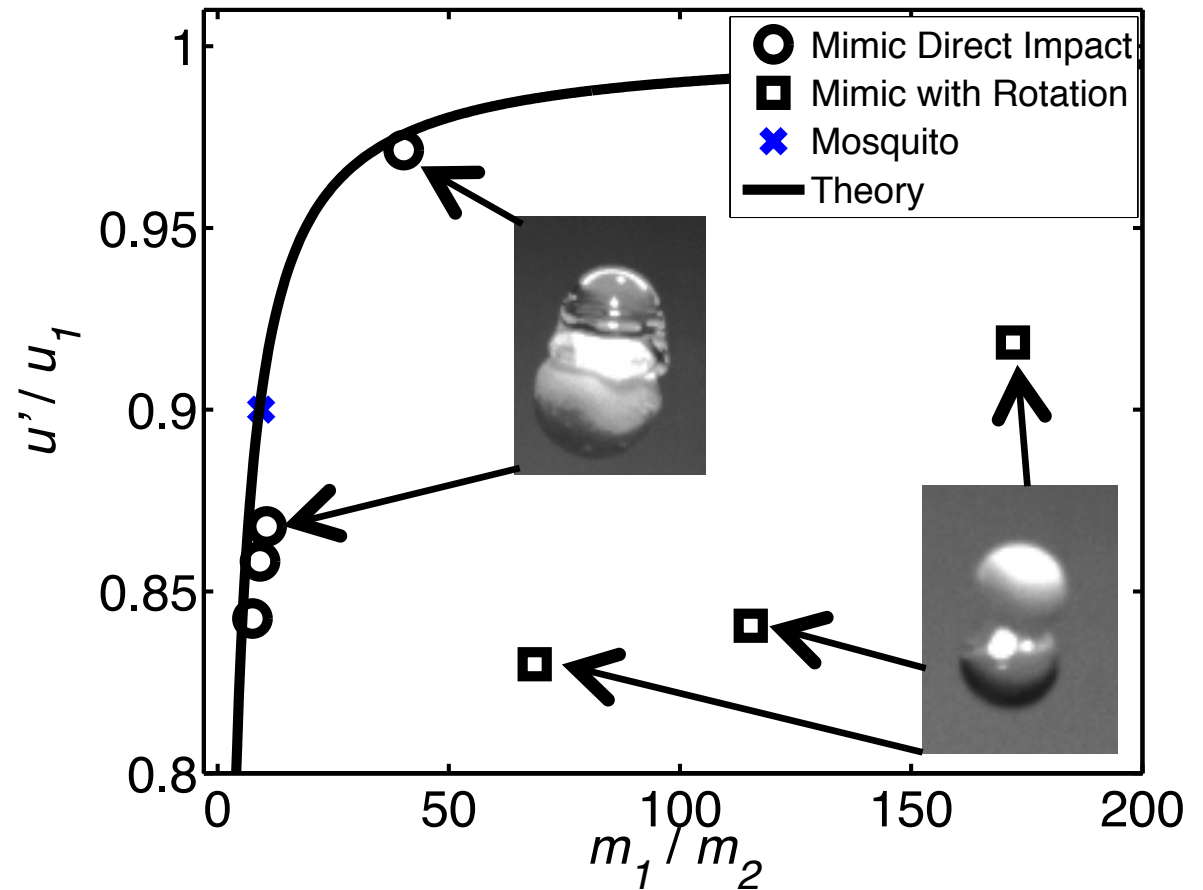
Inelastic Impact



Inelastic Impact

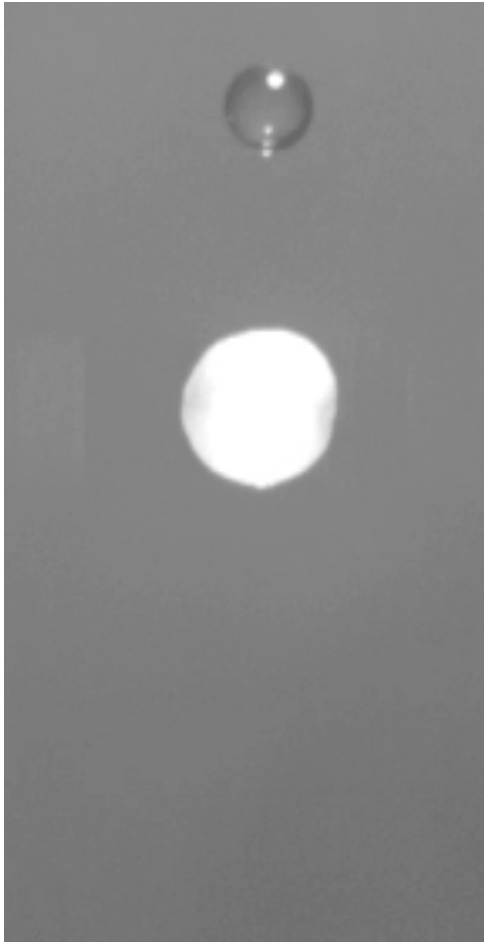
$$\frac{u'}{u_1} = \frac{m_1}{m_1 + m_2}$$

$$\frac{m_1}{m_2} = 1 - 50$$



Mosquitoes are so lightweight, raindrops lose very little momentum (2-17%) upon impact.

Small drops slow down more



Drizzle

Small Drop ($m_1 / m_2 = 9$)

→ large deformation



Rain

Large Drop ($m_1 / m_2 = 100$)

→ slight deformation

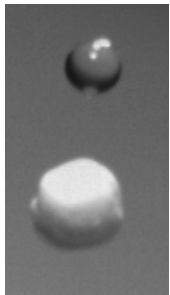
Collisions with body of high inertia



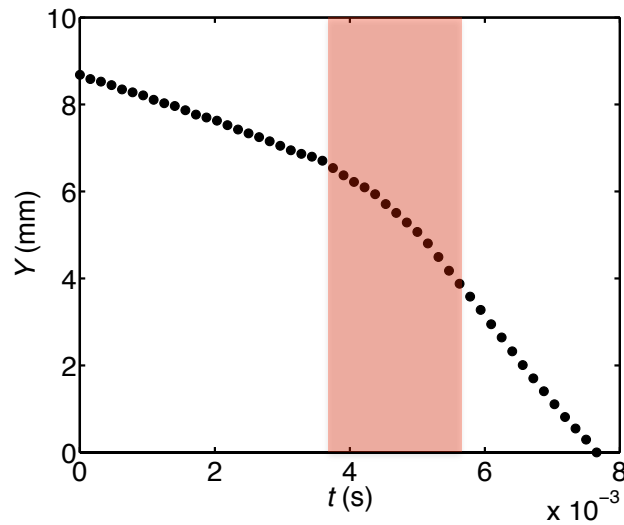
Terminal velocity raindrop on a solid surface: $F \sim m_1 u_1 / \tau \approx 5 \times 10^4$ dynes
which is 10^4 times the weight of a mosquito.

Larger drops apply more force

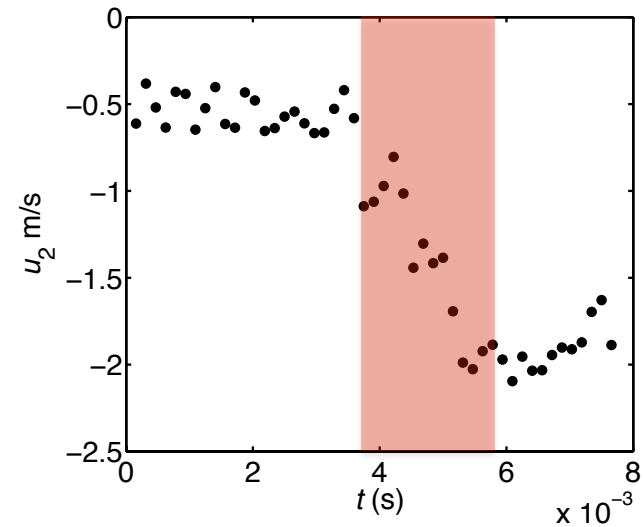
Small drop



Position

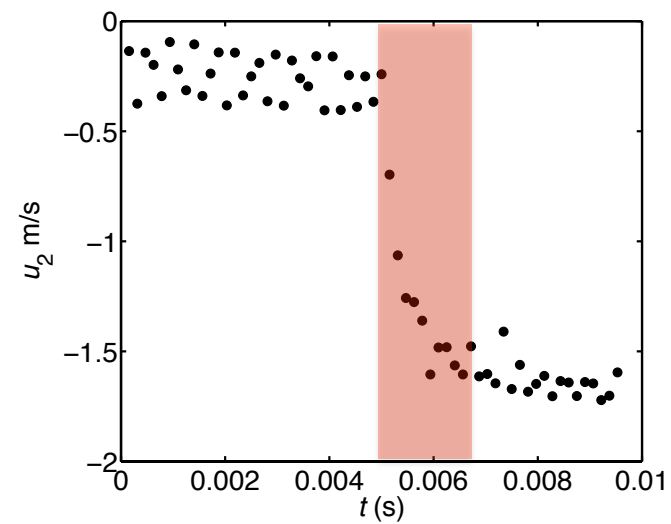
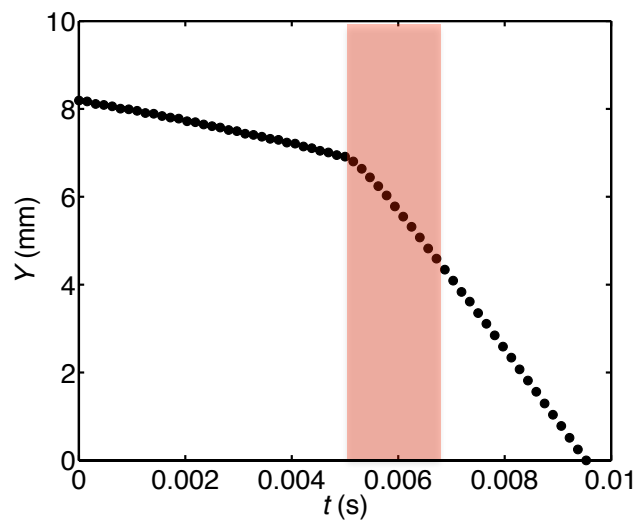
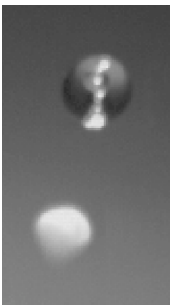


Velocity



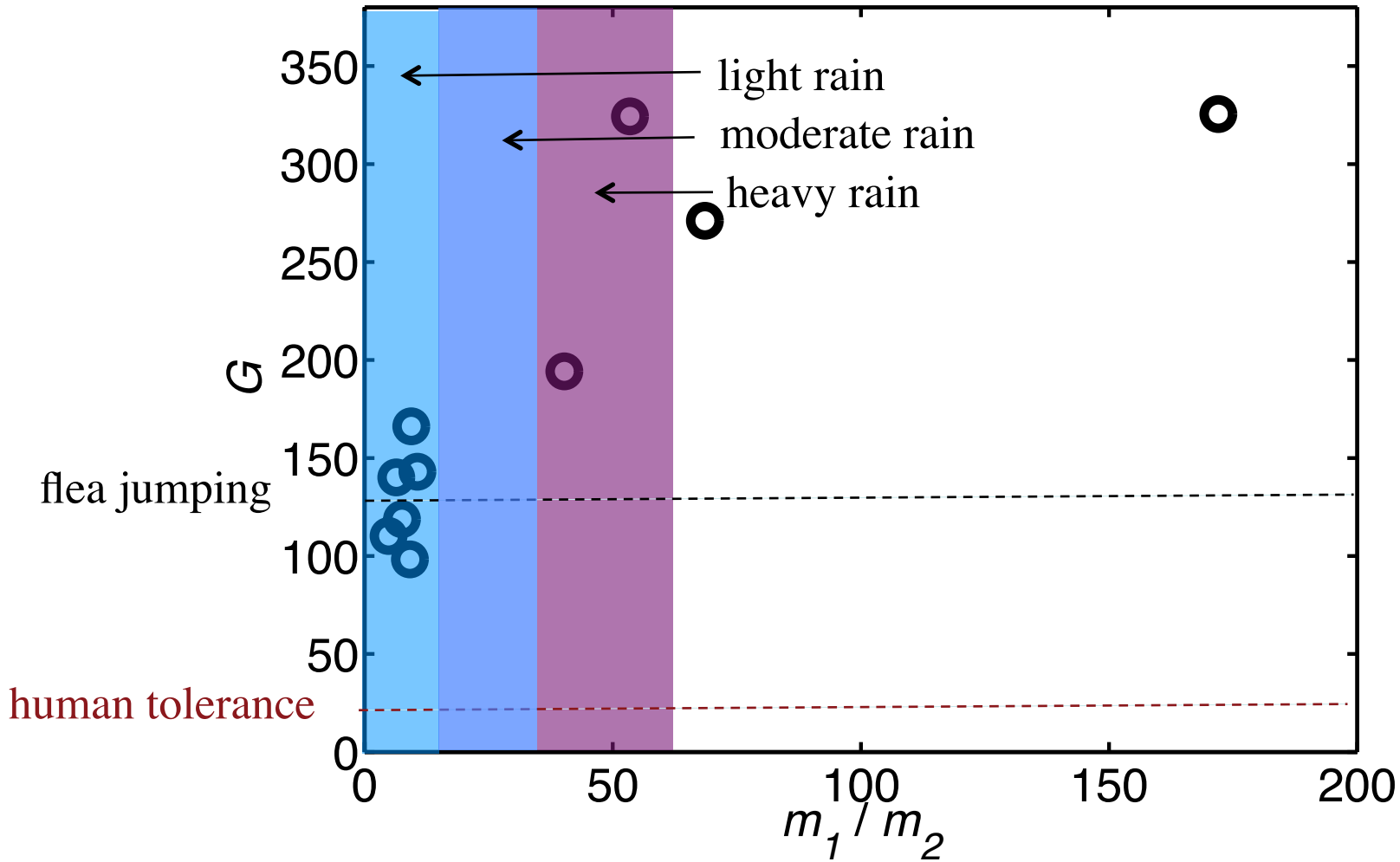
#g's = 65

Large drop



#g's = 95

Mosquitoes Accelerate 50-300 G



$$F = 2 \text{ dynes} \times 300 \text{ G} = 600 \text{ dynes} \approx 0.61 \text{ gf}$$

Max force: 4000 dynes (flight still possible) – 10,000 dynes (death)

Dimensionless Acceleration

$$F = u' m_2 \quad \tau = 0.5 - 1.8 \text{ ms}$$

