Averages: Still Flawed

The Fishing Competition

Supplement for the break after the fifth segment

In the fifth segment of the ***Averages: Still Flawed*** module, we discuss the fishing competition between Zan and Dan. During the break, we suggest that you ask the students whether they think this could even be possible. At first, we hope they think this is crazy, that you should just take the average of the averages. What we'd like to demonstrate is that the numbers behind the averages are critical and may tell a different story if not interpreted correctly.

For instance, in this case, Zan is less skilled at fishing than Dan at each location individually. But, for both Zan and Dan, one of the locations (the lake) yields significantly more fish per visit than the other (the river). Zan is smart, so the lake is where she focuses her efforts. Dan spends too much time fishing at the inferior location (the river), and that's why Zan catches more fish overall than Dan!

To see ultimately what happened using the numbers, you may have to guide them to work this out via a structure like the one below.

	Dan	Zan
Catching fish in a lake	__ / __ = 89%	__ / __ = 83%
Catching fish in a river	__ / __ = 43%	__ / __ = 33%
Catching fish in both a lake and a river:	__ / __ = 57%	__ / __ = 73%

We would suggest guiding them by asking them "So, what do we know?", which includes not only the percentage success rates shown in the segment but also the 30 total trips for Dan and Zan. With that information, the total number of successful trips can be calculated as 17 and 22 for Dan and Zan, respectively.

	Dan	Zan
Catching fish in a lake	__ / __ = 89%	__ / __ = 83%
Catching fish in a river	__ / __ = 43%	__ / __ = 33%
Catching fish in both a lake and a river:	17 / 30 (57%)	22 / 30 (73%)

From here, trial and error could prove successful, but if not, guide them in particular to the 89% and the 33% for Dan catching fish in a lake and Zan catching fish in a river, respectively. These percentages should be familiar enough that they will see some options.

For instance, for Dan catching fish in a lake, hopefully they see that the only fractions that could lead to 89% are either 8/9, 16/18 or 24/27. In order for the percentage of Dan catching fish in a river to work, this forces the answer to be 8/9.

For Zan catching fish in a river, the percentage of 33% has options ranging from 1/3 to 10/30. The option of 10/30 should clearly be out, which will hopefully lead them to try 1/3 and then 2/6, leading them to the right answer and the table below:

	Dan	Zan
Catching fish in a lake	8 / 9 (89%)	20 / 24 (83%)
Catching fish in a river	9 / 21 (43%)	2 / 6 (33%)
Catching fish in both a lake and a river:	17 / 30 (57%)	22 / 30 (73%)

So, who won? There is certainly an argument that Dan is better at fishing at each location, but Zan won this particular competition. If Dan had spent more time fishing at the lake, perhaps he would have won.

For more information and other examples on this topic, we recommend searching online for “Simpson’s Paradox”, which discusses similar examples in more detail.