- Édouard Lucas
- (1842-1891)

http://faculty.evansville.edu/ck6/bstud/lucas.html

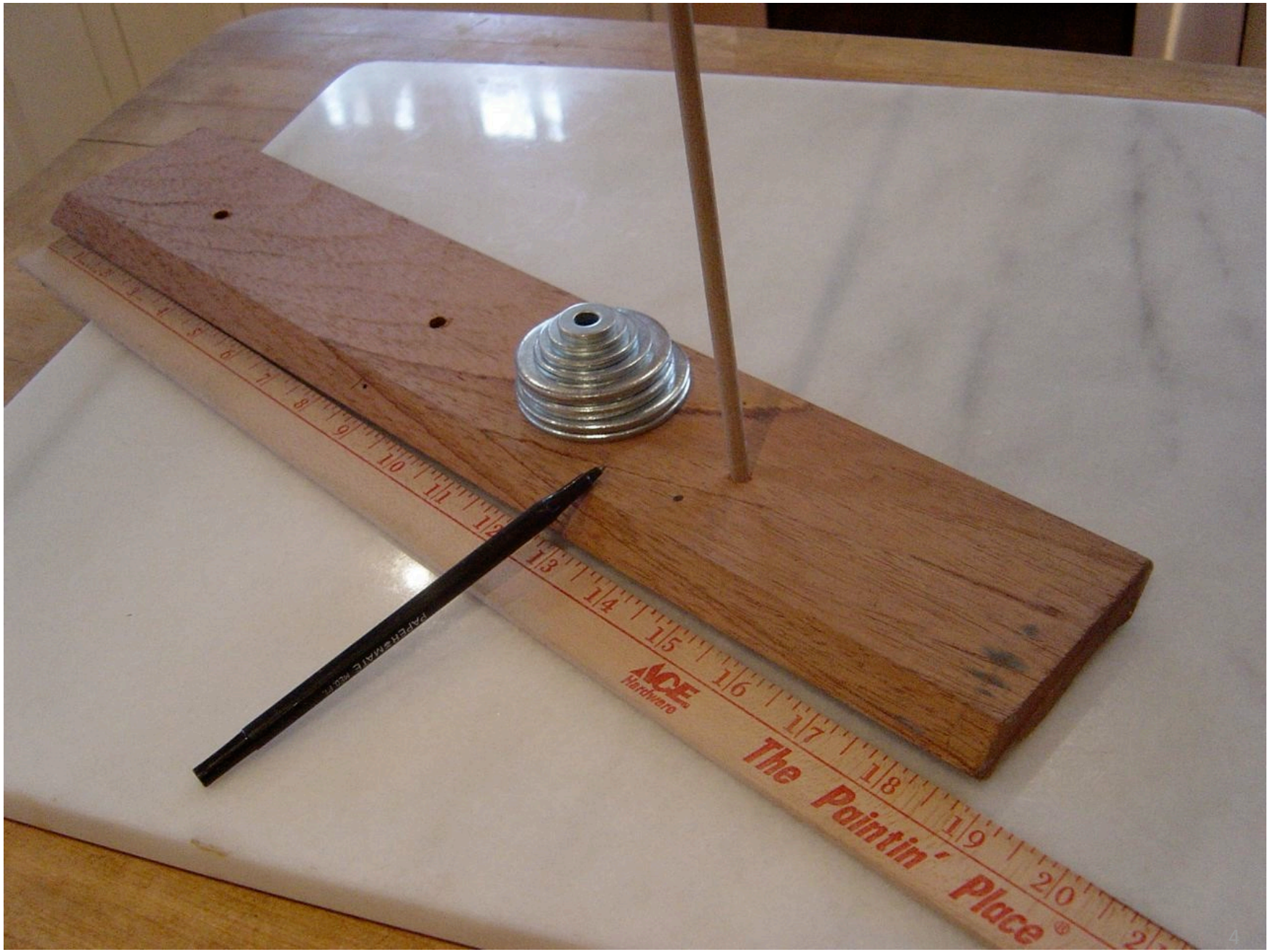
Rules for Moving Disks

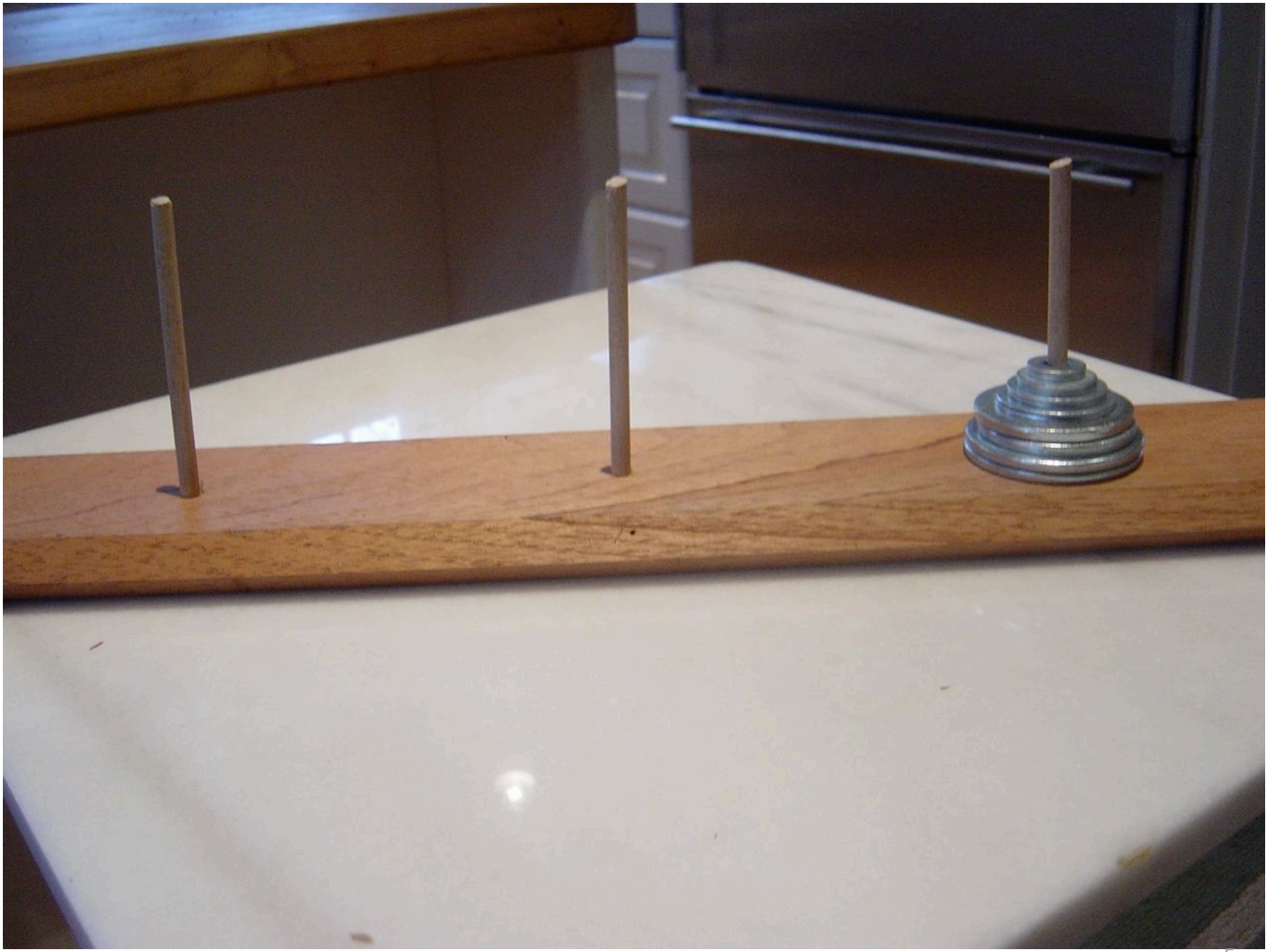
1) **We can only move one disk at a time.**

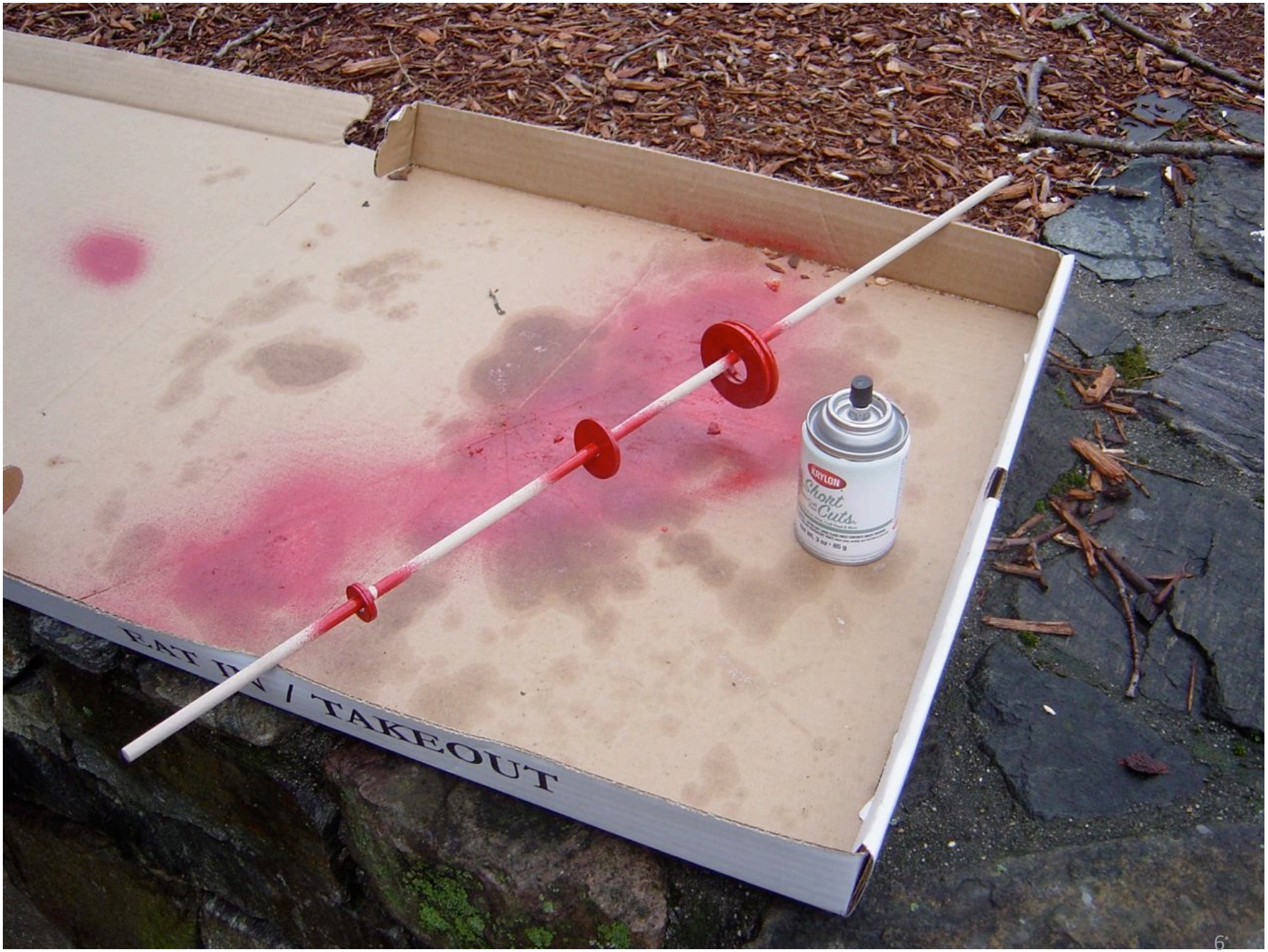
   We can move it from its current tower to either of the other two towers, assuming we obey Rule #2.

2) **Any stack of disks must always be in ascending order, with smallest on the top and largest on the bottom.**

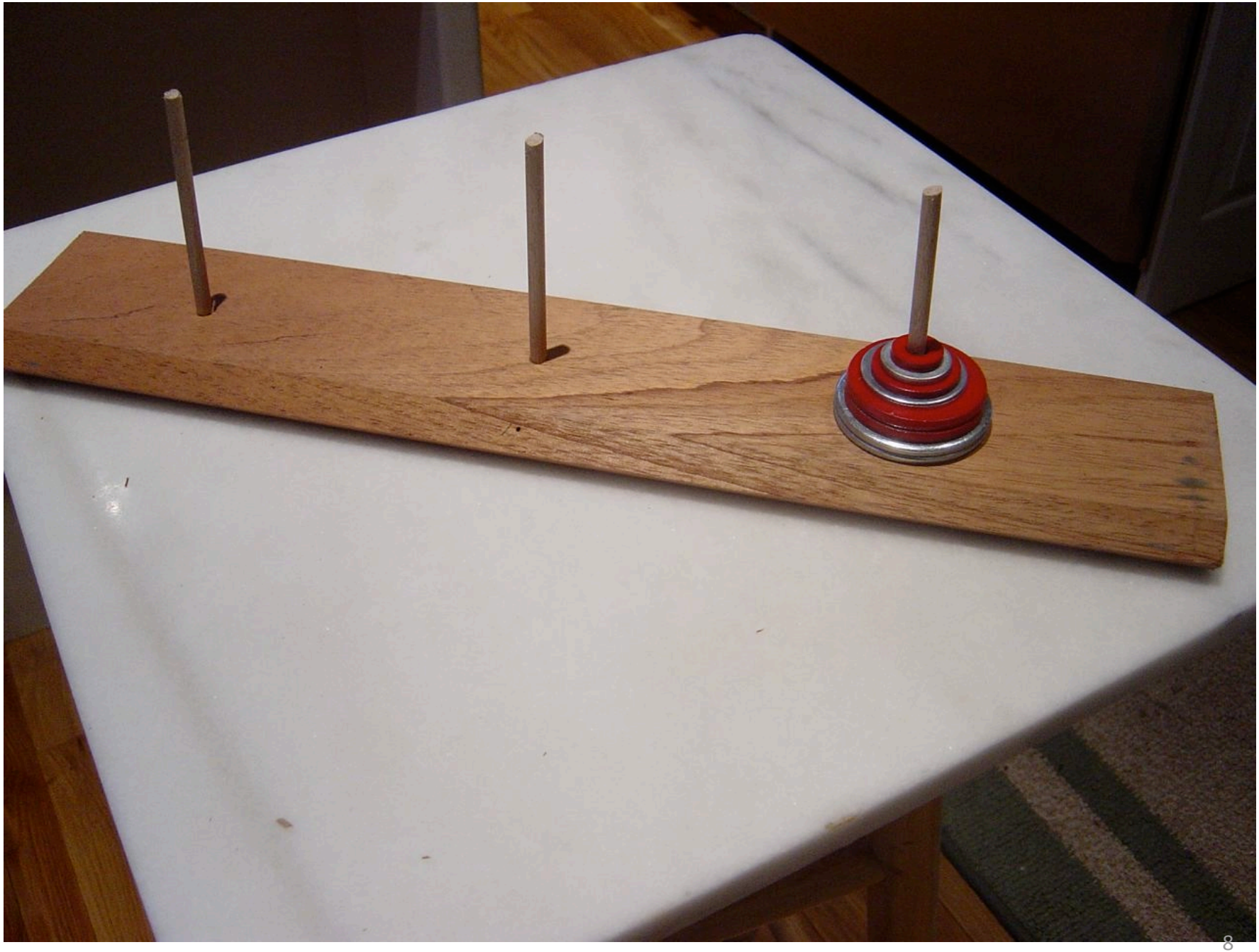
Making a Towers of Hanoi – at Home


ACE Hardware
The Paintin' Place




TAKEOUT
KRYLON
Short Cuts


N / TAKEOUT



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N	2^N	2^N-1
1	2	1
2	4	3
3	8	7
4	16	15
5	32	31
6	64	63
7	128	127
8	256	255

Sequence of Moves for $N = 3$

- 1. Small to Right.
- 2. Medium to Middle.
- 3. Small to Middle.
- 4. Large to Right.
- 5. Small to Left.
- 6. Medium to Right.
- 7. Small to Right

First Note from Dr. 4 to Dr. 3

- "Please do the *N* = 3 problem, placing the top 3 disks currently on Tower #1 onto Tower #2, the Middle one. Then pass everything back to me."

Second Note from Dr. 4 to Dr. 3

- "Now please do the *N* = 3 problem again, this time moving the 3 disks on Tower 2 (Middle) onto Tower 3 (Right). Then pass back to me."

Note from Dr. 5 to Dr. 4

- "Please do the *N* = 4 problem, placing the top 4 disks currently on Tower #1 onto Tower 2, the Middle one. Then pass everything back to me."

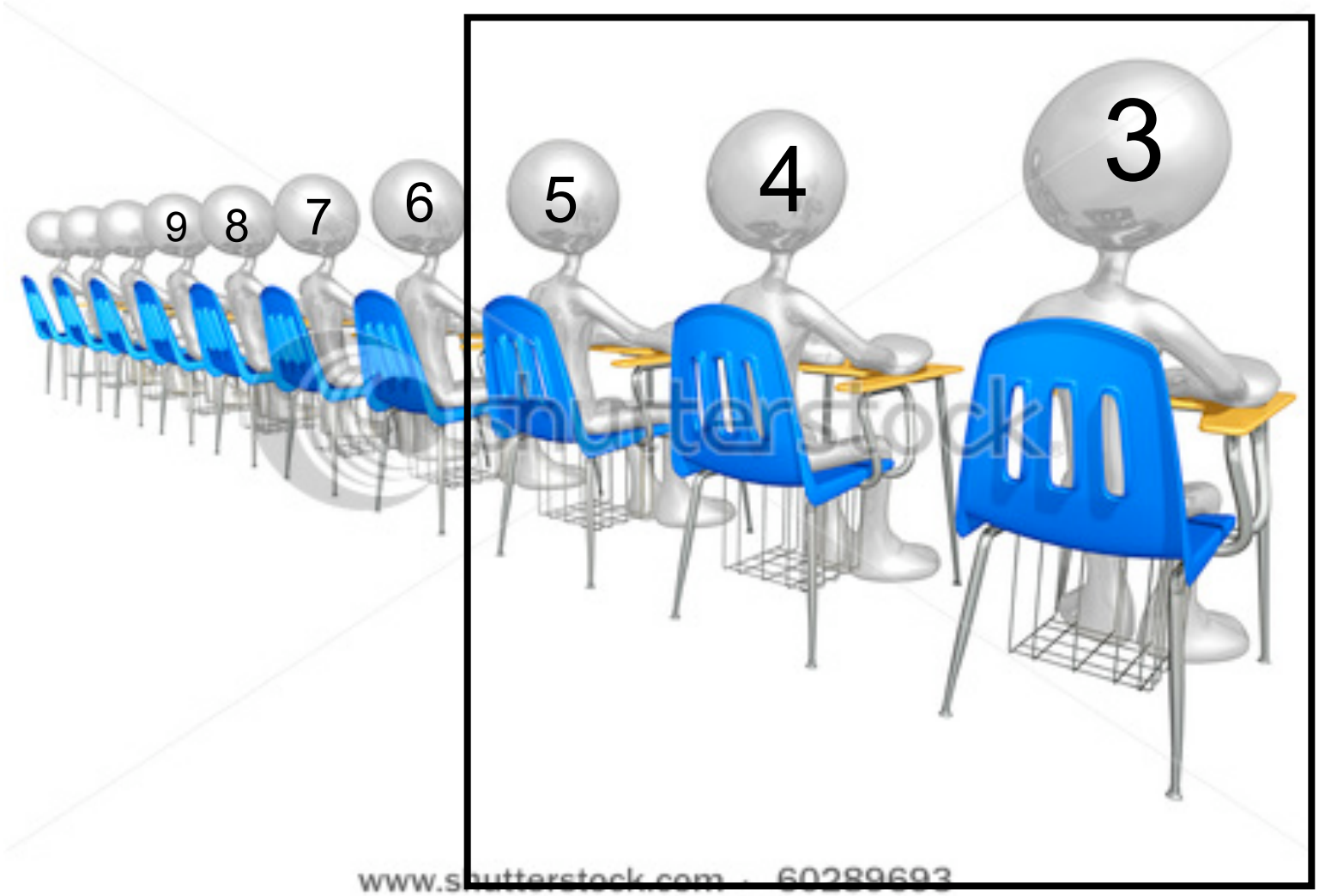
Second Note from Dr. 5 to Dr. 4

- "Please again do the *N* = 4 problem, placing the 4 disks currently on Tower #2 onto Tower #3, the Right one. Then pass everything back to me."

The Good Drs.

http://image.shutterstock.com/display_pic_with_logo/54820/54820,1283470473,5/stock-photo-students-in-a-classroom-sitting-at-desks-60289693.jpg

The Good Drs.



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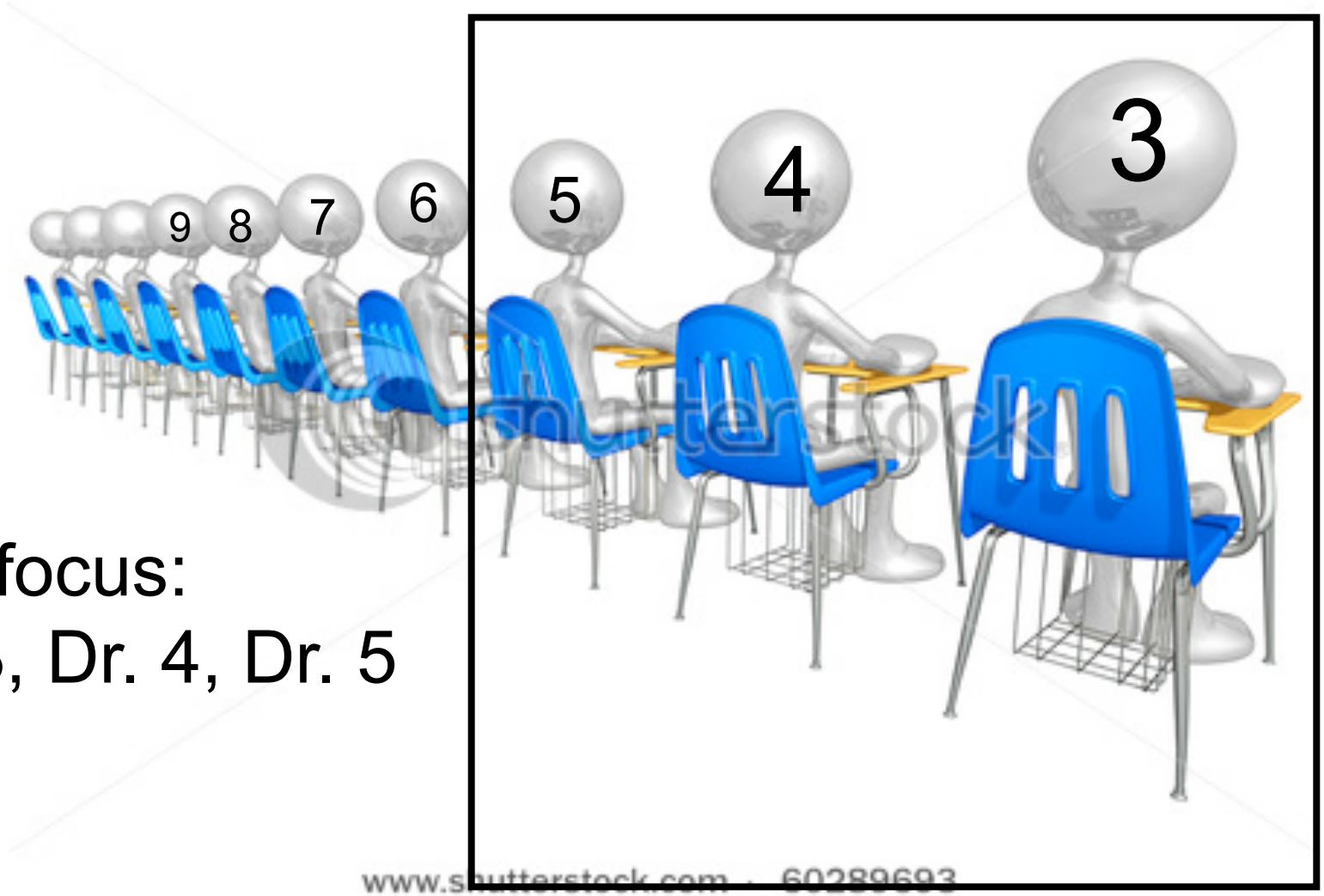

Our focus:
Dr. 3, Dr. 4, Dr. 5

http://image.shutterstock.com/display_pic_with_logo/54820/54820,1283470473,5/stock-photo-students-in-a-classroom-sitting-at-desks-60289693.jpg

Recursion is a method of defining functions in which the function being defined is applied within its own definition.

Factorial: An Example of Recursion

- By definition,
  - $N! = N(N-1)(N-2)\ldots(3)(2)(1)$
- Recursion with Factorial:
  - $N! = N(N-1)!$

- Sol(4,1,3) = Sol(3,1,2) plus Sol(1,1,3) plus Sol(3,2,3).
- Or,
- (Best set of moves to move 4 disks from Tower 1 to Tower 3) equals
- (Best set of moves to move 3 disks from Tower 1 to Tower 2) plus
- Movement of largest disk from Tower 1 to tower 3 plus
- (Best set of moves to move 3 disks from Tower 2 to Tower 3)

The 4-Disk Problem

- We have 4 disks:
  - Small
  - Medium
  - Large
  - Super Large
- Sol(4,1,3) = Sol(3,1,2) PLUS Sol(1,1,3) PLUS Sol(3,2,3)

Sol(3,1,2) = (Set of moves to move 3 disks from Tower 1 to Tower 2)

- 1. Small to Tower 3.
- 2. Medium to Tower 3.
- 3. Small to Tower 3.
- 4. Large to Tower 2.
- 5. Small to Tower 1.
- 6. Medium to Tower 2.
- 7. Small to Tower 2.

PLUS

Sol(1,1,3) = “Movement of ‘Super Large’ from Tower 1 to Tower 3”

PLUS

Sol(3,2,3) = (Set of moves to move 3 disks from Tower 2 to Tower 3)

- 1. Small to Tower 3.
- 2. Medium to Tower 1.
- 3. Small to Tower 1.
- 4. Large to Tower 3.
- 5. Small to Tower 2.
- 6. Medium to Tower 3.
- 7. Small to Tower 3.

The COUNT Operation

- COUNT{Sol(3,1,3)}=Number of moves required to move 3 disks from Tower 1 to Tower3.
- Sol(3,1,3)=
  - 1. Small to Right.
  - 2. Medium to Middle.
  - 3. Small to Middle.
  - 4. Large to Right.
  - 5. Small to Left.
  - 6. Medium to Right.
  - 7. Small to Right

COUNT{Sol(3,1,3)}=7.

Counting Moves

COUNT{Sol(4,1,3)}=

COUNT{Sol(3,1,2)}+

COUNT{Sol(1,1,3)}+

COUNT{Sol(3,2,3)}.

Counts: Using our Hypothesis

- COUNT{Sol(*N+1*,1,3)} = COUNT{Sol(*N*,1,2)}+ COUNT{Sol(1,1,3)}+ COUNT{Sol(*N*,2,3)}, or

- COUNT{Sol(*N+1*,1,3)} =

$$[2^N - 1] + 1 + [2^N - 1] =$$

$$2*2^N + 1 - 2 = 2^{N+1} - 1.$$

Proof by Induction

Final Challenge Problem

- N = 64 disks.
- Monks correctly moving one disk per second, every second.
- How long for them to complete this **Towers of Hanoi** problem?
- One day, one week, one month, one year, ten years, 100 years, or longer?

Examples of Trail Markers

http://www.dwelement.com/userfiles/image210_lg.jpg
http://blog2.bibleplaces.com/uploaded_images/c9a5b936cd51_9237/IsraelTrailmarkeratMachteshRamontb110702007.jpg