Flaws of Averages

BLOSSOMS Module

Supplement to Flaw of Averages Closing Segment:

"Song and Dance"

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As you will find in the video, the closing segment summarizes the three Flaws of Averages that we present along with one last illustration of the Flaws of Averages for your and your students' enjoyment: Rhonda tap dancing and Dan singing.

If a student questions you about how we calculated the average number of taps that Rhonda tapped and the average note that Dan sang, here is how we did it:

Rhonda's Average Number of Taps

For calculating Rhonda's average number of taps, Rhonda counted the number of taps that she tapped in our first 'song and dance' and found that she made 61 taps. We then timed how long the first 'song and dance' lasted, which was 12 seconds.

Thus: $\dfrac{61 \text{ taps}}{12 \text{ seconds}} = 5.08\bar{3}\dfrac{\text{taps}}{\text{second}} = 305\dfrac{\text{taps}}{\text{minute}}$

We then used a metronome to find how fast 305 taps per minute would be, which is the pace at which Rhonda is tapping on the video.

Dan's Average Note That He Sang

When calculating the average note that Dan sang, we ran into Flaw of Averages #1 and #3 along the way.

Our first step, though, was converting the notes into a numerical sequence.

Dan's opening note was a B, which we've set to have a value of 1, and the highest note he sang (in the short segment) was a B an octave higher, which we've set to a value of 13. The rest of the notes and their corresponding are filled in to the table below.

Note	B	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
Value	1	2	3	4	5	6	7	8	9	10	11	12	13

Overall, Dan sang 22 notes, with the following distribution:

Note	B	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
Value	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of Notes Sung	5	0	0	5	1	2	1	6	0	0	1	0	1

With the value of each note set and the frequency with which he sang each note, we calculated the average note that Dan sang as follows (leaving out the notes that he didn't sing):

$$\frac{1*5 \ + \ 4*5 \ + \ 5*1 \ + \ 6*2 \ + \ 7*1 \ + \ 8*6 \ + \ 11*1 \ + \ 13*1}{22} \ = \ \frac{121}{22} \ = \ 5\frac{1}{2}$$

Here is where we come to Flaw of Averages #1: "The average is not always a good description of the actual situation". For the notes that Dan sang, we defined 13 different notes and equated them to the integers of 1 through 13. However, the average value of $5\frac{1}{2}$ is not an integer, so there is no note out of the 13 defined notes that corresponds with the average note that Dan sang. In fact, it falls exactly halfway between the notes of D# and E (values 5 and 6, respectively).

Rather than try to sing a pitch halfway between these notes, Dan chose to sing a D# (value 5) for the average note in the video.

Your students may next ask 'why not take the average note with respect to time?' This is indeed another way to calculate the average note that Dan sang and illustrates our Flaw of Averages #3: "The average depends on your perspective".

The tables below show the number of beats that Dan sang each syllable and the note that was sung for that syllable as well, which will lead to a different calculation of the average note that Dan sang:

Syllable	Hear	the	beat	of	Rhon-	da's	feet
Note	B	B	B	D	B	D	F#
Value	1	1	1	4	1	4	8
Number of Beats	1	0.5	3	1.5	1	1	7

Syllable	to	the	song	we	love	the	me-	lo-	dy	of
Note	F#	F#	B	A	F#	E	D	D#	E	F
Value	8	8	13	11	8	6	4	5	6	7
Number of Beats	0.5	0.5	1	1	1	1	0.5	0.5	0.5	2.5

Syllable	For-	ty	Se-	cond	Street
Note	F#	F#	D	D	B
Value	8	8	4	4	1
Number of Beats	1	1	0.5	1	2.5

There is a lot of computation that must be done to calculate the average from this perspective, and to do that accurately we used a spreadsheet to do the following calculation:

$$\text{Average Note Dan Sang} = \frac{\displaystyle\sum_{\text{all notes sung}}\left((\text{Note Value}) * (\text{Number of Beats})\right)}{30 \text{ Total beats}}$$

$$\text{Average Note Dan Sang} = \frac{1*1 + 1*0.5 + 1*3 + 4*1.5 + \ldots + 4*0.5 + 4*1 + 2.5*1}{30}$$

$$\text{Average Note Dan Sang} = \frac{167}{30} = 5\frac{17}{30}$$

With an average of $5\frac{17}{30}$, we again fall in between D# and E, although in this case we are slightly closer to the note E (value 6).

Using our table structure to illustrate the outcomes:

The Average…	From the Perspective of…	Yields a Value of…
Note that Dan sang	The number of notes he sang	$5\frac{1}{2}$
Note that Dan sang	The length of time (beats) he sang each note	$5\frac{17}{30}$

Since the calculations yield almost the same result, this may not be as clear of an illustration of this Flaw of Averages as Rhonda's Dance Class Example or the Child's Travel Time Example (see the supplement to Flaw of Averages #3: "The average depends on your perspective").