Video on Blossoms/Quadratics

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Notes on the questions suggested before video pauses

After part 1: The parabola $y = -x^2 + 4x + 9$ opens downward because of $-x^2$. Since $y = 9$ at $x = 0$, the parabola must cross the x axis twice – before and after $x = 0$. The formula for the roots produces $2 \pm \sqrt{13}$, giving practice with square roots.

After part 2: The parabola $y = x^2 - x$ factors into $x(x-1)$. Then $y = 0$ when $x = 0$ and also when $x = 1$ (two real roots).

The parabolas $y = x^2 - x + 1$ and $y = x^2 - x + 2$ have no real roots. You can see that $x^2 - x + 1 = 0$ is impossible since *all real x are smaller than* $x^2 + 1$. (Why? Because if $|x| > 1$ then x^2 is bigger, and if $|x| < 1$ then 1 is bigger.) The quadratic formula gives the complex roots of parabolas:

$$y = \frac{1 \pm \sqrt{-3}}{2} \text{ and } y = \frac{1 \pm \sqrt{-7}}{2}.$$

After part 3: The area with sides x and $50 - x$ is $y = -x^2 + 50x$ with slope $-2x + 50$. Then the area is maximum where the slope is zero, at the top of the parabola. This top point has $x = 25$ and the rectangle is actually a square.

After part 4: A square is the best of all rectangles but not the best of all shapes. *That honor goes to a circle.* The circumference going around the circle is $2\pi r$ and the area is πr^2:

$$2\pi r = 100 \text{ meters and } \pi r^2 = \pi \left(\frac{100}{2\pi}\right)^2 = \frac{2500}{\pi} \approx 795\,.$$

This circular area of 795 easily defeats the rectagular area of $(25)^2 = 625$.

Question for superstars about two parabolas $y = ax^2 + bx + c$ and $Y = Ax^2 + Bx + C$. What condition on the six numbers a, b, c, A, B, C means that the parabolas never touch?