Growing Exponentials: A Teacher's Guide

Exponential growth is keenly applicable to a variety of different fields ranging from cell growth in biology, nuclear chain reactions in physics to computational complexity in computer science. In this lecture, through various examples and activities, we have tried to compare exponential growth to polynomial growth and develop an insight about how quick the number can grow or decay in exponentials. Dear teachers, you have a major role in making this lecture a success with your students, especially through motivating the activities in between the different video segments. We have included few suggestions on how to conduct the activities, but you may have better ideas on how to present the material!

We begin with the classical Indian tale about the chessboard to motivate the power of exponentials. At the end of the segment, we ask the students whether the king would be able to give the smart mathematician his reward. We suggest that the students draw a 3x3 or a 4x4 matrix to simulate a chessboard, and then write the number of rice that should be placed on each square of the matrix. Then, they could start summing up the first two numbers, then the first three numbers, etc. This should help the students catch the pattern and hopefully come up with the answer $2^{\text{square number}}-1$. The second segment introduces the problem of tearing and stacking paper. We suggest you split the students into small groups and try the experiment through tearing a sheet of paper. Once they have an intuition of how the height of the stack of paper grows, you may try to help them relate this problem to that of the Indian story to come up with the answer. Tables for the powers of 2 are supplied to the students to make sure they do not spend too much time with numerical calculations.

In the third and fourth segments, we introduce the notion of functions and graphs to emphasize the growth of exponentials visually. Our assumption is that students are familiar with graphs and can roughly estimate the point of intersection of two functions. Tables for powers of 2 are again provided to avoid the hassle of calculations. The linear payment on the n^{th} day is just 10,000n. If the students have a graphical calculator, it would be instructive to supplement this activity with an exercise of plotting graphs and finding their intersections on graphical calculators. The fourth

segment involves comparing polynomial functions with the exponential function. If your students are not familiar with the term “polynomial”, it is a good idea to introduce this notion to them prior to this segment to make it easier for them to follow. The fourth segment asks the students if they could find a polynomial function that would beat the exponential. You could carry out this activity by splitting the class in two halves and have each half challenge the other to find such a function and then alternate. One they get the intuition that exponentials would eventually become bigger than any other polynomial, help them see that actually, if we were to limit the game to 30 days only, an appropriately high degree polynomial would actually perform better than the exponential.

At the beginning of the fifth segment, we introduce convergence of sequences using the Tootsie roll example. We compare the exponential function to the polynomial functions through the ratio test. This could be an unfamiliar topic to your students. We suggest you prepare them for the notions of limits and the convergence through taking the ratio of the two functions. We have included some calculus links to websites that we believe would be helpful to present this material. In the same activity we introduce Moore’s law which involves taking power of fractions. We suggest reviewing this notion at the beginning of the fifth activity to make it easier for students to perform such calculations.

What we have presented in the teacher’s guide are only suggestions, and you definitely have more knowledge and experience on ways to make this lesson more accessible and interactive for your students. We hope that you enjoy this video lesson as much as we have!