Teacher's Guide

Segment	Contents
Exercise 1	**After Defining rational and irrational numbers, the students are asked to solve the and exercise.** The first exercise of **the lesson** is in the form of 6 cases which should take at most 5 minutes. The purpose of the exercise is 1- To learn how to extract the integral and fractional parts from any number. 2- And consequently distinguish visually the difference between rational and irrational numbers by looking at the fractional part of each case. Note that the first 3 cases of the exercise correspond to rational numbers whereas the last 3 correspond to irrational numbers. Remark: Given that calculators have limited (rounded) digit spaces, the instructor should help the students obtain corresponding infinite representations, particularly in the first 3 cases of rational numbers.
Main Question of Lesson	The instructor should meticulously state the **Main question of the lesson** and afterwards he/she is recommended to invite the students to think of a probable answer and write it on a piece of paper.
	Then the instructor must make sure that the student must understand the representation and notation of the fractional part of a number, since the rest of the lesson is totally dependent on these notions.
	The instructor is recommended to present the 2 parts of the Main theorem, **(the "if" part and the "only if" part)**, **while** stressing on the notion of logical equivalence "if and only if".
	Then the Plan of the Proof of the "only if" part is **presented**, **and** the instructor should stress that the proof includes 2 cases **or statements**. The first is that of a terminating fraction while the second is distinctly that of a non-terminating sequence with a repeating pattern.
	The Proof and example of statement 1 are straightforward.

Exercise 2	In the proof of statement 2 of the “only if ” part, the obtention of the identity resulting from the multiplication of f by 10^k should be stressed and illustrated in **Exercise 2.**
Exercise 3	The proof of the “if” part is first based on the Euclidean division theorem. The instructor should stress this concept in particular the notion that the remainder belongs to a finite set that includes zero. The procedure is self explanatory in the video and should enable the students to answer the question regarding the termination of the representation of the rational numbers. The students are given 3 minutes to **solve Exercise 3**.
	The answer to exercise 3 in addition to an example of a terminating sequence representation should **allow students** to complete the exercise that follows.
Exercise 4	Exercise 4 illustrates the proof **of the "if" part** and should be completed in couple of minutes .
	Then the instructor must insist on the fact that infinite representations of rational numbers result from the fact that all remainders in the successive multiplication by 10 and division by n algorithm are never zero and take values between 1 and (n-1).
	At this point, the instructor assists the students in the understanding of the Pigeon hole principle and the example of 10 pigeons and 9 holes.
Exercise 5	**Exercise 5** of the 3 pigeons and 2 holes is a very straightforward application of the Pigeon Hole **Principle**.
	The application of the Pigeon Hole Principle to the non-terminating sequences is probably the most “abstract” part of the lesson and consequently the hardest part. In case the mathematical level of the students is not sufficient for understanding the general proof, the instructor can omit that proof and limit the explanation to the 6/7 example.
Exercise 6	**Exercise 6 should be** easily done by the students and the example of 6/7 could be conducive to understand the general proof.
first elements of the answer to main question	**The introduction of the Main Question's answer** is intuitive and should allow the students to perceive that there are 2 infinities one for the rational numbers and another for the irrational numbers with the second one much bigger than the first infinity. The rigorous proofs of these facts **are based on the Countability of the rational numbers and the uncountability of irrational numbers.**

	To obtain countability of the rational numbers, **a sequence of subsets {R_n} is introduced**. Examples of sets {R_n} are given for n=1,2,3 and 4.
Exercise 7	**In Exercise 7,** the students should be able to define R_5 **which is** 1/6 and 5/6 since in 2/6, 3/6 and 4/6, the numerators and denominators have gcd>1.
	The student should be assisted in understanding that by defining R through the {r_n}'s, **he\she** obtains a one-to-one relation between R and the set of Natural numbers.
	The proof of uncountability of the set of irrational numbers, making this set a bigger one than the set of rationals, is based on **Cantor's "diagonal" argument** .
Answer to main question	Finally, the answer to the main question of the lesson is obtained through the fact that the **Probability to have a number f picked at random between 0 and 1**, (0<f<1) is the ratio of ∞(Rationals) over ∞(Irrationals), ∞_1/∞_2, with $\infty_1 << \infty_2$ and this is a very small number.