Blossoms Free Fall Teacher Guide

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"This is the year 2410 and we are at Mission Control on Earth about to receive a communication from an expedition launched into space five years ago. Listen carefully.

Voice fade-in "Planet P Explorer to Mission Control."
"We have dropped a pebble from a height of 36 feet.
It took 2.4 seconds to hit the ground.
Atmospheric conditions like your moon: no air or other gases present.
Please confirm. Over"

Mission Control: "Read you loud and clear. We will assign the task of analyzing that data to our class of master math students. It is common knowledge that a falling object on Earth speeds up at a rate of 32 feet per second for every second it is falling. With a little focus and effort, you can determine the gravitational constant on Planet P from the given information.

We will pause now for groups in your class to confer and will return after at least one group, using a chart and/or graphs, has presented an explanation of the calculations and answered questions to the class's satisfaction."

Teacher hits "pause" for groups to try their hand. Hints as needed:

1. The average falling speed is total distance dropped divided by the time it took to fall.
2. The speed at at which the pebble crashes to the ground can be derived from the equation Average speed = (initial speed + terminal speed)/2
3. The acceleration constant on Planet P G_p = Change in speed divided by the falling time.

Teacher hits "play".

Mission Control: "Our technicians prepared the following explanation and chart.

At the instant the pebble is dropped, time $t_0 = 0$ sec; distance fallen $d_0 = 0$ ft;
height off ground is starting height minus distance fallen $h_0 = (36–0)$ or 36 ft.
and s = 0 ft/sec (since it is dropped from rest)".
[Note: The average speed ($\overline{s}$) is left blank since average is not defined over '0 time'.]

Time elapsed (t)	Average speed ($\overline{s}$)	Falling speed (s)	Distance (d)	Height above ground (h)
Secs from drop	from drop to now	at that instant	fallen	at that instant
0	xxxx	0 ft/sec	0 ft	36 ft–0 ft = 36 ft

"Take a few minutes to make sure that this chart (so far) is equivalent to yours."

Teacher hits "pause" for class to do its thing.

Teacher hits "play".

Mission Control: "Next, our technicians filled in the line when t = 2.4 (hitting ground).
Try it yourself, then see if our technicians get the same results as you do."

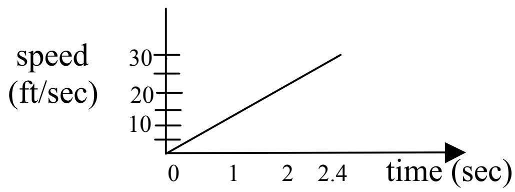
Teacher hits "pause" for groups to try their hand..

Teacher hits "play".

Mission Control: "Here is our reasoning and the 2.4 sec. line on our chart.

At the instant the pebble hits ground t = 2.4 seconds;
distance dropped, d = 36 ft;
height above ground, h = 0.
average speed for the first 2.4 seconds (36ft / 2.4 secs) $\bar{s}$ = 15ft./sec.

The object starts from rest (s = 0) and gains speed at a constant rate as it falls (due to gravitational attraction). The pebble's speed is graphed below.



For linear relations like falling speed, the average speed, $\bar{s}$ = (0 + s)/2
Since we know the average speed over the first 2.4 seconds is 15 ft/sec and v_0 = 0, we get 15 = (0 + s)/2. Solving for s, we get s = 30 ft/sec (the speed at 2.4 sec)

Time elapsed (t) Secs from drop	Average speed ($\bar{s}$) from time of drop	Falling speed (s) at that instant	Distance (d) fallen	Height above ground At that instant (h)
0 sec	xxxx	0 ft/sec	0 ft	36 – 0 = 36 ft
1				
2				
2.4 sec	36ft/2.4sec=15ft/sec	30 ft/sec	15(2.4)= 36ft	36 – 36 = 0 ft (hits the ground)

Take your time to make sure that this chart (so far) is equivalent to yours. Also try to calculate the acceleration by dividing the change in falling speed by the change in time."

Teacher hits "pause" for class to do its thing.

Teacher hits "play".

Mission Control: "Acceleration is the change in speed divided by the change in time.
The acceleration on Planet P: a = (30-0) / (2.4-0) = 12.5ft/sec <u>each second</u>
That can be written $a = \frac{12.5 \text{ ft / sec}}{1 \text{ sec}}$ or 12.5 ft/sec^2
Now that we have the gravitational acceleration, you can calculate the falling speed at any time by multiplying the time the pebble has been falling by this acceleration constant.
s = at. Take a few minutes to fill in the speed the pebble is falling at t = 1 and t =2.
Go ahead and fill in the rest of the table.
Teacher hits "pause" for class to do its thing.

Teacher hits "play".

Here is what our technicians came up with.

Time elapsed (t) Secs from drop	Average speed $(\overline{s})$ from drop to now	Falling speed (s) at that instant	Distance (d) fallen	Height (h) at that instant
0 sec		0 ft/sec	0 ft	36 – 0 = 36 ft
1	6.25 ft/sec	12.5 ft/sec	6.25(1)=6.25 ft	36 – 6.25 = 29.75 ft
2	12.5 ft/sec	25 ft/sec	12.5(2)=25 ft	36 – 25 = 11 ft
2.4 sec	36ft/2.4sec=15ft/sec	30 ft/sec 15 = (0+?)/2	15(2.4)= 36ft	36 – 36 = 0 (hits the ground)

If you experiment on Earth, you will discover that it takes 1 second for a pebble dropped from a height of 16 ft (4.9 m) to strike the ground. Going through the same series of calculations as above, you will derive that famous acceleration constant on Earth, $g = 32$ ft/sec^2 (9.8 m/sec^2)

If you imagine doing this experiment on a "very large" planet, the gravitational force will be greater than it is on Earth and much greater than we found on the "small" planet we used as a sample above. Choose your own height from which the pebble is to be dropped and make up the time it takes to fall to the ground. Follow the steps above to do the appropriate calculations. Good luck. Over and out."

Notes to the teacher:

Extensions -- For a class that has had some Algebra, it would be a challenge to fill in a variable row in the table by putting "t" as an entry in the "time" column and filling the rest of the row with functions of t. The thinking process is as follows.

> At any time, t, the speed is $s = at$ (on Planet P, 12.5 t), (on Earth, 32 t ft/sec or 9.8t m/sec)
> The average falling speed $\overline{s} = (0 + s)/2 = ½\ s = ½\ at$
> The distance fallen $d = \overline{s}\, t = ½\ s\ t = ½\ at^2$ (on Earth, 16 t^2 ft or 4.9 t^2 m)
> The height t seconds after being dropped is $h =$ Initial height $– ½\ at^2$

The formulas for a dropped pebble for any acceleration constant "a", are:

- instantaneous speed of the pebble at time t: $s = at$
- average speed of the pebble from the time it was dropped $\overline{s} = ½\ s = ½\ at$
- distance the pebble has fallen during the first t seconds is $d = \overline{s}t = ½\ at^2$ and
- height above the ground at time t (dropped from an initial height of h_0) $h = h_0 – ½\ at^2$.

For a class that has had some data collection experience, the experimental falling time should be the average of several stop watch trials.