Teacher's Guide: Using Geometry to Design Simple Machines

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Thank you for considering using this video module for your class. This is a teacher's guide for use with the video. There is also a trailer at the end of the video meant to help you. I suggest you both read the teacher's guide and also watch the video trailer. There are some things covered in more depth in the teacher's guide, such as the gepmetry topics. Some other things are shown more effectively in the video, such as the hardware we suggest you use in the classroom.

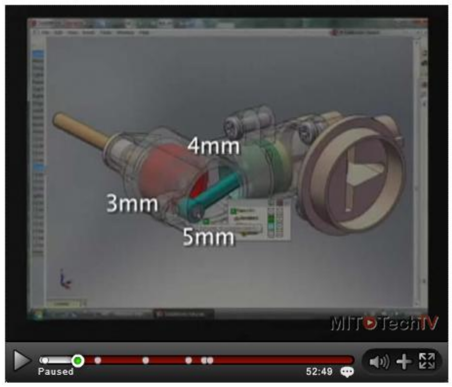
The topic of this video is design of machines and it can be seen as an application of geometry including triangles and circles. This module does not require any formal mathematics prerequisites other than exposure to concepts in geometry like angles in triangles and radii of circles. In-depth solution of some of the challenges requires geometry content such as Pythagoras' theorem and some concepts from trigonomerty. These concepts are used in ways that may be unfamiliar to students because we are interested in how geometries change over time, what engineers call kinematics. But kinematics is discussed in a simple way and is not prerequisite to learning from this video.

The topics in the video are easier to grasp if you can get some direct experience with them. Because of this, I strongly suggest you bring some meterials to class to allow your students to build shapes that can move. One good choice is "peg board" that is available in hardware stores. I cut segments of pegboard by sawing down the center between a row of holes and then I cut these strips into various lengths. Using these strips, you can model mechanisms. To connect two links of a mechanism, join two strips with a pin -- the holes in the peg board can be used to join pieces together by sticking a nail or dowel into the hole in two pieces of peg board. This will hold the two pieces together but allow rotary motion. If peg board is not available to you, cardboard or heavy paper and thumb tacks could be used instead. Please see the trailer to the video for more detail on this hardware.

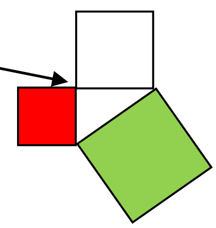
As the teacher of the class, you should feel free to stop the video wherever you like to ask your students questions, to clarify points from the video, or to add your own examples. However, there are several points in the video that were specifically designed as stopping points and where challenges are posed to students. Tthe rest of this teacher's guide is structured as notes to expand on those pauses in the video.

Pause #1

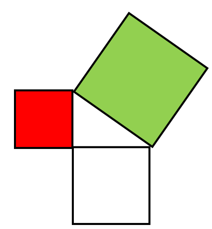
At this point in the video, a toy engine was discussed and a computer model was used to show how it moves. A challenge posed to the students. Say the distance from the center of the crankshaft to the end of the connecting rod is kown to be 3 units (such as millimeters) and the length of the connecting rod is known to be 5 units. What if the toy engine were placed in a specific configuration shown below so that the end of the connecting rod is 4 units from the center of the crankshaft. Could you tell me, given this information, exactly how the connecting rod is positioned with respect to the other parts of the engine?



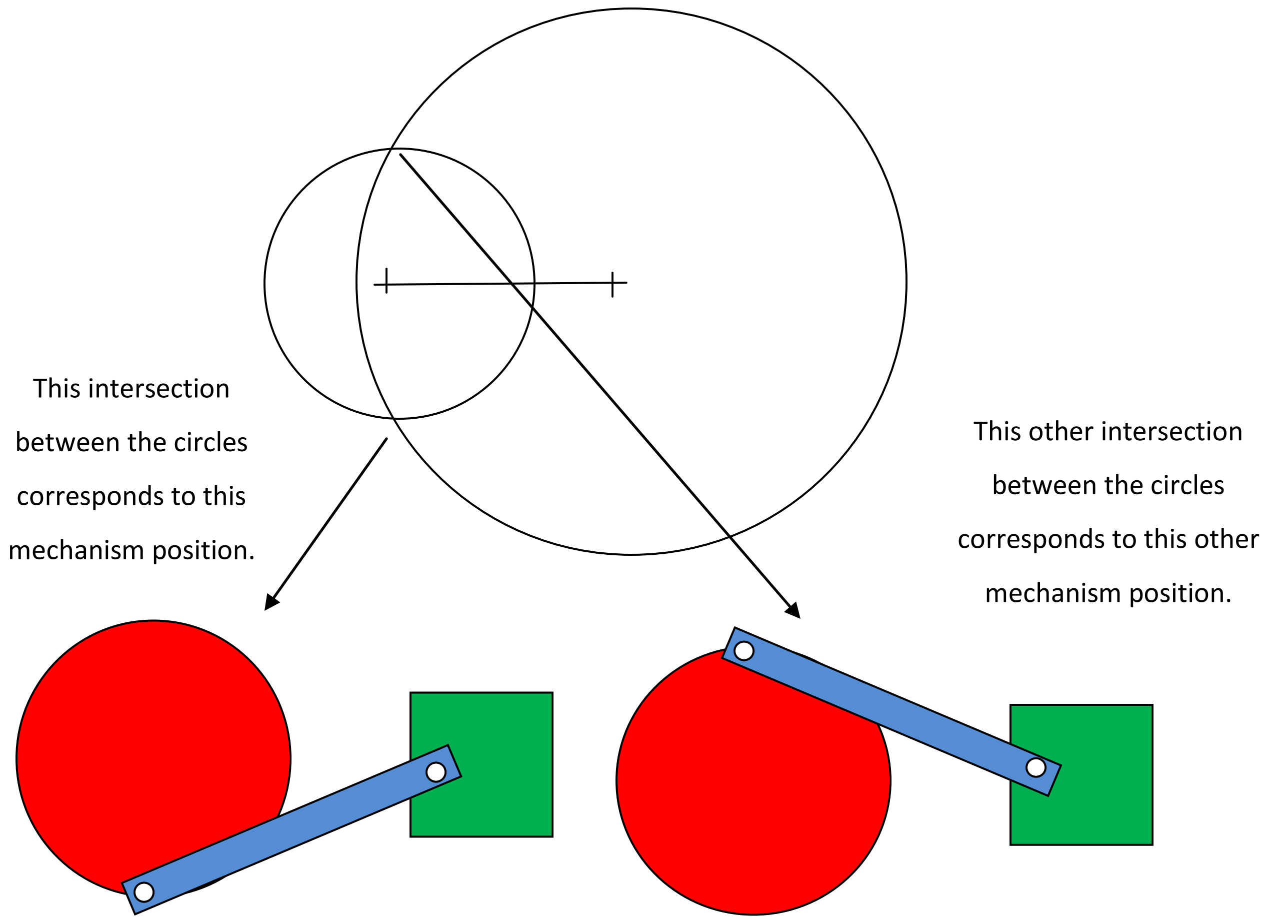
Three lines within this engine define a triangle. The lines have lengths 3, units, 4 units, and 5 units respectively. We know from Pythagoras' theorem that a triangle with sides of these lengths is a right triangle since $3^2+4^2=5^2$. A good figure to draw at this stage is shown below. The area of the red and white squares summed together is equal to the area of the green square if the red and white squares form a 90 degree angle where they connect.



Given this fact, we can answer the question, "Could you tell me… exactly how the connecting rod is positioned with respect to the other parts of the engine?" The answer is "Yes, mostly." In this case, there are two possibilities for the location of the crankshaft. The crankshaft could be ponting straignt down, but given the same link lengths given in the problem statement, the crankshaft could also be pointg straight up instead.



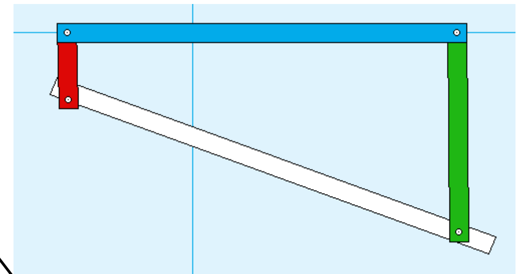
So, for the particular lengths we gave in the problem statement, we can figure out that the crankshaft is at exactly 90 degrees. But it is also more generally true that no matter what lengths we give, as long as they are consisent with the constraints needed to form a trinagle (see the "Broken Stick" video for more detail) we have completely determined the position of the mechanism by defining its three link distances. You can explain this another way and reinforce some lessons from geometry even if you don't want to cover Pythagoras' theorem. First, we know that the crank pin (the joint on the left end of the connecting rod) has to lie on a circle of known radius centered on the middle of the crank shaft (in red). You might draw that circle with a compass. Now, we also know the location of the piston and so we know the location of the pin on the right end of the connecting rod. The connecting rod keeps its two ends the same disatnce apart at all times, so the left end must be on a circle centered at the right end with a radius of the length of the rod. You might draw that circel with a compass too. You could show that the two circles you've drawn intersect at <u>two</u> points. Those are both legitimate solutions to the set of geometric constraints.

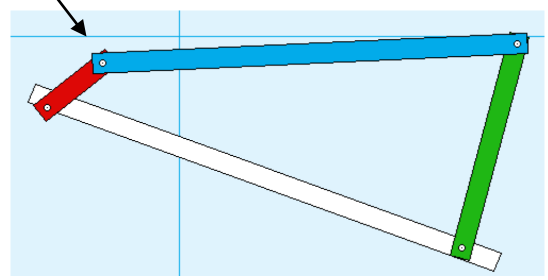


Pause #2

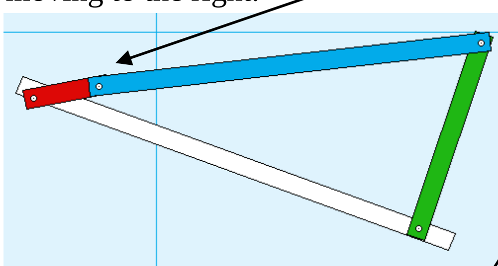
The questions posed were:
What will happen if I manipulate this short, red bar on the left and make it go around and around? What will happen to the end of the long, green bar on the right? How is that similar to or different from the mechanism in the engine that we saw just a few minutes ago?

We started with the mechanism looking like this.
If the red bar is turned clockwise, the top of the red bar travels to the right. The part of the blue bar that is connected to the red bar also moves to the right. This, in turn, pushes the other end of the blue bar (where it's connected to the green bar) to the right (and every other element of the blue bat too).
The key thing is that if the blue bar is solid and stiff, the distance from each point on the bar to any other point on the same bar must remain constant during motion. This is often called "rigid body motion".



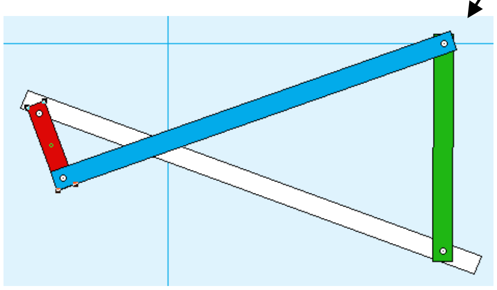


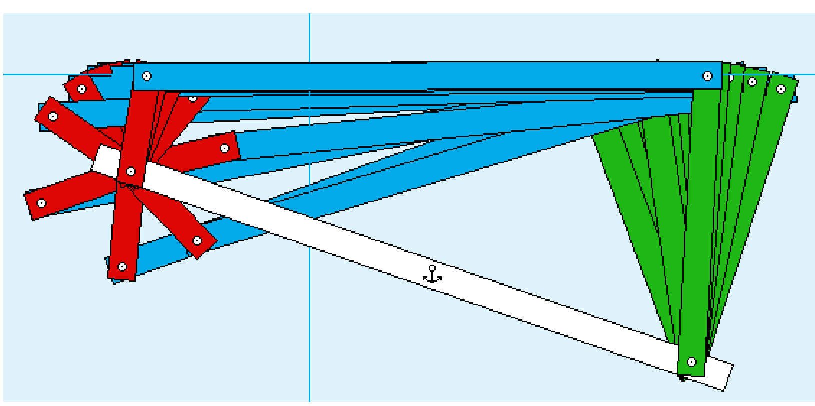
But soon after, the red and blue links become aligned. At this point, which we call a "toggle point", the green bar stops moving to the right.

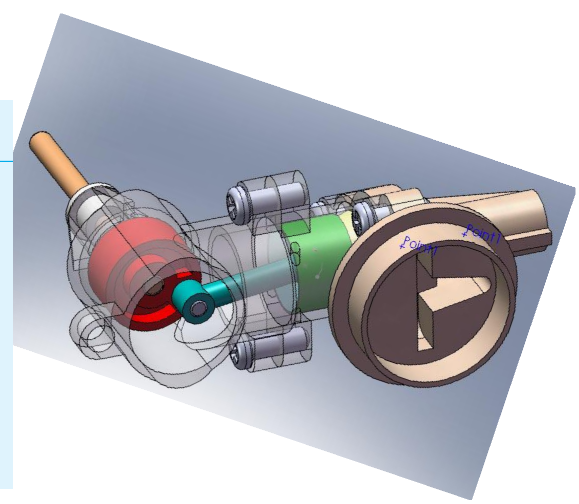


That's because the motion caused by the red bar is at right angles to the blue bar. At that instant, the blue bar is rotating about its joint at the end where it attaches to the green bar.

Next the green bar begins to move to the left. As the red bar goes around and around, the green bar goes back and forth. Since the angle through which the green bar turns is only about 25 degrees, the arc of motion is fairly straight. This invites a comparison to the engine. The piston on the engine moves back and forth in exactly a straight line. The top end of the green bar approximates the same motion.





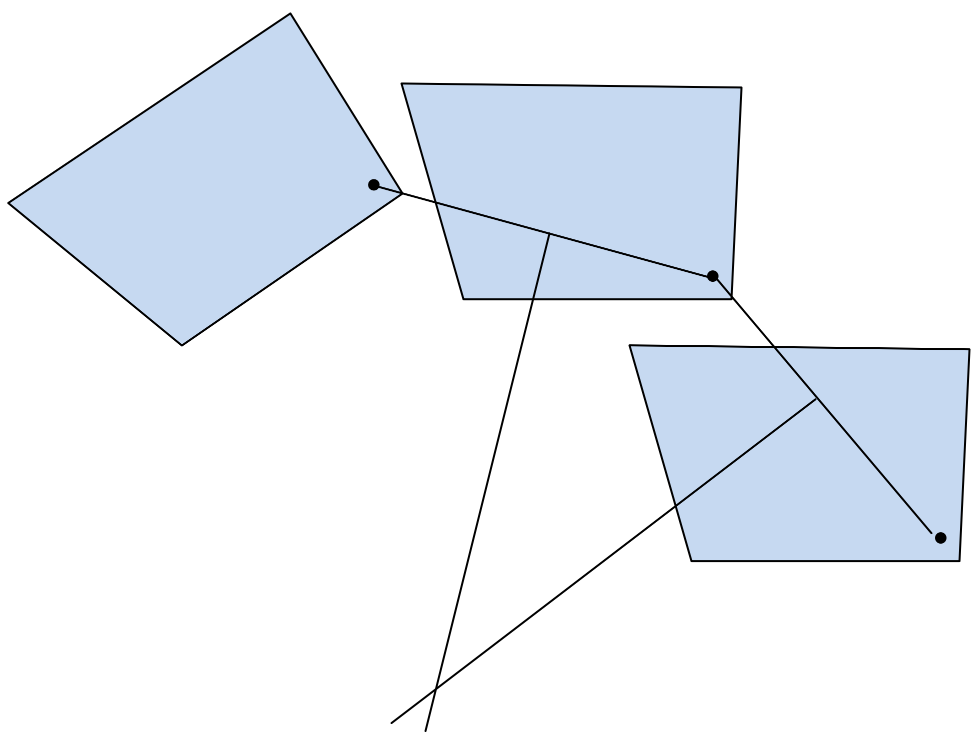


Pause #3

The question posed was: “How can we define the desired geometry of the four bar mechanism that will guide the bed of the dump truck?”

One approach is sketched out here. We have three positions of a container. Focus on just the connecting point on the lower right. As the container moves, so does the connecting point. We have three points that define the desired motion of just that one spot on the container. Three points should uniquely define a circle.

There are many ways to find the center of a circle give three points, but here’s one quick and easy way. Draw a line bwteen two points. Find the middle. Draw a line at a right angle. Do the same for the next two points. The place where those two lines intersect is the middle of your circle and therefore the location of a pixed bearing point on the bed of your dump truck.



Now, you can do the same thing for the other side of the mechanism. On the left side we have a point we chose to connect the bar to the container. Move the container through its three desired positions and we get three desired points. Draw lines, bisect the segments, form perpendicular lines, and find their intersection. Now you have the fourth bar in your mechanism.

Pause #4

The question posed was -- What if there is a barrier over which the payload must be lifted. The design we just prototyped seems to overlap the barrier and might therefore impact it during operation. What might we do as designers to overcome this problem?

One good answer is to change the shape of the links without changing the locations of the attachment points. You see, as long as you keep the attachment points in the same place, the object's path remains unchanged – the shape of the material conecting the two points does not affect the motion. In this case, to avoid the collision, you could shape the link like a "V" and bend it around the obstacle.

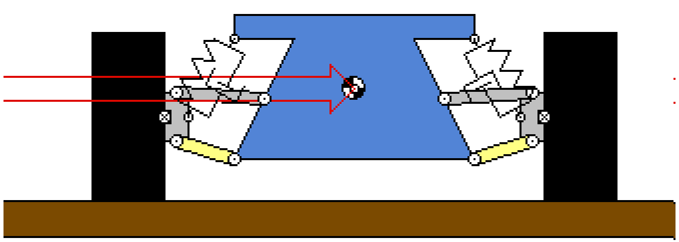
Another interesting option is to change the location of the left side attachment point. We chose the point on the payload to put the pin somewhat arbitrarily. If we reconsider this choice, we need to repeat the process of finding the three locations of the attachment point and the center of a circle containg those three points. The center of that circle will be the location of our new link position. This new geometry might avoid the collision with the barrier.

Pause #5

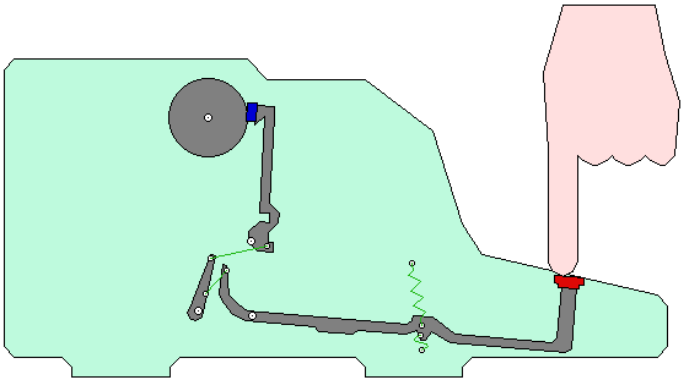
This final pause is a broader summarizing point. All of this aterial is meant to be of practical use in real life. So, the challenge posed to the stduents is: "Using the skills introduced in this video, what machine could you make that would be useful in your community?"

There are surely to right and wrong answers to this challenge, but here are some examples that might help students if they get stuck:

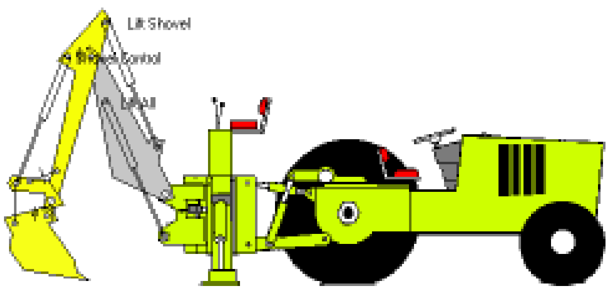
A car's suspension and steering have mechanisms that guide the motion of the wheel as you go over bumps and make turns.



In a typewriter (not so common anymore), a mechanism guides a stamp with a letter to strike the page.



In construction equipment a mechanism guides the motion of a scoop such as the one on this back hoe.



In a sewing machine, a mechanism guides the needle and thread in such a way as to make stitches that are tied together and don't readily unravel (e.g., see http://home.howstuffworks.com/sewing-machine1.htm)