Teacher's Guide: Broken Stick Experiment

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Thank you for considering the Broken Stick Experiment for your class. I hope you and your students enjoy the experience. This short teacher's guide is written as a supplement to the video Teacher's Guide included at the end of the video segments. So, if you have not yet viewed that, it would be best to view it now, and then return here! Thanks!

This module does not require any formal mathematics prerequisites other than first year simple algebra. It is not usual textbook material and requires the students to think through the exercise from "basic principles." I have performed the Broken Stick Experiment live three times with high school '10^{th} graders' in Lexington, Massachusetts, all students taking a course in Geometry. In these classes, I simulated the breaks between video segments by asking the students to come up, out of their seats, to take the chalk from my hand, and to perform the next step in the solution to the problem on the blackboard. This seemed to work well, except for the day that the saw slipped when I was cutting the yardstick, and blood spurted from a finger! Luckily, a female student carried a first aid kit, she administered first aid to me, and the class continued on schedule! So, if you use a yardstick or meter stick and intend to cut it with a saw, PLEASE BE CAREFUL! I have also performed the Broken Stick Experiment about eight times in MIT classes, both undergraduate and graduate classes in applied probability. But I should emphasize that for high school use of this BLOSSOMS Learning Video, no formal exposure to probability is needed. All that is required to solve the problem is covered in the video. Those who want to delve deeper into the problem, and who might want to tackle some of the stretch problems we have posted on our web site, would need to study applied probability more formally. These advanced problems are even suitable for college level classes, as we have used them at MIT.

Regarding the need for a yardstick or meter stick, we have received good feedback from high school teachers who have used uncooked dry spaghetti instead of a wooden stick. Spaghetti is inexpensive, readily available in most places, and allows each student to have her/his 'own stick' to break! If each student breaks her/his own spaghetti stick, then you need to concern yourself with how they find random numbers, and how they scale them to the length of the spaghetti stick and mark them on the spaghetti stick. Else, since the human brain is not good at generating random numbers, one might find lots of nearly equilateral triangles made with spaghetti!

You as teacher in charge of the class can stop the video at any time. There is no need to wait until the pre-planned end of any given video segment. Such stoppage may be required sometimes to keep the students attuned to what is happening in the video and to understand the developing logic. For example, we have received feedback from high school teachers using this BLOSSOMS module that the students sometimes feel that I move too quickly after identifying the respective lengths of the three stick (or spaghetti) pieces and saying that none can exceed 1/2 yard in length, else a triangle could not be formed. Quickly after I argue that the 3 stick lengths are, respectively,

x_2, $x_1 - x_2$, and $1 - x_1$ (assuming here that $x_2 < x_1$), I write down the 3 required inequalities,

$$x_2 < 1/2$$
$$x_1 - x_2 < 1/2 \text{ and}$$
$$1 - x_1 < 1/2,$$

and very quickly show on the blackboard how these inequality constraints can be plotted onto our square of possible experimental outcomes. Some teachers have found it useful here to stop the video to make sure that all the students understand what is happening. It may help to plot the 3 inequalities as equality equations first and then to argue that acceptable points must be on some specific side of each drawn line. What we are doing here is finding the intersection of 3 half-planes, each half-plane corresponding to one of the inequality constraints, and the intersection of the three half-planes comprising the set of points that simultaneously satisfy all three constraints (meaning that a triangle can be formed).

We welcome your feedback. Please feel free to contact me at rclarson@mit.edu. If you and your students develop new twists and turns to this BLOSSOMS video module, please send them this way – together with your solutions! - and we will post them to this web site. Have fun and, please, cook the spaghetti before you eat it! ☺

http://www.avantisavoia.com/image/store/0/productImages/33-spaghetti.jpg