

BLOSSOMS VIDEO LESSON TRANSCRIPT

Fantastic Factorials Transcript

Assalamualaikum and greetings
I'm Shawalni
I'm a mathematics teacher at one
of the High School
in Kuala Lumpur, Malaysia
Throughout this video
you will be able to learn
how we can organize various objects
in many different ways
and different in a situation
by using the mathematical concept of factorial.

Noah, I want to seat here
No, you can't
Adam, Can I seat here?
No, you can't
Stop fighting
When we left
you will exchange seat on our journey later
Ok dad

As you can see
to get the spot they like in the car
especially by the window
No one wanted to sit in the middle
I need help to determine the
order that will satisfy them all
Can you help me?
Now, you will be working in groups
Each group will be given cards
labeled Adam, Bakar and Noah
Try to arrange the card
to find the number of possible
arrangements of seating

The answer to our first activity is 6
Did you get it right?
Let me explain to you using supported video
acted by Mrs. Bashirah's children in the car
In this video, A represents Adam
B represents Bakar
and N represents Noah
The number of arrangement is 6

Now let's continue to the next scene
Mom.. I cannot open the bag
I have a secret code set for
the combination log of the bag
The clue for the code is that the code uses only 4 digits
of even numbers 2, 4, 6, and 8 and without repetition

Could you please help the kids
to figure out the code
for the combination lock?
Please work in your group
to solve the code
for the combination lock
Each group will be given cards
labeled 2, 4, 6 and 8
Try to arrange the card
to find the number of possible
arrangements to unlock the luggage bag
I will see you in a while

Did you get the answer
to the combination lock?
Let's watch this video
for the answer

Mom, I got it
Well done son
what is the code?
6 4 2 8

As you can see
Bakar managed to figure out the answer
during his 15th attempt
May I use this camera, mom?
Sure Adam but be careful
let's go out and take pictures
Ok Dad

Can you give us a suggestion
on all the possible seating arrangements
for the photo session?
Please work in your group
to help Mrs. Bashirah
in finding the possible seating
arrangements for the photo session
Each group will be given card labeled
Mom, Dad, Adam, Bakar and Noah
Try to arrange the card to find
the number of possible arrangements

to capture the picture in different positions
This require "mom" and "dad"
not to stay at only one position, at the center
they have to also switch places with their children

The answer to the numbers
of possible seating
arrangements for the
photo session is
120

What will happen if we have to arrange
5, 6, 7
or more objects?
Can we still use the same method
as suggested earlier to find
the possible number of arrangements?
it can be done
but there will be too many answers
Can we list down all the possible arrangements?

Do you know that we have another method
to find the solution to the problem
We can use a
Multiplication Rule

We have 1 place to filled in
if we have an object
the place can be filled in 1 way only
this is one factorial

If one thing can be done in m ways
and a second thing can be done in n ways,
then the total number of ways in which the two
things can be done together
or in succession is $m \times n$
This principle can be extended to the
case where more than two things have to be done

We started with two objects at two places
The first place can be arranged either
object 1
or object 2
2 ways
if an object we've put in the first place
2nd place can only be filled with one way
here multiplication rule can be used
two events occurring in sequence
then the number of ways is

if one thing can be done in m ways
a second thing in n ways
a third thing in p ways
then the total number of ways
in which these thing can be done together
or in succession is $m \times n \times p \times \dots$

to compile 3 objects in three places
The first place can be arranged either
object 1 or 2 or 3
namely 3 ways
if we take first object to the first place
second place can only be filled in 2 ways
either by 2nd object or 3rd object
Next the third place
can only be filled by one way only
multiplication rules can be used here
three events occur in sequence
then the number of ways is
 $3 \times 2 \times 1 = 6$

How about to organize
4 objects in 4 places
and 5 objects in 5 places?
Try to solve the problem
Together with your teacher.
We will meet shortly

Thus we can conclude that
If 2 objects at 2 places
 2×1 equal to $2!$
If 3 objects to the 3rd places
 $3 \times 2 \times 1$ equal to $3!$
If 4 objects to the 4 places
 $4 \times 3 \times 2 \times 1$ equal to $4!$
If 5 objects to the 5 places
the number of ways is
 $5 \times 4 \times 3 \times 2 \times 1$ equal to $5!$

Thus if n objects to n places
the number of ways is
 $n \times (n-1) \times (n-2) \times \dots$
 $\times 4 \times 3 \times 2 \times 1$
are known as
 n factorial

Therefore
 if we want to arrange n different objects
 we can use this formula
 $n! = n (n-1) (n-2) (n-3) \times \dots \times 4 \times 3 \times 2 \times 1$
 Let us do an experiment
 teacher will have to invite students
 one by one to the front of the class
 and appoint them each with role of
 Adam, Bakar, Noah and Nancy
 You will also have to prepare a blanket
 or large sheet of paper
 for the next activity
 Have fun

Let us discuss
 About the activity that you just did
 Adam
 Represents one way
 Adam
 plus Bakar
 represent 2 ways
 When Adam
 Plus Bakar
 Equal to 2
 When multiply by Noah
 the answers is 2 ways
 Similarly, if the position is changed
 Noah plus Bakar and multiply by Adam
 the answers is still 2 ways
 Again, if we change the position
 Noah
 plus Adam
 and multiply by Bakar
 you will get the same answer of 2 ways
 this can be formulated to

this can be formulated to
 $6 + 6 + 6 + 6 = 24 = 4 \times 3! = 4!$

Now, try to imagine
 If we had to add another student
 This time we use the formula
 Proof by Induction

Congratulations, students
 With your blanket exercise
 what you were really doing is something called
 Proof by Induction
 With 3 students behind the blanket

and one outside,
didn't have to rearrange
the students behind the blanket
You knew from your previous work
that the number of ways
to arrange those 3 students
was $6 = 3!$
And you had one outside the blanket
One by one
you exchanged the visible student
with one who was invisible
behind the blanket
You found that there were
4 ways to put an individual student
outside the blanket
and for each there were $3!$ Ways
to arrange those behind the blanket
In that way you discovered that there are
 $4 \cdot 3! = 4!$
distinct ways to arrange 4 students
So what have you done
in your class activity?
with your blanket exercise
You've discovered Proof by Induction
Let's talk about Proof by Induction

Mathematical "proof by induction" sounds complicated but it really isn't!

We deal only with positive integers, 1, 2, 3, and so on

You start with a simple base case and show that your result is true, as we have shown in this BLOSSOMS video.

Then you say, "Suppose my result is true for some value of N , say $N=3$, then I can show it is true for the next N , in this case $N = 4$."

This method, applied first from a small value of N , then to the next value of N , and so on and so on, then proves that your result is true in general.

It's pretty powerful, pretty neat, and usually reserved for college students. But let's do it here! It's fun with Fantastic Factorials!

Proof by induction says,
Suppose for some value of N
the number of distinct ways
to arrange N different objects is $N!$
We have already shown this to be true
for small values of N
in particular $N=1, 2, 3$ and 4

Now consider an
 arbitrary value of N
 something like $N=99$, for instance
 Suppose we have shown
 it to be true for that value of N
 This means that there are
 $N! = 99!$
 way to arrange 99 different objects
 Then we want to show that
 it is also true for $N+1$
 for instance for the number 100
 How to formulate this?
 Place the N objects (students)
 behind the very large sheet
 and we know that the number of
 distinct ways to arrange them is $N!$
 That is using what we call the
 inductive hypothesis
 And we have the $N+1$ st object
 call him Rahim
 standing outside the sheet to the left
 But now we place Rahim behind the sheet
 and we take Noah out
 standing to the left
 Do this for for all $N+1$ students
 one by one
 For each student outside the sheet
 there are $N!$ ways
 to arrange the N different
 students behind the sheet
 But there are $N+1$ times we do this
 that is each of the total of $N+1$ students
 get his or her turn outside the sheet
 So, just as we showed before
 there are $N+1$ times that we do this
 and for each there are $N!$ ways
 to arrange students behind the sheet
 If we sum
 $N!$ a total of $N+1$ times
 we get
 $(N+1)N! = (N+1)!$
 This is proof by induction
 You did this standing in front
 of the class with the sheet
 We can start
 with $N=4$
 our result shows
 that then there are $5!$
 ways to arrange five students

Then, since our result
is good for five students
applying our result
it must be good for six
And then 7
and 8
and so on
ninety-nine students
You see
this proof by induction
wasn't so complicated
And welcome to a
college math argument

In relation to the topic of arrangement
I will leave you with some homework
You will have to solve the
task given by your teacher
and discuss them in your next class
Meaning, we will not discuss
the answers to your homework here
but you will have to do it
with your teacher in the next class

You can apply what you have learned
through this lesson in your daily life
For example
while you were arranging things in your home
like arranging furniture in the living room
posters
pots, pans and even
photo frames like what
Mrs. Bashirah is doing now
Not only this
factorial can be useful
for your other daily life practices
For example
to arrange books in your classroom
or library
at the window display
arrange cars in a showroom
and arrange luggage in the car bonnet
I hope you have enjoyed the lesson
and I really hope that this video
manage to assist you with learning
concept of factorials

This video is intended to relate
everyday life factorial topic
Students are expected to apply
the principle of multiplication
listing all the possibilities
and the proof by induction
Through these videos
students are exposed to cooperative learning
and role playing in class
In addition to cards labeled technique
teacher can also lead students to organize
their position in the class
so that all students feel
different sides of seat
Not only that
teachers can use any objects
that can be stacked like books, tables and chairs
Happy exploring factorial topic