

# Measuring Distances in the Milky Way

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[00:00:23.07] PROFESSOR HAJJAR: Betelgeuse, or Alpha Orionis, is the second brightest star in the constellation of Orion, the hunter. It's a red super giant and a very massive star. It's located at a distance of 640 light years from us, which means that light traveling at 300,000 kilometers per second would take 640 years to reach us.

[00:00:47.42] Betelgeuse is 120,000 times brighter than the sun. Now, how did you get that information? And how did you get the distance to that star?

[00:00:58.16] Hi, I'm Roger Hajjar, astronomer at Notre Dame University in Lebanon, the Middle East. I'll be your guide on our galactic trip. Now, assume that you're trapped in the class. Not a nice start for a galactic trip, right? We're exactly in this situation when it comes to stars. We are trapped on earth and would like to measure the distances of stars without a meter stick. After all, no one is long enough to reach Alpha Orionis.

[00:01:30.66] So how could we do it? That is the question. Anyone has an idea? Let's scratch our heads and get to the palm tree.

[00:01:41.37] So if you're looking at the segment, it means you found it. It's the properties of a triangle. So let's figure it out.

[00:01:49.02] Here is our palm tree. It represents the star to which we would like to measure the distance. Two of the students will be located at two different spots. Both of them, with the palm tree, will form a triangle.

[00:02:01.08] Now, get the appropriate equations that will allow you to calculate the distance from the measurement of the baseline and the two angles. And then, we will build a small device to be able to measure the angles. For that, you will need a cardboard disk, two straws, a protractor, and a pin.

[00:02:43.24] Now that you've found the equation, you will need to proceed with your measurements. Remember that the measurement is just a comparison between two quantities, one you call a unit, such as the tile on the floor, and one is what you want to measure, like the baseline. In that situation, you will get the baseline in number of tiles. And if you don't have a meter stick, that's not a problem at all.

[00:03:05.90] Now, choose a unit, and let us does identify two baselines, one about half of the other. Both should be on the same line. In other words, you should end up measuring the same distance.

[00:03:16.71] Start by measuring the two baselines in terms of the unit you chose. Then, with each one standing on both sides of the baseline, you will measure the angles by pointing one straw to your friend and one to the tree, each in its turn.

[00:03:36.17] Let each pair measure the distance for each baseline separately, then combine all your measurements for each baseline to calculate the average and standard deviation. The standard deviation is some kind of spread on your measurements. More on that later.

[00:04:00.52] As you see, the different values you obtained for the angles are slightly different although you stood at the same positions. You notice a spread around the mean distance you found. This is an uncertainty on the measurement. In other words, your distance could be thought of being anywhere between the mean minus the standard deviation and the mean plus the standard deviation. You never have an infinitely accurate value. And these are important limitations.

[00:04:28.66] So what are the limits of our technique? Suppose we are measuring the speed with a ratio of distance over time. Now, we have uncertainties on the distance, and we have uncertainties on the time. We're going to denote the uncertainty on the distance by  $\Delta l$ , and the uncertainty on the time by  $\Delta t$ .

[00:04:54.92] Now, if you want to calculate the uncertainty on the speed, the first thing we do is calculate the log of the velocity. The next step is to differentiate that equation. And that would give-- now, we need to convert that into an uncertainty. To do so, we just-- notice that we converted a negative sign to a positive side. That's because errors always add up.

[00:05:45.38] Now, try to figure out how to calculate the uncertainty on your equation. For that, you will need to estimate the uncertainties on your measurements, the baseline and the two angles. To give you an example, since we chose tiles to be the units of our measurement for the baseline, you cannot really say that the baseline is 7.51923 tiles. You might probably be able to say that the distance is known to  $1/2$  or  $1/3$  of a tile, but no better. Once you estimate the uncertainties on your measurements, and you find out how to calculate the uncertainty on the distance, come back to me.

[00:06:55.65] This is the equation you should have obtained. Each term is labeled by a letter, the distance, the baseline, and the angles. You would have probably estimated the uncertainty on the angles. I, personally, estimate them to be around 3 to 5 degrees. After all, your hand was probably shaking, and you took the aims to the tree and to your fellow student in sequence. And that would increase the uncertainty on the angle.

[00:07:26.46] Now, try to estimate what you found for that uncertainty. Play around with the formula. And then calculate the distance,  $d$ , at which the uncertainty would be  $1/3$  of that distance. Do that for both baselines. What do you notice? How does the length of the baseline affect your measurements?

[00:08:05.09] You have discovered that the larger the baseline, the more accurate your measurement, and the farther you could go while providing a good estimate of your distance. We are now on a Ferris wheel. We would like to measure the distance to a person on the playground below far from the Ferris wheel.

[00:08:22.26] Well, what is the Earth really doing? Which leads to the following questions. When we are on Earth and looking at the stars, what are the baselines we could use? And what is longest? Remember, we don't need to measure the angles simultaneously.

[00:08:52.59] We have discovered today the principles of the simplest technique to measure distances. Astronomers use one they call parallax. It's the fact that the relative position an object depends on the point of view of the observer. If you have tried hard enough, you'd have discovered the longest usable baseline, usable for Earth-bound observers, is the diameter of Earth's orbit, 300 million kilometers. That's quite large.

[00:09:23.56] Well, astronomers measure the angle at the top of the triangle, at its summit. You will be surprised to know that the largest such angle measured is 1.5 arcseconds, knowing that one arcsecond is 1 out of 3,600 divisions of a degree. If you think this is small, well, Gaia, the European Space Agency mission, will be able to measure one microarcsecond. That's one millionth of one arcsecond.

[00:09:53.91] Now, Gaia will barely be able to reach the distance to the closest galaxy to ours. And Gaia is not to be launched yet. So how did we measure distances to all the stars around us knowing that our technique does have limits. So what do we do when we reach

the limits of our technique? Well, if you're curious enough, these are two words for you-- spectroscopic parallax.

[00:10:18.34] I hope you enjoyed our trip in the galaxy today measuring distances to the stars. Well, I wish you goodbye and hope to see you again on another galactic trip.

[00:10:30.15] In this module, the idea is to put students in a situation similar to the one faced by astronomers when mapping the universe. They need to look for means to make measurements without what we consider the intuitive way of doing so. The idea is to show them, also, that the techniques used are not fancy or complex, but based on simple and easy applicable and understandable concepts. If today instruments are complex, the basic ideas are simple.

[00:10:57.86] The main activity is to measure the distance to a tree, or it might be a rock, at a certain distance outside of the classroom. From that simple activity, I aim to make students discover a simple trick to measure distance using readily available material. They will also discover that the measurement is basically a comparison between a unit and that what they would like to measure, knowing that this may not need to be international units. And, more importantly, I aim to make them discover that any technique has imitations that depend on the process and the instrument used, which ultimately means that you always need a variety of methods to complete and go beyond any one of them.

[00:11:42.60] In order to proceed through the module, the students need to know basic trigonometric functions and how they relate to the angles and sides of the triangle. They will also need to know how to calculate derivatives of these functions in order to be able to compute an expression for the uncertainties. Ideally, one would be proceeding in a class with a large window to the outside and some clearly available physical object outside. But one may proceed with the module in a completely open or closed space.

[00:12:13.39] Students will have to build with two straws, one cardboard disk and a pin, a small device to aim and measure angles. You will see a sample of the device in the module. They will need a protractor to label the edge of the disk and measure angles. You should prepare material for one device per pair of students.

[00:12:34.00] The module should not take more than 50 minutes to complete if the students have the necessary mathematical background I already talked about.

[00:12:41.88] In the second part, divide the students in pairs and let them find the equation that allows them to measure the lengths of the perpendicular to the baseline given the two angles. Then, build a small device that allows them to measure the angles. It's very important that they do the measurements with only one device, taking the first, then the second.

[00:13:03.64] At the third pause, let each pair measure the angles for the two different baselines. Ideally, the baselines would make an isosceles triangle with the tree or the rock. Let the largest be as close to an equatorial triangle as possible. Each group calculates the distance for each baseline. Then combine them all in front of the class. After the first stop, always in pairs, let them try to find the expression for uncertainty.

[00:13:34.32] The fifth activity is essential. Let them use the same values of the uncertainty they have estimated earlier. For example, 5 degrees for angles and 1 tile for distance. Then let them consider, to makes things simple, that the two angles are equal. They should now increase or decrease the angles and calculate the distance and uncertainty until they find that the uncertainty is  $1/3$  of the distance.

[00:14:01.87] Last is a discussion of the motion of the earth and the fact that you don't need to measure simultaneously the angles. It is important that they discover that we may wait for six months to do the second measurement.