How to estimate the value of π ?

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Hello, everyone，welcome to my class. My name is Renyong Feng and I am from Verakin High School of Chongqing, China. Today, we are going to estimate the value of pi.

First, I am going to ask you a question: “What is pi?” Here, I am not asking about the value of pi, but the fundamental definition of pi, which is independent of any numerical value. Can you answer my question?

Once you have got the answer, can you think of some ways to estimate the numerical value of pi, assuming no one has ever told you about its value and you need to explore by yourselves? Try this, and I believe that you will have fun during this exploration.

<u>Activity 1:</u> What is π? What is the fundamental definition of π?

Now everyone, after the discussion， you must have known the answer! Pi is the ratio of the circumference of a circle to its diameter and this ratio is a constant. Once we know the radius or diameter of a circle, we can use pi to calculate its circumference or area. Today we are going to answer this question: how the value of this constant is estimated. Well, let me try to do it first.

According to the definition of π, pi is the ratio of the circumference “*l*” of a circle to its diameter “*d*”, so we can use the formula $\pi = \frac{l}{d}$ to get the value of pi as long as we know the circumference “*l”* and the diameter “*d”* of a circle. Now I will show you my method. Here, I have a tire with normal pressure. I am putting some wet paint on its outside edge. Then I’m rolling the tire forward carefully. When you are rolling the tire, you should try your best to follow a straight line. Now you can see that two marks are left behind us. So you can imagine that the distance between these two marks should be the circumference of the tire. Now, let’s measure this distance between these two stains. The distance I measured is around 2.14 meters. Next, let’s measure the diameter of the tire. First, I put the tire to the edge of the flowerbed as closely as possible. Then I put a set square close against the tire and try to keep the side of the set square parallel to the edge of the flowerbed. Through this way, I can get the diameter of the tire, which is around 67 centimeters. According to the results we have gotten, the circumference of the tire “*l*” is 214 centimeters, and the diameter “*d*” is 67 centimeters. Therefore, the approximate pi value equals to: 214 divided by 67, which is around 3.19.

Are you clear with it? It is very easy and you can do it too! If you don't have a tire right now, you can also use round cups that are available to you. For example, pick a cup and then twine a paper tape onto it for one round to get its circumference. Then measure out its diameter with the rulers in your pencil box. SO you can easily estimate the value of π. Now the time is yours. Try it by yourselves.

<u>Activity 2:</u> Students use the materials around them to estimate π value.

How was your exploration? Was it fun? Did you paint and roll the tire to estimate the value of pi? How close is your estimate to the true value of pi?

To improve the accuracy and reduce the errors, we can repeat the measurements for several times and calculate the average.

Are there any other ways? Now your teacher will give you a paper with a coordinate system on it. Look, there are 100 tiny squares and a one-fourth arc of a circle on the paper. Now I would like you to estimate the value of pi with this paper? The time is for you to think about it. We'll be back soon.

<u>Activity 3:</u> Students estimate the value of pi with the grid paper.

All right, have you got the answer and how? Did you count the number of the squares outside and inside the arc? How are these numbers related to the value of π? Now let's discus it: how do we use this paper to get the pi value?

Now let's assume that the radius of this sector is 1. So we can get the accurate area of this sector: S equals to ¼ * pi * r^2, equals to 1/4*pi. Next, let's use the total area of these tiny squares to estimate the area of the sector. We can get the area of the sector by counting the number of the tiny squares that fall inside the sector. Because the area of each tiny square is 1/100, if we get the number of the tiny squares inside the sector, we can easily get the area of this sector. To make this process simple, we can count the number of the squares ***outside*** the sector, which is easier to do. So let's do this, 1, 2, 3……..21, total is 21. When I was counting, for each square crossed by the arc, if the bigger part of the square falls outside the arc, then this square is included into the squares outside the sector; if the smaller part is inside the sector, then it was counted as the squares outside the sector. So the total number of the tiny squares that fall outside the sector is 21. I use "*m*" to represent this number, which equals to 21. Therefor the tiny squares that fall inside the sector is 79. So the area of the sector is around 79 * (1/100), which is 0.79. That means: the value of (pi/4) is around 0.79. So the value of pi is around 3.16.

All right, we've got the value of pi with this graph. It is simple, isn't it? Let's try it one more time. You will get another paper from your teacher with a 20 ×20 grid. Please use this paper to estimate pi value again. Let's see, this time, is the estimate of the value of pi better or worse compared to the pi value we got from the last activity.

<u>Activity 4:</u> Students use 20×20 grid paper to estimate the value of pi.

Are you done with it? What's your π value when you use a 20×20 grid?

When the number of the squares increases to 20×20, the estimated value of π is 3.15, which is closer to the true value 3.14. So you may jump to a conclusion that the more and the smaller the squares are, the more accurate pi value will be. Theoretically, it's right, but in practice, it will be more and more difficult to count when the squares become more and smaller. Then what can we do if we want a more accurate value of π?

Let's try another method, which is more fun. Have you ever played the dart-throwing game? Do you like it? Have you ever seen people throwing darts blindfolded? Next we will try the method of dart throwing, instead of counting the squares. Hey, can you please come here and have a try? This is the target, a square with the side-length of 2 and with a circle embedded within it. Now I will blindfold you, and then you start to throw a dart to this target for 20 times. Let's see what result we can get. OK, you can start right now.

Would you like to try this method? Now it is your time, you need to throw the dart for 20 times, and record the number of the times the dart falls inside and outside the circle. If the dart misses the square target, please throw it again until the dart lands inside the square for 20 times. When you are done, please think about this question: can we estimate the value of pi by the number of the times the dart falls inside and outside the circle? Moreover, please be extremely careful when throwing the dart because dart throwing could be dangerous. Your teacher may ask you to use a rubber dart or magnetic dart as shown in my hands instead of the dart with a metal tip.

<u>Activity 5:</u> Students use the method of dart throwing to estimate the value of pi.

Is this method interesting? But can we estimate the value of pi by this method?

Here is the square target with the side length of 2 and a circle imbedded within it. We throw the dart randomly to this target for "*n*" times, and write down the number of times the dart falls inside the circle, which is represented by "*m*". So according to the principles of a geometric probability model, the ratio of the area of the circle to the area of the square should approximately equal *m/n*, therefore, the area of the circle is approximately 2×2×*m/n*=4*m/n*. Because we know from geometry that the area of the circle precisely equals to pi×1^{2}, the value of pi is estimated at around 4*m/n*. According to our results, 17 out of 20 times the dart falls inside the circle. By the formula we got above, the value of pi is around 3.4. This result looks a little far away from the true value. The reason is that our dart throwing is not stable. If we want a more accurate pi value, we have to repeat this experiment for many times, which means that we need to throw the dart a very large number of times.

With the development of information technology, the computer has become a powerful tool to help us solve many practical problems. Have you ever seen coin-tossing simulated by a computer? If dart throwing can also be simulated in the computer, it will be much easier for us to get a large amount of experimental data conveniently and quickly. But how can we use a computer to simulate dart throwing? Think about it. You can discuss with your neighbors and ask your teacher for help. I will give you some time and we will be back soon.

<u>Activity 6:</u> How to use a computer to simulate the dart-throwing process to estimate the value of pi?

Welcome back! Have you figured out how to use a computer to simulate dart throwing? How did you do that? Next, I will show you how I use a computer to simulate the process of dart throwing. Let's see if my method is different from yours.

First, we put the dart in a coordinate system. Because we have a circle and a square, we coincide the center of the circle with the coordinate origin. Because the graph is congruent in four quadrants, we only need to think about the case in the first quadrant. The landing spot of a dart in the first quadrant can be described by coordinates *x,y*. Because the radius of the circle is 1, the values of *x,y* should be between 0 and 1. If $x^2 + y^2 > 1$, then the point with these coordinates should be ***outside*** the circle. Otherwise, the point is ***inside*** the circle. So, if we ask a computer to generate "*n*" sets of *x,y* values, then "*n*" points with the coordinates *x,y* are generated to simulate the dart throwing for "*n*" times. The value of "*n*" can be set to 100, 500, 1000, even 10,000. Then we take the integral value of $\left[x^2 + y^2\right]$, which means the integral part of a number. For example, the integral part of 1.5 is 1; the integral part of 0.23 is 0, which is different from round off as we usually talk about. So by this way, we can get some numbers, either 0 or 1. Last, we sum these numbers, 0 or 1, which can be done by summation formula. Therefore, the result from the sum is "*m*", the number of the points that fall outside the circle, so the number of the points that fall inside the circle equals to (*n-m*). Then we can use $S = \pi \times 1^2 = 4 \times \frac{n-m}{n} \Rightarrow \pi = 4(1 - \frac{m}{n})$ to estimate the value of pi.

All right, have you understood how to use a computer to simulate dart throwing to estimate pi value? Please use computers to try it by yourselves. You can also discuss with your teacher about the details. I am looking forward to hearing the good news from you.

Hi everyone, did you have fun in this class? In this lesson, we used four methods to estimate the value of pi.

1. The four methods use different mathematical thoughts, and all of them are able to estimate pi value.

2. Let's sum up the advantages and disadvantages of each method. The first method, tire-rolling method is a direct method, which is the simplest way of measurements in physics. If we can reduce the errors by using the knowledge in physics, this method can get stable results directly. The second method, square-counting method, uses the mathematical method of approximation. Because it is difficult to count very tiny squares, the precision and accuracy of the result may not be good. The third method, dart-throwing method, is lots of fun. However, the result is not stable because of the randomness of this method, which is also one of its disadvantages. The fourth method, using a computer to simulate lots of dart throwing, is fun and efficient. This method can obtain plenty of data, so the result is quite accurate. Also the method is very convenient.

3. Please continue to improve the above four methods and think if there are any other ways to get a more accurate value of pi. Please try to do these after class.

Thank you for your attention and participation! Have fun learning! Goodbye!

Teacher's Guide

Hi, teachers, thank you for using this video lesson. Next, I will give you several suggestions about this lesson.

1. We used four methods to estimate the value of pi. On the one hand, we'd like to engage the students to estimate the value of pi by themselves, on the other hand, we want to arouse their interests of learning the geometric probability model and help them better understand this geometric probability model.

2. We don't require the students to get a very accurate value of pi, especially in the experiment of dart throwing, which has bigger errors and is only intended to stimulate the learning interests of the students. Here, I need to remind you that when throwing darts, you should be very careful.

You can use these magnetic darts as I used to make the experiment safer. You can also think of the other ways to substitute this experiment.

3. If you demand high accuracy of the value of pi, you should use the methods of tire rolling and computer simulation. Using a computer to estimate the value of pi should be the best method because we can get the data up to 10,000 and then calculate an average to get an accurate value of pi.

4. Through the personal participation, the students can understand that many factors can affect the result during their measurements. At the same time, they are encouraged to keep trying to improve the methods and find the new ways to estimate the value of pi.

5. If computers are not available in your schools, you can print out the data sheet that is available on the BLOSSOMS website for the students so that they can calculate the value of pi according to the data sheet.

6. When giving this lesson, the teacher can have the groups of students compete with each other and compare which group can get a more accurate result, have a better method or use simpler materials. By doing these, the students' enthusiasm in learning can be inspired.

7. The detailed steps and a demo video showing how to use a computer to simulate dart-throwing is available under the category, "For Teachers" on the web page of this lesson. Please watch that video for more information.

Thank you again for using this video lesson. If you have any other questions about this video, please feel free to contact me through the MIT BLOSSOMS website. All the best in your work, bye!