FREE FALL
By John Bookston

Hello! My name is John Bookston and I'm so glad to have you here with us at Space Central on the exciting news of the landing on Planet Platypus. The spacecraft Einstein Ramanajan has been in contact with us and is now calculating the gravitational constant on that planet. We are here in a suburb of Boston, Massachusetts at the Arlington High School where I teach mathematics. Thank you for joining us.

Well, I received a message this morning that the astronauts had landed. They are very excited. They have communicated to us the experiment which they did this morning, dropping a pebble from a height of 36 feet and timing very carefully; that it hit the ground on planet P in 2.4 seconds. I now will communicate to them and in several minutes, after my voice gets to them and their reply gets back to us, we will hear what they have to say and the excitement in their voice.

This is Space Central, Planet P. Do you read me? I got your message this morning clearly and am delighted by your soft landing, by your experiment and the data calculations that we are doing at this end to determine the gravitational constant on Planet P. I have with me this morning a classroom full of future astronauts who are going to help us and I look forward to having you say hello to them when my voice gets to you and yours gets back to me.

Hello again! While we're waiting for the astronauts to get our message on Planet P and for them to convey a message directly back to you and for that message to reach us back here, let's go through the calculations that the astronauts did earlier today after doing this simple experiment. They have a tower constructed that is 36 feet high and from that height they dropped a pebble and timed it as it fell to the ground. This is on a planet not too different from our moon. There is no atmospheric interference with the pebble's fall. The only force on it is the force of gravity. They found that it took 2.40 seconds for the pebble to strike the ground. From this simple experiment we can, and you will be going through the calculation for figuring out what the acceleration constant is, what the force due to gravity is on Planet P. So what did they do? They said, "We can find the average speed that the pebble experienced during the entire period of its fall by looking at the total distance that it fell divided by the total time of its fall." (writing equation on board) This comes out to 15 feet/second and that will turn out not only to be an easy calculation, but to be a very crucial calculation in moving on from here.

The second step that they took and you can take right now, is you can see that if the only force operating on this pebble is gravitational, the force is constant and the speed will increase at a constant rate. There will be a linear function. At time 0 the speed of the object when you open your fingers to drop it is 0. We know that at time 2.4 the speed of the object is… We don't know the speed of the object when it hits. That's what we're looking for up here. But we do know that the average speed between time 0 and 2.4 is 15 feet/second. It becomes clear just by looking at a graph, a linear function has its average at the midpoint and so if this is 15 feet/second, the speed at which it crashes to the ground must in fact be 30 feet/second. Another simple calculation, understanding of the fact that the pebble is going faster and faster and faster, gaining speed at a constant rate and striking the ground after 2.4 seconds. We know its average speed here so we calculate the terminal speed. This is a very good place for us to stop for a moment and to have you in the classroom go through these steps, whether with your own experiment, you can make up a Planet Q and drop a pebble from 100 feet and say it takes five seconds to hit the ground, or any other numbers you wish. Go through a calculation of average speed during its fall and then instantaneous speed at the instant that it hits the ground. I'll be back with you when your teacher tells me that you as a class understand this part and are ready to move on.

Welcome back! I certainly hope that your calculations in class with your own numbers came out. You understand that the calculation is quite simple, even though it might not have appeared that way when someone originally said, "Well, they dropped a pebble and it hit the ground in 2.4 seconds and they dropped it from 36 feet." The next calculation, the calculation of the constant acceleration is just as easy. Acceleration is a constant. (writing on board) The pebble goes faster after the first second, goes faster by that much again after the second second. Faster by that much again after the 2.4 seconds, well, not by that much again, but every instant in time it's gaining speed and gaining it at a constant rate. Well, we know that the total speed gained over here is 30 feet/second. It went from 0 to 30 feet/second. So the change in speed is 30 feet/second. If we divide the change in speed by the change in time, that is the definition of acceleration. And the change in time was 2.4 seconds. I will do the calculation here and then I will take a break to allow you and your teacher to go through that calculation and to say, "I can do that on any planet by just knowing how much speed was gained over how much period of time." In this case that comes out to 12.5 feet/second, each second, which could be written per second or we write it as seconds squared in the denominator. So we have now calculated the acceleration constant on Planet P as 12.5 feet/second squared. Take your time. Make sure that THIS one is OK because when I come back we'll fill in an entire chart and you will be able to do this for any planet gravitational constant.

Hello Space Central! We read you loud and clear! This mission has been spectacular. Thank you for your congratulations and we have completed our calculations. I hope that ours are the same as yours. We calculated the gravitational constant to be 12.5 feet/second squared. We have another 47 planets to visit before returning to Space Central. We should be back in eleven of your years. I understand that you are being visited today by a classroom of potential astronauts. I hope that they follow through with their mathematical knowledge to do what I am doing. The view is spectacular. This is a wonderful mission. I wish them only the best in the rest of their year in mathematics. Over.

Welcome back! To review what we have done so far very quickly. We dropped a pebble from 36 feet. It took 2.4 seconds to hit the ground. We figured out that the average velocity during its fall was 15 feet/second. That the instantaneous velocity at the moment the pebble hit the ground was 30 feet/second. And we went to the next step of figuring out that the acceleration had to be 12.5 feet/second using the fact that it changed speed from 0 to 30 feet/second in 2.4 seconds. Using the acceleration constant, we know how fast that pebble is falling at any moment in time. I have created a table here and I will fill it in with you. I don't think you'll have any problems doing this.

At time 0 you drop the pebble and the instantaneous speed is 0. During the first second of fall, we know it's going to gain 12.5 feet/second. Each second it gains 12.5 fee/second. So after 1 second, the speed of the pebble is 12.5 feet/second. After 2 seconds it has gained another 12.5 feet/second. After 2 seconds it is falling at a rate of 25 feet/second. And we know that after 2.4 seconds it is falling at a rate of 30 feet/second. That is its speed as it hits the ground.

(pointing to the speed column on the chart)
This is its speed as the stopwatch hit 2 seconds. This is the speed as the stopwatch hit 1 second.
(pointing to the average speed column on the chart)
We know that the average speed from the time it's dropped until any time T here is just going to be twice the speed it, I'm sorry, it's going to be half the speed it is at that time. The average of 0 and 12.5 feet/second is 6.25 feet/second. The average between time 0 and time 2 is the average of 0 and 25, which is 12.5 feet/second. And of course we already figured here that the average between time 0 and 2.4 seconds is half of the terminal speed of 30 feet/second, which was 15 feet/second. This column now gives us the ability to calculate how far the pebble has fallen -- during each of those -- at any instant in time.

During the first second the average speed of the pebble was 6.25 feet/second and it's been falling at that average speed for 1 second, so it has fallen 6.25 feet. If it has fallen 6.25 feet from its initial perch of 36 feet, it will be at a height of 29.75 feet. As the stopwatch passes the time 1 second from drop, the pebble is here (pointing to the ciagram of the tower) if we might look at it. It has not fallen very much. It's at 29.75 feet. It has fallen only 6.25 feet. Let's do the same thing at the instant that the stopwatch says 2 seconds. During the first two seconds its average speed was 12.5 feet/second. And it had been falling at that average speed for 2 seconds, so it has fallen 25 feet. If it fell 25 feet and it started 36 feet above the ground, its current height at the time the stopwatch says 2 seconds, is 11 feet. Initial height minus how far it had fallen, and we know that at 2.4 seconds it is down to hitting the ground. This type of chart is what Sir Isaac Newton used to determine gravity when it wasn't given to him -- as it's given to most school students -- as a formula. And it is a very straightforward set of calculations. I now will take time for your teacher to go through the chart with you, for you to become comfortable with it, and then we will do the much more difficult (not really) task of applying what we just learned to problems dealing with free fall.

Welcome back! The basics are now entirely covered and if you go practice those a few times -- planets of your own choosing, making up the height and the time it took for the object to fall to the ground -- you will be ready for solving, applying this knowledge to every day free fall here on earth. Unfortunately I can't take you to Planet P where you could try your own experiment because it is very time consuming and I don't think either of us would like to spend that amount of time. However, I can apply what we just learned to a strange planet from someone else's perspective. They came, they landed on earth, they did these calculations and here is what they would have found.

If they had dropped the pebble from 16 feet high, it would have taken exactly 1 second or very close, 1.00 second to hit the ground. They then would have calculated the average speed that the pebble took, the speed at which it hit the ground and then the acceleration here on earth (writing on board) and they would have gotten a value of 32 feet/second each second [squared]. Using this constant of acceleration we now are going to pose problems and see if we can analyze the situation. Instead of dropping a pebble from a certain height, I am going to launch a pebble using a slingshot, into the air, and tell you only that it went up and came back down in exactly 24 seconds. The question is what was the velocity that it had to have at the moment of launch in order to go up here and return down in 24 seconds? And the second question is going to be how high did the pebble reach? What was the maximum height of the pebble? If you were just starting on that from scratch, if you turned off the video right now, would you be able to use the information that you had from before to calculate these two pieces? I'm going to give you at least one hint and that is whatever time it took to get to the top, it is losing speed at 32 feet/second each second until it gets to a speed of 0 at the top and then it is gaining speed at 32 feet/second each second until it hits the ground. So this must be in terms of time exactly half of the 24 seconds, or this must be at time 12 seconds. Now I'm telling you that I dropped a pebble and it took exactly 12 seconds to return to ground. I know the acceleration constant is 32 feet/second each second and that tells me that after 12 seconds it has gained 32 feet/second each second for 12 seconds. And I know the speed then at which it crashes back down to earth. Because of the symmetry here it must have gained or lost from launch exactly the same amount of speed, in the same amount of time. So it must in fact have been launched with this exact same velocity. I will let you go through this example before we calculate the height at which, the maximum height that the pebble reached. Take time now and we'll come back for that final segment when you are ready.

This has been a terrific opportunity for me to be with you today. I feel very good and I wouldn't be surprised if while the tape was off, if some of you tried to figure out what the maximum height was making a table of your own. I'm going to go through my calculations now. Again, it is nothing more than straightforward logic. We have not used a single formula and will not use one. If you use the hint that I started with, at time 12 seconds the pebble has reached its maximum height. It was losing speed at 32 feet/second each second it was rising. It was then gaining 32 feet/second each second it was falling and so at any given height it would be going up at exactly the same speed that it was coming down on its fall. At 12 seconds we are trying to find the height. The distance it's fallen will be 0 because I am starting at 12 seconds and doing everything from 12 seconds on. The instantaneous speed at its peak, it slowed down to 0 and started falling. The average speed does not make any sense. This is going to be at time 12, because this is going to be from time 12 on. At time 13, after one second, its instantaneous speed will be 32 feet/second. Its average speed will be 16 feet/second. Its distance fallen will be… Well, let's write these in. At time 13 seconds, it's instantaneous speed will be 32 feet/second. Its average speed from time 12 on, from the time it began falling, it's been falling for one second. It went from 0 to 32 feet/second. It's average speed is 16 feet/second and how far has it fallen? Average speed 16 feet/second, over a time period of one second it has fallen 16 feet. Now we could put in 14, 15, 16, all the way down to 24. We don't need to do that any more because all I'm going to do is tell you that its instantaneous speed after 12 more seconds when it hits the ground had to be what we calculated over here to be a time period of 12 seconds with a gain of speed of 32 feet/second each of those 12 seconds. (writing on board) It is hitting the ground at the speed 384 feet per second, which answered our first question, what was the speed at which is was launched? It had to be launched at a speed of 384 feet/second in order to take 24 seconds on earth to rise and fall. So we know at time 24 its instantaneous speed is 384 feet/second. Its average speed between 12 seconds and 24 seconds is exactly half of that because it started at 0 at the top. So that's 192 feet/second. If its average speed over these 12 seconds was 192 feet/second, we know that it has traveled a total distance of 192… let's just do the calculation here. 192 feet/second average for 12 seconds, a total of 2304 feet. That is the distance that it has fallen which tells us in the exact same symmetry that that's the distance that it reached as a maximum height.

I certainly hope that this type of step by step approach, no formulas, only minimal calculation, will be inspirational for you in the sense that you can do very complicated problems if you get to be a good problem solver, if you get to think logically and to do your best, neat tables, diagrams of what it is that you are trying to solve. And I wish you nothing but the most fun and enjoyment in your math classes! Thank you for letting me join you today.

Colleagues! It is such a pleasure to be able to share my excitement about teaching and teaching without formulas and teaching for understanding with you. The whole purpose of my joining in you and trying this lesson in front of you is to get as much excitement and activity on the part of students and to minimize the amount of formulas and the amount of teacher chalk and talk that we engage in, in our daily lives. When I was at a math conference, a statewide math conference and a national math conference I did a poll of a few, nothing statistically significant, a few of my colleagues, and asked what they knew and how they would teach gravitational free fall. Their answers were uniformly, "Oh, I think there is a formula about negative 32, no negative 16, T^2 Oh I can't remember what the formula is but I'd have to look it up. Or I'd refer them to my science colleagues because I never did understand gravitational free fall." The idea that came up in BLOSSOMS here at MIT to have mini-sessions, to have breaks where your class does the work, to have you be able to interact with your students in the normal way and for me to be nothing more than a bit of a ham here, is in my estimation a very nice way of sharing lessons which seem to work for us with you. If you have questions about the materials that are provided with this, if you have questions about how it works with students or what to do with particular kinds of students, please don't hesitate to email back here to BLOSSOMS or directly to me: bookston@gmail.com. Thank you so much!