The Mathematics of Voting Transcript

Hello, my name is Andy Felt. I'm a professor of Mathematics at the University of Wisconsin-Stevens Point. This is Chris Natzke. Chris is a student at the University of Wisconsin-Stevens Point.

Today we're going to talk about different methods of voting.

Now many people are surprised to learn that there are several different methods of voting, and the outcome of an election can be decided by what method you choose. For an example, Professor Arnold Barnett of MIT and Professor Alex Belenky have shown that with the current system used to elect the United States president, as little as 24% of the population could elect the president.

Today we're going to run an election in the classroom. And so first we're going to meet the candidates. There are four candidates. And I want you to meet them.

Hello. I am candidate A of the best smiley face competition. You better believe that I'm going to win this.

Hello. I am candidate B for the best smiley face competition. I hope you all will vote for me.

Hello, I'm candidate C for the best smiley face competition. I hope you will make the right choice in this election.

Hello, I am candidate D for the best smiley face competition. Here, no one can stop my good looks.

Now that you've met the candidates, what we're going to do is ask you to rank them. Each person in the class should rank them.

And so for example, maybe you liked candidate B the best. Then you would rank B as the first choice. And then maybe your next choice was candidate D. And then you would maybe have the order B, D, A, C.

Or maybe someone else liked the order A, C, B, D. Rank them from most favorite to least favorite. And now the teacher can pause the video.

Welcome back. While you were casting your votes, we held an election here. And these are the results.

You can see that three people voted A, B, C, D. Three people preferred A, C, B, D. One person liked the order A, D, B, C, and so on. And you can see the results here and here.

For example, two people liked C the best, B second best, A third best, and D fourth best, and so on. So then the question is how do we decide the election?

And this is where we want to talk about the different methods. One of the most common methods of deciding the election is called plurality. And plurality is very simple. All you do is take the person's first choice, and the candidate who has the most first place votes, wins.

In fact, most of the time, the voter only indicates their first choice. So they wouldn't rank them. They would just say I like C the best, for example.

Now Chris is going to count up and show you who wins our election by plurality.

So going through our election numbers and such, we see here that three people have chosen A, B, C, D, three people A, C, B, D, and one person for A, D, B, C, meaning with plurality, candidate A has seven votes for it. For B, we have two in the first column, one in the second, two in the third one. That would give us five.

Then, moving onto our next set, we have for C, we have one, two, and one. That'll give us our four. And then for D we have one, six, and three, which will give us 10 votes overall.

So altogether here, we have A with seven, B with five, C with four, and D with 10. So using the plurality method of voting, D has won the election here with our data.

Now to mention, we have D with 10 votes out of 26 total votes. So D has won with the plurality, but he has not won with the majority. What the majority is, if you get half or more of the total votes.

So looking at the data, you can see that maybe everybody else didn't like D, but D is still the winner because that's still the most votes by plurality. So we have a few questions for you to try and answer during the break.

Do you think that candidates A, B, C, and D, are similar? Why D won in our election? And is there anything undesirable about the plurality outcome?

Now if you take this time to calculate who won plurality in your election, this is where the teacher can pause the video. And to allow the class to answer the questions.

Now maybe during the break that you took the time, you discussed some of the major weaknesses that exist in plurality.

One of the major weaknesses of plurality that can happen in an election with three or more candidates, what often happens is that the mainstream candidates wind up splitting the votes of the mainstream voters. And what can happen is an unusual candidate then, one that distances himself from the others, can wind up winning a plurality election.

That is exactly what happened in our election. Candidates A, B, and C, were similar candidates maybe more mainstream candidates. And they wound end up splitting a majority of the vote. And it allowed candidate D, kind of an unusual candidate, to win the election. And this shows where plurality is weak even though it's widely used in many different countries for government elections.

So in an attempt to compensate for this weakness and plurality, a method called instant runoff was devised. And instant runoff works like this. If there are no candidates that win a majority of the votes-- remember that a majority means that the candidate has received more than half of the votes-- then the weakest candidate, the candidate with the fewest number of first place votes, is eliminated.

In our election, the highest vote-getter was candidate D. But 10 votes is not a majority. It is only a plurality. In order to get a majority out of 26 votes, he would need 14 votes, or more than half.

So because D did not win a majority, candidate C is eliminated. The reason candidate C is eliminated is candidate C received the fewest first place votes.

So in instant runoff, candidate C is eliminated. And candidate C's votes are not then thrown away. What happens is candidate C's votes are sent to other candidates.

This voter liked A second best. And so this voter's vote will be cast for A in instant runoff.

So A now has eight votes. The seven votes represented over there plus this one vote.

These two voters liked B second best. And so B wins those two votes when A is eliminated, giving B seven votes.

This one person liked D second best. And so that vote goes to candidate D.

So in instant runoff, the new vote totals are eight for candidate A, seven for candidate B, and 11 for candidate D. Now you may notice that nobody has won a majority still. The highest vote count is D, but he only has 11 out of 26 votes.

So because there is no majority candidate yet, instant runoff goes through a second round of elimination. And beginning our second round of elimination, looking at the

voting totals with 11 for D, eight for A, and seven for B, we see that in this round, B will be the next one to be eliminated.

So in this case, we have here, you see, two votes here, one vote here, and C was eliminated here, so we go down to his third pick. So these five votes from here will all go to A, giving A a total of 13.

Then going back, we have to redistribute some of the votes from C once again. So going from here, you see two votes here went to B. These now go to A instead. So A's total from there will end up being 15.

Now with our new totals here, we have A with 15 votes, D with 11 votes. A has enough votes to have the majority. A wins the election with the instant runoff method of voting.

Instant runoff is a good method. It's also known as plurality with elimination, because you start out with a plurality round, and then you eliminate candidates until there is a majority winner. In our case, A won.

This method is used in several places. One is in Australia. It is used to elect the lower house of representatives. It is also used to elect the president of Ireland.

Now I'd like you to pause the video in a moment and answer the following questions. I'd like you to determine who won your election the one that you held in your classroom using instant runoff method.

I'd like you to find out, is it the same candidate as won earlier with plurality? Or is it a different candidate, just like in our election?

And if it was a different candidate, are you happier with this result? Do you think this is a more fair result? Now you can pause the video.

Welcome back. Hopefully you discussed some of the advantages to instant runoff.

For example, in our election, the people who voted for candidates C and B did not waste their votes. Their votes wound up counting for candidate A and getting candidate A elected. So people who liked candidate C and candidate B could vote for their favorite candidate and yet still not waste their votes.

So in an instant runoff, your second and third choices can really count. It's a nice thing about instant runoff.

Next we're going to talk about a method that you may not have heard of before. And that method is the Borda Count.

The way the Borda Count method for voting works is that for each place that you rank somebody, points will be awarded to that candidate. So in our example with four candidates, a first place vote is worth three points, second place two points, third place will be one point, and last place would be no points. At the end, you tally up all the points, and the person with the most points is the winner of the election.

So let's see who won our election using the Borda Count. It's kind of a complicated process.

If you think about candidate A and the number of points that candidate A got, candidate A was the first choice of all seven of these voters. And so using the Borda Count method, that gets a three times seven points. Because each of those seven votes was a first place vote. So they're worth three points.

If you look at these two voters and this voter, candidate A was their second choice. And so that gives A two points each. Let's see if A got any more second place votes. A got a second place vote here and a second place vote there. So that's two plus one is three, four, five second place votes for A.

Second place votes are worth two points. Third place votes are worth one point. And let's see, where does A get a third place vote? Here A got two third place votes. Two more here. Six third place votes there. So that's two plus two plus six. 10 third place votes. And each third place vote is worth one point.

And in our election, fourth place votes are worth no points. So A gets no points from being the fourth choice.

I should mention that these point values are valid for four person elections. In a 10 person election, the first place would be worth nine points, second place would be worth eight points, and so on. So you adjust it depending on how many candidates there are.

Let's see how many points A has. 21 plus 10 is 31 plus 10 is 41 points.

Let's add up how many Borda Count points candidate B has. Candidate B has five first place votes each worth three points. How many second place votes does candidate B have? He has three second place votes there. Two here and none here. So that's five second place votes. Five second place votes.

And the number of third place votes for candidate B? Three plus one plus one plus three. So that's eight third place votes.

So the number of points. Each third place vote is worth one point, remember? So 15 points from first place votes, 10 points from second place votes, eight points from third place votes. That's 25 plus eight is 33 points for candidate B.

You can see this method is a bit difficult to calculate the winner, but in some sense it's also fair. Candidate C got four first place votes each worth three points. Second place votes for Candidate C, nine here, second place votes for candidate C.

Three here and two there. So that's 14 second place votes for candidate C. Many people like candidate C second best.

How many third place votes did candidate C get? One here, three right here, two right there. And so that's six. So 12 plus 28 is 40 plus six is 46. Wow.

And candidate D, our plurality winner. How many Borda Count points did candidate D get? Well he got 10 first place votes.

How many second place votes did candidate D get? One here, one here, and nothing else. So two second place votes.

And how many third place votes? One here and one here. So two third place votes.

Look at this. D got 10 first place votes, two second place votes, two third place votes, for a total of 34, 36 Borda Count points.

So look at our totals. 36, 46, 41 for A, 33 for B. Candidate C is the winner. Candidate C, who got the least number of first place votes. Candidate C, who was the first to be eliminated by instant runoff, has won the Borda Count election.

And conversely, candidate D, who won our plurality election, his support faded. And he came in third place in the Borda Count election.

Now I'd like you to pause the video in a moment and discuss the following questions. How is it that candidate C managed to win the Borda Count election in our election? How would you describe the support for candidate C versus the support for candidate D? Which one has broad support, and which one had support of some, and was disliked by most of the others?

I want you to calculate who won your election using the Borda Count. Find that out. Was it a different candidate than the other two elections methods?

And discuss with your class and with your teacher what you like and don't like about the Borda Count method. What are the advantages and disadvantages? Pause the video now.

Welcome back to the video. A Borda Count method really is a different method than what you're used to seeing, and a little more complicated when deciding on who wins. And it really drives home the point that every vote really matters. As far as the points and where they're distributed.

Now with this method, it also gives all the moderate candidates, who appeal to the large audiences, a much better chance to win the election. They have a little bit of an advantage here. This type of voting method is used for American college football, for voting for the winner of the Heisman Trophy, and the Eurovision song contest.

Now all through out the video today we've gone through several different types of voting methods. We have our plurality voting where it's quite simply the most first place votes win.

We have our instant runoff where you eliminate the candidates with the least first place votes and redistribute them based on how people have ranked the votes. And the Borda Count where you award points for each first place, second place, third place, and so on, for the votes. And decide the winner by who has the most points.

This concludes the classroom portion of the video. We hope you enjoyed the video. We thank you for your attention.

We'd like you to discuss with your classmates some issues. What was your favorite method of voting? Why did you like that method the best?

Even experts disagree on what the best voting method is. And so it's common to have disagreement. I'd like you to discuss that with your candidates. Thank you very much.

Take care and good luck.

Welcome to the teacher comments portion of the video. This part of the video is for the teacher only. It will give some hints and comments on the various parts of the video.

First of all, thank you very much for choosing our video, and it's a very important topic we think. And hopefully it's an interesting one for your students.

The prerequisites for the students are very simple. All they need to understand is simple arithmetic.

In the introduction, Chris mentioned that the US president can be elected by a very low percentage of the population. There is a reference to that work in the teacher packet, in case some of your students would like to investigate that further.

In the first break, you will be gathering the data from your class. Your class will be voting on the four candidates: candidate A, candidate B, candidate C, candidate D. Each student should rank the four candidates first, second, third, and fourth.

You can gather the data in several ways. One is to make photocopies of a ballot that is included in the teacher packet. That's one good way because all of the data will be

written then. And you can then go over it several times to make sure that your data is correct.

Another way to do it if you don't want to photocopy, or if you don't have access to a photocopier, is to have the students decide on an order for themselves, and then close their eyes and ask the class, who chose candidate A as their first candidate? Who chose candidate A as their second candidate? And so on.

You should write all of the data on a board like we did, for the class to see all the data. Also, if your data doesn't turn out to be interesting enough, for example, if one candidate actually has a majority, all three methods will return the same candidate. Then you can use an alternate data set that is available and in the teacher's packet. So we have a second set of data that you can use if yours does not turn out to be interesting enough.

The time during the second break and such might be real quick, real easy, focusing on the weaknesses of plurality. There should be a good bit of discussion between the classmates and such. The winner of the election should be fairly obvious, seen very quickly.

But then the third break should take a little bit of time. Watch the class because not everybody is going to be familiar with instant runoff type of voting and such. Just to make sure that everybody's understanding how it's calculated, where the votes are going to.

Of course, depending on how the votes come out, you might only have one round of elimination versus the two that we've shown here as part of the video. The discussion itself should be a bit longer, especially if you have two different winners now with the different methods of voting and such.

During the fourth break, you will be calculating the winner of your election using the Borda Count method. That method is the most complicated of the methods. It requires careful understanding of the method. And so you should keep a close eye on your students and make sure they understand how to add up the points involved with the Borda Count.

And of course during the fifth break and such, be expecting lots of discussion, people working together, trying to discuss about the different methods and such. Hopefully everything is going OK up to this point.

One thing that's optional for you to do after the end of the video, is you could even hold an election to decide the class's favorite method of elections and holding them. And you can try using all three different methods to see what ones come out on top. And have a little bit of fun with it at the end.

This concludes the last part of the teacher's portion of the video. We hope you've enjoyed the video. Hopefully it's helpful for you and your class, and that you have a better understanding of different methods of voting and common places that they're going to be used. So have a good day.

Thank you and goodbye.