SUBMITTING TO AUTHORITY:

AGREEING TO FOLLOW EX ANTE UNKNOWN ORDERS

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Abstract

Though contracts under which workers agree to follow ex ante unknown orders in return for constant hourly wages are very common, we do not have a satisfactory rationale for their use. We here show that such contracts can be asymptotically efficient. Players do not need to know the set of possible orders nor ever observe each other's payoffs; they just need to know the distributions of costs and values. This allows them to sustain a relational contract in the form of a quota mechanism even if costs have infinite support. The mechanism lets the players identify inefficient orders such that they can avoid implementing them. (*JEL* D01, D02, D86, J41, L14) (*Key Words:* Authority, Incomplete contracts, Employment, Quota mechanisms.)

I. INTRODUCTION

In a recent job posting MIT was looking for a salaried academic secretary to

- perform a variety of administrative tasks for MIT.nano,

- assist with conference and event planning and coordinating,

- provide complex administrative office support,

- coordinate conference rooms bookings,

- maintain and update portions of facility website content,

- act as the primary point of contact and customer success representative, and

- support existing users with administrative inquiries, registration, and onboarding activities.

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The job offered has a lot of richness combined with extreme simplicity. By taking it, you agree to follow orders without knowing what they will be and yet the hourly pay is the same no matter how onerous individual tasks are. Think about the second item: "Assist with conference and event planning and coordinating," This can be something as simple as booking hotel rooms but may also be much, much more perhaps involving travel. Beyond not knowing what the tasks will be, you also will not know how highly the boss values specific tasks just as she will not know how burdensome you find them. There is no possibility of contracting based on tasks or their values and costs. An additional complication is that some of the tasks may be so costly that they really should not be done: If you have an injury that makes it painful to carry heavy things, if your child has a doctor's appointment, if you feel unsecure about how to do something, etc.. In such cases you might ask the boss to be relieved and she may or may not grant that. In the vast majority of cases you will, however, simply follow whatever order you are given and receive the same hourly pay no matter what. One might wonder why anybody would take such a job, but we know that these contracts are extremely common.

The purpose of this paper is to explain why they are used. In the environment described above we show the existence and asymptotical efficiency of a Weak Markov Perfect Bayesian Equilibrium in which orders are given and most often followed in return for a fixed per-period wage. The efficiency result does not depend on players having any ex ante knowledge of the set of possible tasks, only that they know the probability distributions of their costs and values. The other critical assumption is that orders are issued frequently such that the inter-task discount rate is small. There is very little systematic evidence about the frequency with which orders are given, but the data we do have is quite compelling. In particular, Guest (1968) followed 56 foremen in an auto plant and found that they attended to a new issue every 48 seconds and spent an average of 26.4% of their time face to face with direct reports. This suggests that the average foreman gave a new order every three minutes and, if they had ten direct reports, that the average worker received a new order every half hour.

Model and main result

In each period one of infinitely many tasks can create value for the boss and she learns its identity at the start of that period. The other player, the worker, can perform the task though he

incurs a cost in the process. Costs and values are private information of the worker and the boss respectively and ex post unobservable. To implement all efficient tasks and eliminate those for which cost exceeds value, the players use a quota mechanism. The mechanism divides time into blocks with equal numbers of periods/tasks and a per-block quota for each player. Specifically, the worker may, in each block, make a maximum number of unverifiable claims to the effect that the current task has high costs. If such a claim is made, the boss may accept it and withdraw the order but could also make an unverifiable counterclaim to the effect that the task has high value, implying that it should be performed in spite it's the high cost. Counterclaims about a task can only be made after the worker has made a claim about that same task and the number made per block is also subject to a quota. Using their knowledge of the cost and value distributions, players can rely on the law of large numbers to assess the veracity of the claims by comparing the actual frequency of high cost (value) claims with theoretical frequencies over the number of tasks in the block.[1] Claims and counterclaims are restricted to "high" or "low" and values are either high or low, but costs are drawn from a distribution with infinite support.

The proof is complicated because the actual and theoretical frequencies differ, because discounting means that the players will be tempted to make false claims in the early part of a block, because the actual numbers of high costs and values differ between blocks, and because of the infinite number of possible costs. The first two complications vanish as blocks become longer and interest rates go to zero – hence our efficiency result is only asymptotic. The third complication is much harder to deal with since the meaning of high and low costs change over time as functions of worker strategies, the number of claims already made, and the number of periods/tasks remaining in the block. Specifically, if n periods remain in the block and the interest rate is zero, a worker who has k claims left will be tempted to make one if the cost of the current task is larger than the expected value of the k'th highest out of n draws from the cost distribution. Compared to the case in which the worker claims high iff the cost is higher than the lower of the boss's two values, this adaptive strategy makes him better off. It is not an equilibrium for the worker to use that strategy if interest rates are positive, but as they go to zero, it becomes an increasingly good approximation to the actual equilibrium strategy. The boss's

[1] The critical role played by the law of large numbers is consistent with the stylized fact that this type of contract is not used for individual tasks, for tasks that take a long time to complete, or when new tasks only are needed infrequently.

equilibrium strategy approximates honest use of counterclaims as the block length goes to infinity.

So the prevalent practice of paying fixed wages at constant intervals can be asymptotically efficient and the behavior prescribed by the equilibrium behavior, in which the worker normally follows orders but occasionally asks to be relieved, mirrors casual observation as well.

Literature

The paper contributes to the literatures on authority, incomplete contracts, quota mechanisms**,** and bilateral trade.

In spite of the widespread use of incomplete contracts with submission to authority and constant payments, we still do not have models that reflect the full complexity of order taking. The first paper on the topic is probably Simon (1951), who suggests that workers take orders in return for fixed payments when they are roughly indifferent between the tasks covered by the agreement. In the present paper, the costs can have a very large variance as long as their distribution is known. Another early contribution is Alchian and Demsetz (1972), who use an analogy to a customer buying products from a grocer. The latter situation differs from that considered here for two reasons: Each product has its own price and the set of products is finite and known ex ante. In the more recent literature several authors, e. g. Baker, Gibbons, and Murphy (2002), have like us looked at the employment relationship through the lens of relational contracting but make stronger informational assumptions, use bonuses, and often focus on moral hazard. The work closest to ours is Bolton and Dewatripont (2005, Ch. 12.5) who also allow payoffs to be ex post unobservable but assume that the boss has superior ex ante information about the worker's costs and is disciplined, by threat that the worker could quit, to pay ex post bonuses as compensation for costly tasks. All of these papers fail to explain use of constant payments. Similarly, constant payments have long been explained by risk sharing, for example by Baily (1974) and Marcus (1984), but those models do not explain submission to authority.

The study of incomplete contracts was pioneered by Grossman and Hart (1986) and Hart and Moore (1990), who assume that contracts necessarily are incomplete but allow ex post bargaining (renegotiation) such that pay can vary from task to task. Another stream, prominently associated with Williamson (1975, p. 66), assumes that agents are boundedly rational and that

contracts necessarily are incomplete. He explains the prevalence of simple contracts by bounded rationality and the possibility that more complicated contracts could be exploited by opportunistic agents.[2] In contrast to this, the mechanism in the present paper does not rely on bounded rationality and is asymptotically optimal while still being very simple. Even though the parties cannot contract on contingencies, they can solve the problem by exploiting their statistical knowledge about the distributions of costs and values.

The paper makes a small contribution to the recently growing literature on quota mechanisms. The first example was probably Townsend's (1982) analysis of a repeated risk sharing problems in which one player reports private information about their current wealth in every period.[3] More recently Jackson and Sonnenschein (2007) and Ball, Jackson, and Kattwinkel (2022) derived some general efficiency results in games in which information arrives simultaneously, while Renault, Solan, and Vieille (2013), Escobar and Toikka (2013), Frankel (2016), and Ball and Kattwinkel (forthcoming, Section 8.2) consider dynamic games such as ours. The case we consider is arguably closest to that of Escobar and Toikka (2013) except that our players move sequentially and one of them (the worker) has an infinite type space.[4] To the best of our knowledge this is the first paper on quota mechanisms in which one player has infinite type space. If only one of two players has an infinite type space, the mechanism works as if it was finite, since all that matters is the interval in which their type falls. However, the equilibrium messaging strategy of the player with an infinite type space is very different and quite interesting. As described in Lemmas 4 and 5 below, the decisions about when to send messages are solutions to a complicated dynamic optimization problem and have subtle implications for the efficiency of equilibrium play. Compared to other applications it is worth emphasizing that ours is particularly natural. Since the quotas are defined periods, there is less question about the availability and comparability of the individual elements and the high frequency with which orders are given makes it natural to focus on vanishing interest rates. It should also be mentioned

[2] Consistent with this theory, Levin and Tadelis (2010) argue that a contract on time is easier to administer than a contract on quality and use this to explain several empirical regularities.

[3] Radner (1985) is another early contribution.

[4] In our application it is clearly a realistic assumption that costs have infinite support and our treatment of that case constitutes a small theoretical contribution. The existence proof relies on the fact that the player with finite type space has a dominant strategy such that the opponent can be assumed to maximize against that. We are then able to characterize that maximum in enough detail to derive a limiting efficiency result as the discount rate goes to zero.

that our mechanism is extremely simple while much existing literature on with multidimensional types proposes very complicated mechanisms (Skrzypacz and Toikka, 2015). Finally, in contrast to much of the literature on quota mechanisms, the behavior described in the present paper is an equilibrium. It does not depend on anybody insisting that a quota mechanism is used.

The model can also be interpreted as depicting bilateral trade largely through relabeling: The tasks, the worker, and the boss could be widgets, a seller, and a buyer, respectively. It seems less natural to assume payment in non-trade situations, as is done in the present paper, but that would be a very minor change in the proofs and boundaries in the price. The mechanisms suggested in this literature are typically more complicated than that proposed here, but also more powerful. (See e. g. Skrzypacz and Toikka, 2015, and Athey and Miller, 2007).[5] Relative to this literature the main contribution here is not asymptotic efficiency, but that our mechanism is extremely simple and, in the case of labor services, realistic.

The basic model and the main results are presented in Section II, an extension to a richer value space is analyzed in Section III, and a discussion concludes the paper in Section IV.

II. MODEL AND ANALYSIS

Time $t = 1, 2, \ldots$ is discrete and infinite and nature randomly chooses one of an infinite number of tasks at the start of each period. The task can be performed during that period but not later. The boss (she) derives value from having tasks performed and the worker (he) can do it but incurs costs in the process. The value of task t is v_t where $v_t \in \{v, V\}$ and its cost is c_t where c_t is drawn from the distribution F with support $[0,V]$. Using f to denote the density of F, we define $c \equiv \int_0^v xf(x)dx/F(v)$ and $C \equiv \int_v^V xf(x)dx/[1 - F(v)]$, such that c is the mean cost if costs are below v, and C is the mean cost if costs are above v. The probability of $c_t > v$ is $1 - F(v) \equiv p$ and the probability of $v_t = v$ is q. Values and costs are independently drawn and trade is inefficient iff $c_t > v$ and $v_t = v$ such that the first best expected per-period surplus is $(v - c)(1- p)q + (V - C)p(1 - q) + (V - c)(1 - p)(1 - q) \equiv \Gamma^*$. Value and cost realizations are, even ex post, known only to the boss and worker respectively, although F, v, V, and q are common knowledge. Each player

[5] Fuchs (2007) and Angelucci and Orzach (2023) present simple relational contracts with bunching of trades but do not use quota strategies.

may make an unverifiable claim ("high" or "low") in each period. There is transferable utility, the players are risk neutral and discount future payments at the same rate $r > 0$ per period.

Before playing the game defined below, the players agree on a way to determine the wage. This can happen in each period or before period *1* and wages can be paid in each period or only in period in which the worker performs a task. In order to fit the example in the Introduction, we will look at the case in which a wage w_t is paid in every period.[6] Therefore, if *I[t]* is an indicator function equal to *1* if there is trade at t and *0* otherwise, the boss and worker aims to maximize $\sum_1^\infty (1 + r)^{-t}(I[t]v_t - w_t)dt$ and $\sum_1^\infty (1 + r)^{-t}(w_t - I[t]c_t)dt$, respectively. At each t, the worker decides which, if any, claim to make, whether to perform the task, and whether to terminate the game. The boss decides which, if any, claim to make, whether to pay the worker, and whether to terminate the game. When making these decisions, each player knows the public history (claims made, trades consummated, and payments made), as well as the history of their private information (costs for the worker and values for the boss). Participation is at-will, and if either player terminates the game, both players get zero in all future periods. Beyond possible steps related to wage determination, they play the following stage game each period t:

1. The boss learns the identity and value of the next task.
2. She orders the worker to perform it and he learns its cost.
3. The worker may or may not make a claim. If he does, he can say "high" or "low".
4. The boss may or may not make a claim. If she does, she can say "high" or "low".
5. The worker may or may not perform the task. If he does, the boss gets v_t and the worker incurs the cost c_t.
6. The boss may or may not pay the agreed-upon per-period wage w_t.

We will describe a mechanism with order taking and constant payments that asymptotically implements the first best outcome in a Weak Markov Perfect Bayesian Equilibrium. In our mechanism the players negotiate constant per period wage w before period *1*. We assume that the agreed upon wage lies in $(c[1-p] + Cp[1-q],\ v[1-p]q + V[1-q])$ such that it does not give all the expected first best surplus to one of them. Other mechanisms can also implement the first best in games with the same payoffs but do not explain order taking and constant pay. In addition, they

[6] Similar results hold, under slightly different conditions, if the wage is contingent on trade, but I feel that the stated assumption better reflects actual practice.

may have hidden costs. One possibility is that the parties engage in independent negotiations every period. It is reasonable to assume that they bargain efficiently but it is also reasonable to assume that the individual negotiations are not completely costless (because of the associated delays and other costs). These costs will be unimportant in many settings particularly if costs and values are high and periods are long. They will, however, cumulate to a non-trivial factor if individual trades are small and frequent as is often the case for trades in human asset services. In such circumstances, as when an executive needs a stream of different services from her secretary, it seems reasonable to conjecture that the surplus achievable with trade-by-trade negotiation is less than Γ^*. Alternatively, the traders could try to write a complete contingent long-term contract, but since the number of possible tasks is infinite (or very large), this cannot be done. We will, however, show that they can agree to a relational contract that exhibits the main characteristics of the employment contracts such that (1) the boss gives an order every time a new task is needed, (2) the worker is paid an ex ante agreed-upon constant wage in every period, (3) in the limit, the worker obeys the order unless trade is inefficient, and (4) the contract is at will.

The idea is to use *quota strategies* as a point of reference. These strategies are defined on blocks of tasks (or equivalently, periods) as follows: A block consists of N periods. The worker can claim that cost is "high" up to pN times per block. If the worker makes a claim, the boss can respond with the counterclaim that value is "high" up to $(1 - q)\, pN$ times per block, and if either trader exceeds their quota the parties terminate their relationship and get their outside options (which we have normalized to zero) in all future periods. So the quota strategies add a final step to the stage game:

1. The boss learns the identity and value of the next task.
2. She orders the worker to perform it and he learns its cost.
3. The worker may claim that costs are high by announcing C. If this causes him to exceed his quota, the boss ends the game.
4. If the worker makes a claim, the boss may make the counterclaim that value is high by announcing V. If this causes her to exceed her quota, the worker ends the game.

5. The worker may or may not perform the task. If he does not perform the task after announcing high costs and not getting a counterclaim, the game continues. Otherwise the boss ends it.
6. The boss pays the agreed-upon per-period wage. If not, the worker ends the game.
7. After N periods, a new block commences and the counting of claims and counterclaims restarts from zero.

With the use of such quota strategies, we aim to show the following:

Proposition: *There exists a sequence of Weak Markov Perfect Bayesian Equilibria e'(r), such that the expected per-period surplus, Γ[e'(r)], converges to the first best as* $r \to 0$.

It is helpful to start with the non-equilibrium *simple quota strategies* defined as follows:

(a) If possible, the worker claims high cost the first pN times $c_t > v$. If he, towards *the-end-of-the-block,* has k claims left when k tasks remain in the block, he ends by making k straight claims for a total of pN regardless of his actual costs in those periods. (So workers use up their quotas exactly; in some cases by making false claims and in others by not making true claims, both at the very end of a block.)

(b) If possible, the boss responds to a worker's claim by making the counterclaim that value is high whenever $v_t = V$. If she, towards the *end-of-the-block*, has m counterclaims left when the worker has m claims left, she ends by making m straight counterclaims for a total of $(1 - q)pN$ regardless of her values in those periods. (Bosses know that they will get pN chances to counterclaim and they will use up their quotas exactly; in some cases by making false counterclaims and in others by not making true counterclaims.)

(c) The worker follows the order unless he has claimed high cost and the boss has not made a counterclaim.

The proof works by first showing that the simple quota strategies asymptotically implement the first best as blocks get longer and discount rates get smaller (because the frequencies with which high cost and value occur converge to p and $1 - q$, respectively). We next characterize a pair of

equilibrium quota strategies and show that their expected per-period surplus asymptotically dominates that from the simple quota strategies, and thus converges to the first best, as discount rates get smaller and blocks get longer.

Lemma 1: *If $\Gamma(e_p, N)$ is the expected per-period surplus when both players follow the simple quota strategies with block length N, then, for every $\varepsilon > 0$ there exists $r^*(\varepsilon) > 0$ and a function $N(\varepsilon, r)$ such that $\forall\, r < r^* : \Gamma(e_p, N) + \varepsilon > \Gamma^*$*

Proof: If in each block there are exactly pN realizations where costs are greater than v and exactly $(1-q)pN$ of these where per-period values are V, then the simple quota strategies are $(v - c)(1- p)q + (V - C)p(1 – q) +(V - c)(1 – p)(1 – q) = \Gamma^*$ per-period. However, since the cost and value realizations are random, there will in many blocks be deviations from the mean and consequently some end-of-block payoff losses. As described in the previous paragraph, these come in four variants: (i) If the worker has to make false claims, some trades with expected gains $v - c$ will be missed, (ii) if the worker cannot make enough claims, some trades with expected gains $v - C$ will be made, (iii) if the boss has to make false claims, some trades with expected gains $v - C$ will be made, and (iv) if the boss cannot make enough claims, some trades with expected gains $V - C$ will be missed. As is obvious, only one of (i) and (ii) and only one of (iii) and (iv) can happen in any one block. Relative to the first best, losses are uniformly bounded (by a constant depending on F, v, V, q) and the only inefficiency comes from end-of-block forced (counter) claims. If $X \sim Bin(N, p)$ is the number of high-cost realizations in a block, the number of forced "wrong" claims is $\left| X - Np \right|$ (up to a constant factor accounting for the four cases). The expected total block loss is then bounded by $L\mathrm{E}\left| X - Np \right|$ for some finite L and the per-period loss is therefore $L\mathrm{E}\left| X - Np \right| / N$. Using $\mathrm{E}\left| X - Np \right| = \mathrm{O}(N^{1/2})$ (e. g. by Chapter VIII in Feller, 1950), the per-period inefficiency is $\mathrm{O}(N^{-1/2})$, so for any ε we can pick N large enough.

QED

Remembering that the inefficiencies are caused by histories with surprisingly many or surprisingly few high cost or high value realizations, we now proceed to show that there exists a weak Markov Perfect Bayesian Equilibrium in which expected per-period surplus converges to the first best as discount rates get smaller and blocks get longer.

Lemma 2: *The game has a Weak Markov Perfect Bayesian Equilibrium.*

Proof: Within a block, the payoff relevant state is $(N - t, k, m)$ where N-t is the number of periods left, k is the number of claims left, and m is the number of counterclaims left. Note that this is finite. Suppose that the candidate value functions are $W_b(N - t, k, m)$ for the boss and $W_w(N - t, k, m)$ for the worker. The boss, upon observing v_t and a claim, picks

$$Max\{W_b(N - t - 1, k - 1, m - 1)(1 + r)^{-1} + v_t,\ W_b(N - t - 1, k - 1, m)(1 + r)^{-1}\}.$$

The worker, upon observing c_t, picks

$$Max\{Q(N-t, k, m)W_w(N - t - 1, k - 1, m)(1 + r)^{-1} + [1 - Q(N - t, k, m)][W_w(N - t - 1, k - 1, m - 1)(1 + r)^{-1} - c_t],\ W_w(N - t - 1, k, m)(1 + r)^{-1} - c_t\},$$

where $Q(N - t, k, m)$ is the probability that the boss does not meet at claim at $(N - t, k, m)$ with a counterclaim.

Now look at the following map from the (W_b, W_w) function space onto itself:

$\Phi: (W_b, W_w) \rightarrow$ [Value at $(N, pN, (1- q)pN)$ from backward inducting the block using terminal value $[(1 + r)^{-1}W_b, (1 + r)^{-1}W_w)]$.

Because payoffs are bounded in each period and the terminal value only enters N discounting steps ask from the start, Φ is a contraction mapping with modulus $(1 + r)^{-N}$. So the Banach fixed point theorem tells us that a unique fixed point (W_b^*, W_w^*) exists.

QED

Lemma 3: *If r is sufficiently low, the simple quota strategy is the boss' best response to any equilibrium quota strategy used by the worker.*

Proof: Given a value of N, Lemma 3 holds if two things are true. First, the boss cannot refrain from making a true counterclaim before the end-of-the-block. She faces the strongest such temptation if the very first task is (high cost, V), the worker makes a claim, and the boss believes that he will react to her making a counterclaim by claiming high costs in each of the next $pN - 1$ straight tasks - the harshest punishment available to him. It is most attractive to stay quiet if she further believes that any further counterclaims will go unpunished. In that case, if she does make the counterclaim, she gets V but then faces $pN - 1$ tasks with an average of $q0 + (1 - q)V$ followed

by *(1 - p)N* realizations with average of *qv + (1 - q)V.* Let us first assume that the block has exactly *pN* high cost realizations and that *(1-q)PN* of those have high value. So there are no end-of-the-block inefficiencies except for the extra counterclaim the boss would have if she does not use it in the first period. In this case we need

$V + \sum_{t=2}^{pN}(1 + r)^{-t}[(1- q)V] + \sum_{t=pN+1}^{N}(1 + r)^{-t}[qv + (1- q)V] > 0 + \sum_{t=2}^{N}(1 + r)^{-t}[(1- p)qv + (1- q)V]$ or

$$V - \sum_{t=2}^{pN}(1 + r)^{-t}qv + \sum_{t=2}^{N}(1 + r)^{-t}pqv > 0. \qquad (1)$$

This is true if *r* is sufficiently small to ensure that getting the early $(1 + r)^{-1}V$ is worth more than the expected cost of delaying the *(1 - p)qvN. (*As $r \rightarrow 0$ this goes to $V + v(1- p)q > 0.$*)* We can, however, not abstract from the randomness that generate end-of-the-block inefficiencies. Following the argument in the proof of Lemma 1, this is bounded by $LE\left|X - Np\right|$ for some finite *L* and the per period loss is of order $O(N^{-1/2})$ such that we for any ε can pick a large enough *N* as long as *(1)* holds.

Second, the boss cannot spend a counterclaim on a *v* realization before the end-of-the-block. Doing so would give her *v* earlier, but she would lose the option to spend the counterclaim on a *V* at some later point in time. The boss would gain most by doing this if the very first task is (high cost, *v*) and the worker makes a claim. By making a counterclaim the boss makes *v* rather than *0*. The smallest loss comes if the boss is out of quota in the last period and therefore has to forego a *V* realization with probability *(1-q)* and a *v* realization with probability *q.* The value of this, at the start of the block, is $(1 + r)^{-N}[qv + (1-q)V]$ so it has to be true that $v < (1 + r)^{-N}[qv + (1-q)V]$. (As $r \rightarrow 0$ this goes to $v < V.$*)*

If *r* meets these two conditions, the simple quota strategy is optimal for the boss and the worker knows that she will play it.

QED

The worker's equilibrium strategy is quite complex but we can offer a rough characterization. Because costs are a continuous variable, he can do better than simply claim iff costs are above *v.* The reservation value above which the worker will claim depends on how many claims are left in his quota and the number of periods left in the block.

Lemma 4: *Fix a natural number $k \leq pN$. Suppose that the worker just learned the cost of a new task, that $N - t < N$ tasks remain in the block, and that he has k claims left. For every $\varepsilon > 0$ there exists $r^*(\varepsilon) > 0$ and a function $N(\varepsilon, r)$ such that $\forall\ r < r^*$: The worker's reservation value for using one of those claims is within ε of the expected cost of the k'th most costly task among the N-t yet to be revealed.*

Proof: There are two problems with using the expected value of the *k*'th most costly among the *n* tasks yet to be revealed. First, depending on the number of counterclaims yet to be made, not all claims will have the same chance of being met by a counterclaim. The problem is that the boss, at the end-of-the-block, may have run out of counterclaims or may have "too many" left such that she will counter every claim even if her value is *v*. However, we can use the same argument as in the proof of Lemma 1: If $X \sim Bin(N, p[1-q])$ is the number of high-cost and high-value realizations in a block, the number of forced "wrong" counterclaims is $\left| X - Np(1-q) \right|$ (up to a constant factor accounting for the two cases). The expected total block loss is then bounded by $L\mathrm{E}\left| X - Np(1-q) \right|$ for some finite L and the per-period loss is therefore $L\mathrm{E}\left| X - Np(1-q) \right| /N$. Using $\mathrm{E}\left| X - Np(1-q) \right| = \mathrm{O}(N^{1/2})$ (e. g. by Chapter VIII in Feller, 1950), the per-period inefficiency is $\mathrm{O}(N^{-1/2})$, so for any ε, a sufficiently large N exists.

Second, since $r > 0$, the future costs should be discounted such that the current nominal reservation value should be lower. This effect will vanish as $r \rightarrow 0$.

QED

Before getting to Lemma 5, it helps to look at an example in which we use the standard result (from the theory of order statistics[7]) that the mean of the *k*'th highest element in a sample of size *n* from $U[0, 1]$ is $(n + 1 - k)/(n + 1)$.

Example. Suppose that $F = U[0, 1]$, that $p = 1/3$, and that the boss has no counterclaims remaining $(m=0)$. The worker has one claim left $(k = 1)$, has just observed c_t, and is now thinking about whether to use the claim. If one further task remains in the block beyond that he will make a claim now if his costs are above $(1 + r)^{-1}/2$ - the expected cost of the single task still to be revealed. If two further tasks remain, the expected maximum cost of the last two tasks and thus the current reservation value goes to $1 - p$ (= 2/3) as $r \rightarrow 0$, and if three tasks remain it goes to

[7] Se e.g. David and Nagaraja, 2004.

3/4. So in this example, the reservation value adjusts downwards (such that more claims are made) if the number of remaining claims divided by the number of remaining tasks *(k/N-t)* is larger than *p*. It adjusts upwards (such that fewer claims are made) if that same ratio is below *p*. So if "too many" claims have been made the optimal strategy is to slow the speed of claiming down, and if "too few" claims have been made, the optimal strategy call for more claims to be made. The pattern from this example, which generalizes, lowers the expected end-of-the-block losses.

Lemma 5: *Compared to that resulting from the simple quota strategy, the equilibrium distribution of claim times differs in two ways: The worker's expected end-of-the-block payoff losses are smaller and claims are made earlier in the sense of first order stochastic dominance.*

Proof: The simple quota strategy requires that the worker, except for end-of-the-block effects, make claims iff his costs exceed *v*. In the example, he can do better by making claim decisions as functions of the number of remaining periods, claims, and tasks. Equilibrium reservation values change over time as the worker reacts to the evolving history of high-cost tasks. If he has made fewer (more) claims than expected, he lowers (raises) the expected reservation cost below (above) *v* and thereby increases (slows) the expected speed, leading to smaller expected end-of-block payoff losses. Additionally, the worker prefers to get his breaks earlier and equilibrium reservation values will be lower because $r > 0$, causing claims to be made earlier in the sense of first order stochastic dominance.

To see this more generally, consider the binary choices in the proof of Lemma 2. The worker's reservation value c_t^* is given by

$$Q(N - t, k, m)W_w(N - t - 1, k - 1, m)(1 + r)^{-1} + [1 - Q(N - t, k, m)][W_w(N - t - 1, k - 1, m - 1)(1 + r)^{-1} - c_t^*] = W_w(N - t - 1, k, m)(1 + r)^{-1} - c_t^*,$$ or

$$W_w(N - t - 1, k, m)(1 + r)^{-1} - Q(N - t, k, m)W_w(N - t - 1, k - 1, m)(1 + r)^{-1} - [1 - Q(N - t, k, m)]W_w(N - t - 1, k - 1, m - 1)(1 + r)^{-1} = Q(N - t, k, m)c_t^*.$$

By Lemma 3, the probability that the boss does not counter a claim (Q), is independent of the state except for end-of-the block considerations. It is clear that $W_w(\,)$ is increasing in k and decreasing in m. However, $W_w(N - t - 1, k, m)$ is bigger than $W_w(N - t - 1, k - 1, m - 1)$ because a countered claim makes the ratio of unused claim to counterclaims larger and thus increases the

value of each remaining claim. So the left side is positive. The key observation is that there are decreasing returns to k such that $W_w(N - t - 1, k, m) - W_w(N - t - 1, k - 1, m) < W_w(N - t - 1, k - 1, m) - W_w(N - t - 1, k - 2, m)$. The reason is that higher numbers of claims are more likely to be wasted or devalued by end-of-the-block consideration. This means that the reservation value is a decreasing function of the number of claims left. The effect of discounting follows from the fact that a larger r gives a lower reservation value in the equation above.

QED

We are now ready to show our main result:

Proposition: *There exists a sequence of Weak Markov Perfect Bayesian Equilibria e'(r), such that the expected per-period surplus, Γ[e'(r)], converges to the first best as* $r \rightarrow 0$.

Proof: We know from Lemma 3 that the boss plays the simple quota strategy and Lemma 5 tells us that the worker adjusts his reservation values to reduce his end-of-the-block losses but also skews them earlier. The fact that the worker adjusts the frequency of claims to reduce end-of-the-block losses has the same beneficial effect on the boss because it means that her opportunities to counterclaim are adjusted as well. She will lose the values of $p(1 - q)N$ tasks either way, but she is less likely to find herself with too many counterclaims at the end and therefore be forced to use some of them on low value realizations. On the other hand the shift to earlier claims hurts her though this effect vanishes as $r \rightarrow 0$. So for sufficiently small r, the boss is better off than if the worker played the simple quota strategy. By revealed preference, we also know that the worker is better off. So for small r, both players are better off in equilibrium than if both played the simple quota strategy. Because Lemma 1 tells us that the surplus in the latter case converges to the first best and the players cannot do better than that, the equilibrium surplus, which is in between the two, will also converge to Γ^* as $r \rightarrow 0$. Since the wage is set between expected costs and expected values, both players gain from continuing the relationship.

QED

III. TRINOMIAL VALUE DISTRIBUTION

The assumed binomial value distribution is obviously unrealistic and we will now illustrate how it can be generalized by analyzing the trinomial case.

The result continues to hold but the messages need to be richer and the blocks longer. In this model, values $v_t \in \{v, V, \mathbf{V}\}$, where $v < V < \mathbf{V}$, and costs $c_t \in [0, \mathbf{V}]$. The probabilities of v, V, and $\mathbf{V}$ are q, Q, and $1 - q - Q$, respectively, and costs are above V with probability P, between v and V with probability p, and below v with probability $1 - p - P$. We will look at quota strategies according to which the worker sends the claim C if $c_t \in [v, V]$ and $\mathbf{C}$ if $c_t \in (V, \mathbf{V}]$, while the boss's counterclaim is V if $v_t = V$ and $\mathbf{V}$ if $v_t = \mathbf{V}$. In this case the worker can claim that costs are above V a total of PN times per block and that they are between v and V a total of pN times per block. The boss can counterclaim that value is $\mathbf{V}$ and V a total of $P(1 - Q - q)N$ and pQN times, respectively. Compared to the binomial case a smaller fraction of tasks on the average falls in each pair of realizations, such that per-cell convergence will be slower and a larger block size is needed.[8]

The simple quota strategies are generalizations of those in the binomial case:

(d) If possible, the worker claims intermediate cost the first pN times $c_t \in [v, V]$ and high costs the first PN times $c_t \in (V, \mathbf{V}]$. If he, towards *the-end-of-the-block,* has $k + K$ claims left when $k + K$ tasks remain in the block, he ends by making $k + K$ straight claims for a total of $(p + P)N$ regardless of his actual costs in those periods. (So workers use up their quotas exactly; in some cases by making false claims and in others by not making true claims, both at the very end of a block.)

(e) If possible, the boss responds to a worker's claim by making the counterclaim that value is intermediate whenever $v_t = V$ and that it is high when $v_t = \mathbf{V}$. If she, towards the *end-of-the-block*, has $m + M$ counterclaims left when the worker has $m + M$ claims left, she ends by making $m + M$ straight counterclaims for a total of $[Qp + (1 - q - Q)P]N$ regardless of her values in those periods. (Bosses know that they will get

[8] The worker's messages still need to specify intervals if the value function has continuous support, but the length of those intervals introduces a loss of surplus in addition to that caused by randomness.

pN and *PN* chances to counterclaim and they will use up their quotas exactly; in some cases by making false counterclaims and in others by not making true counterclaims.)

(f) The worker follows the order unless he has made a claim that was not met by a counterclaim

We will now state the analogs of Lemmas 1-5 as well as the Proposition from Section II. In each case we will indicate how the proof changes from the binomial to the trinomial case, but since the changes are quite small, we will not present complete proofs.

Lemma 1-3: *If $\Gamma(e_{p,} N)$ is the expected per-period surplus when both players follow the simple quota strategies with block length N, then, for every $\varepsilon > 0$ there exists $r^*(\varepsilon) > 0$ and a function $N(\varepsilon, r)$ such that $\forall\ r < r^* : \Gamma(e_{p,} N) + \varepsilon > \Gamma^*$.*

Proof: Compared to the proof of Lemma 1, there are twice as many types of end-of-the-block inefficiencies in this case: The worker may be forced to make two kinds of false claims or he may be unable to make two kinds of true claims. Similarly, the boss may be forced to make two kinds of false counterclaims or she may be unable to make two kinds of true claims. However, all these kinds of per period inefficiency are $O(N^{1/2})$ so we can make them arbitrarily small by selecting a sufficiently large *N*.

QED

Lemma 2-3: *The game has a weak Markov Perfect Bayesian Equilibrium.*

Proof: Compared to the proof of Lemma 2, the value functions have two more arguments, but the domains are still finite and the choices are still binary. The Banach fixed point theorem still applies.

QED

Lemma 3-3: *If r is sufficiently low, the simple quota strategy is the boss' best response to any equilibrium quota strategy used by the worker*.

Proof: The punishment involves claims of high costs and the worker can still claim intermediate costs in the post punishment phase. As a result, the conditions on *r* are more complicated but it is still true that the boss will prefer to follow the simple quota strategy in period *1* if *r* is sufficiently small.

QED

Lemma 4-3: *Fix two natural numbers $k \leq pN$ and $K \leq PN$. Suppose that the worker just learned the cost of a new task, that $N - t < N$ tasks remain in the block, and that he has k and K claims left. For every* for every *$\varepsilon > 0$ there exists $r^*(\varepsilon) > 0$ and a function $N(\varepsilon, r)$ such that $\forall\ r < r^*$: The worker's reservation value for using one of those claims is within ε of the expected cost of the k'th (K'th) most costly task among the $N - t$ yet to be revealed.*

Proof: The proof proceeds in exactly the same way as that in Section II except for the fact that it covers two types of claims.

QED

Lemma 5-3: *Compared to that resulting from the simple quota strategy, the equilibrium distribution of claim times differs in two ways: The worker's expected end-of-the-block payoff losses are smaller and claims are made earlier in the sense of first order stochastic dominance.*

Proof: Also this proof proceeds as that in Section II except for the fact that conditions cover twice as many cases.

QED

Proposition 1-3: *There exists a sequence of Weak Markov Perfect Bayesian Equilibria e'(r), such that the expected per-period surplus, $\Gamma[e'(r)]$, converges to the first best as $r \rightarrow 0$.*

Proof: The exact same argument as in the proof of Proposition 1 applies.

QED

IV. DISCUSSION

We have explained why employees agree to follow ex ante unknown orders in return for a fixed wage. The underlying contracting problem is very complex and yet admits an extremely simple, asymptotically efficient equilibrium. Beyond a fixed wage and submission to orders, the mechanism shares several other properties with typical employment contracts: Trade has to be repeated and frequently so, the contract is open ended, and too much complaining, too many hard

tasks, or refusal to follow an order will lead one of the parties to terminate the contract. As such, the paper could be seen as identifying some conditions under which employees are, and are not, used. More speculatively, if one is willing to define "firms" based in their contracts with labor, rather than capital, the result contributes to the theory of the firm.[9] From the perspective of contract theory, the paper illustrates how an inability to contract on contingencies may be overcome by quota strategies that use statistical knowledge about the distributions of costs and values. The paper also makes a minor contribution to our knowledge of quota mechanisms because one of the players has an infinite type space. Compared to many other quota mechanisms ours is unusually simple and appears as an equilibrium rather than being unilaterally imposed.

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