

**1.010 - Fall 1999**  
**Homework Set #8**  
 Due November 23, 1999

1. Consider the simple model shown below of a flexible structure resisting on two footings. The foundation engineer's concern is in the relative deflection of the ends of the structure.

$$\Delta = \delta_1 - \delta_2 = Qf_1 - Qf_2$$

Assume that  $Q$  is a deterministic load and the flexibilities (reciprocal of stiffness)  $f_1$  and  $f_2$  of the two soil and footing systems are random variables with common mean  $m$ , variance  $\sigma^2$  and covariance  $\rho\sigma^2$  ( $\rho$  is the correlation coefficient, hence a measure of the lateral homogeneity of the soil). Find:

- (a) The expected value of  $\Delta$
- (b) The standard deviation of  $\Delta$

How do you intuitively explain the dependence of the standard deviation on  $\rho$ ? Would you consider more plausible to have positive or negative correlations? Is this in favor of safety (relative to differential settlement) ?



2. The modulus of elasticity  $E$  of a concrete elements produced at a factory is known to have a mean value of 3600 ksi and a coefficient of variation  $v=0.15$ .

In order to reduce uncertainty, the elements are tested with a device that makes measurements of  $E$  with “noise” i.e. the device gives

$$Z = E + \epsilon$$

Where  $\epsilon$  is a random variable independent of  $E$ .  $\epsilon$  has mean value zero and standard deviation  $\sigma_\epsilon = \sigma_E/3$ . Find an expression for the mean value of  $E$  following a measurement  $Z=z$ . What is the conditional standard deviation of  $[E|Z=z]$  ? Compare with the unconditional value

3. Read *Application Example 9.2* and do *Problem 9.2.1*.