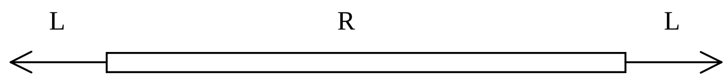
1.010 - Fall 1999
Homework Set #9
Due December 3, 1999

1. <u>Reliability problem</u>. Consider a bar in tension.



The load L and the resistance R are independent random variables with the following characteristics:

$$m_R = 8 \text{ t} \qquad m_L = 8 \text{ t}$$
$$\sigma_R = 1 \text{ t} \qquad \sigma_L = 2 \text{ t}$$

Find the probability that the bar fails,
(a) if L and R have normal distribution;
(b) if L and R have lognormal distribution.

Note: if Y has lognormal distribution, then X = ln Y has normal distribution with parameters

$$m_X = 2\ln(m_Y) - \frac{1}{2}\ln(\sigma_Y^2 + m_Y^2)$$
$$\sigma_X^2 = -2\ln(m_Y) + \ln(\sigma_Y^2 + m_Y^2)$$

Hint: the failure condition may be expressed as either $R - L < 0$ or $\frac{R}{L} < 1$.

2. A contractor estimates that the expected time for the completion of job A is 30 days. Because of the uncertainties that exist in the labor market, materials supply, bad weather coditions and so on, she is not sure that she will finish the job in exactly 30 days. However she is 90% confident that the job will be completed within 40 days. Let X denote the number of days required to complete job A.

   (a) Assume X to be a Gaussian random variable; determine m and σ and also the probability that X will be less than 50.
   (b) Recall that a Gaussian random variable ranges from –inf to +inf. Thus X may take on negative values that are physically impossible. Determine the probability of such an occurrence. Based on this result, is the assumption of the normal distribution for X reasonable?
   (c) Let us now assume that X has a log-normal distribution with the same expected value and variance as those in the normal distribution of part (a). Determine the parameters m and σ of ln(X) and also the probability that X will be less than 50. Compare this with the result from part (a).

3. The Cantilever beam shown below is subjected to a random concentrated load P and a random distributed load W. Assume

   $P \sim N(5\text{ K}, 1\text{ K}^2)$
   $W \sim N(1\text{ K/ft}, 0.2\text{ (K/ft)}^2)$

   a) Determine the mean and variance of the applied bending moment, $M_a=50W+10P$. Assume that loads are correlated as $\rho_{PW}=0.5$
   b) The resisting moment of the beam M_r is statistically independent of the applied moment M_a. Assume that both M_a and M_r are normally distributed and that $M_r \sim N(200\text{ K-ft}, 2500\text{ K-ft}^2)$. Determine the probability of failure of the beam, $P(M_r<M_a)$.