1.010 Fall 1999
Practice Problems (SM, FOSM)

Problem 1 (differential settlement) Consider the simply supported beam in Figure 1.

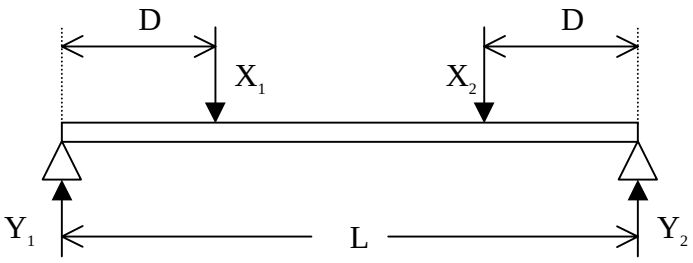

Figure 1. Simply supported beam

The beam has span L. Two loads, X_1 and X_2, are applied at distance $D \le L/2$ from the supports. Under these conditions, the vertical forces applied to the supports are

$$Y_1 = \frac{D}{L}X_1 + \left(1 - \frac{D}{L}\right)X_2$$
$$Y_2 = \left(1 - \frac{D}{L}\right)X_1 + \frac{D}{L}X_2$$

The differential settlement between the supports may be assumed proportional to the difference $(Y_1 - Y_2)$ between the loads. Suppose that the loads X_1 and X_2 are independent and identically distributed, with the same mean value m and variance σ^2. Find the mean value and variance of the load difference $(Y_1 - Y_2)$. Is differential settlement more of a problem for small or large values of D? Why?

Problem 2 (FOSM analysis) If X has normal distribution with mean value m_X and variance σ_X^2, then $Y = e^X$ has lognormal distribution with mean $m_Y = e^{m_X + 0.5\sigma_X^2}$ and

variance $\sigma_Y^2 = m_Y^2\left(e^{\sigma_X^2} - 1\right)$. This result is exact. Calculate in approximation the mean and variance of Y from the mean and variance of X using FOSM analysis. Compare the FOSM mean, variance and coefficient of variation of Y with the exact quantities. Under what conditions are the FOSM results accurate?

Problem 3 (second moment characteristics of vectors) Y_1 and Y_2 are indicator variables for the occurrence/non-occurrence of events (for example, Y_1 and Y_2 might refer to congested or free-flowing traffic conditions in two highway segments). The joint distribution of Y_1 and Y_2 is shown in Figure 2.

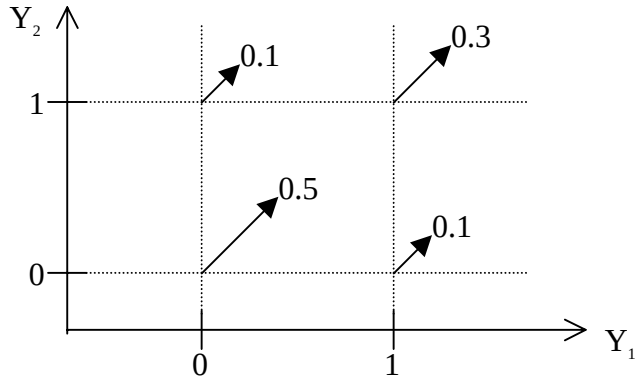

Figure 2: Joint probability mass function of Y_1 and Y_2

Find: (a) the mean value and variance of Y_1 (these are also the mean and variance of Y_2);

(b) the covariance between Y_1 and Y_2. Give a qualitative explanation for the sign (positive or negative) of the covariance.