10.213 Chemical Engineering Thermodynamics
Spring 2000
Test 2 Solution
March 29, 2000

Problem 1 (45 points)

In a process that produces a liquefied stream of ethylene at 15 bar, a low pressure gas at 1 bar and 25 °C (1) is first compressed to 15 bar and outputs at 220 °C (2). This gas is cooled to 50 °C at constant pressure (3) before being compressed to 100 bar (4). This stream exits at 240 °C and compressed to 100 bar (4). This stream exits at 240 °C and is then cooled to a temperature below 100 °C (5). The high pressure, cooled gas proceeds through a throttle valve to produce a stream at 15 bar that is 30 % liquid ethylene (6). The vapor stream is recycled into the process.

a) *Draw the process on the enthalpy-pressure diagram for ethylene using (1-6) for the various states. Your process should start with ethylene at 1 bar and 25 °C and end with liquid and vapor streams at 15 bar. (15 points)*

b) *For the compression of ethylene from 1 bar and 25 °C to 15 bar and 220 °C, what is the efficiency of the compressor? (10 points)*

compressor efficiency = $(\Delta H)_S/\Delta H$ (comparison between isentropic and real process)

for real process, $\Delta H = H(P = 15 \text{ bar}, T = 220\ °C) – H(P = 1 \text{ bar}, T = 25\ °C)$

$\Delta H = 250$ kcal/kg - 160 kcal/kg = 90 kcal/kg

for isentropic case, initial condition: P = 1 bar, T = 220 °C and from diagram

H = 160 kcal/kg and S = 1.45 kcal/kg-K

for isentropic case, final condition, P = 15 bar, S = 1.45 kcal/kg-K and from diagram

H = 225 kcal/kg and T = 175 °C

Thus, $(\Delta H)_S = H(P = 15 \text{ bar}, S = 1.45 \text{ kcal/kg-K}) – H(P = 1 \text{ bar}, S = 1.45 \text{ kcal/kg-K})$

$(\Delta H)_S = 225$ kcal/kg - 160 kcal/kg = 65 kcal/kg

compressor efficiency = $(\Delta H)_S/\Delta H$ = (65 kcal/kg)/(90 kcal/kg) = 0.72

c) *How much cooling (kcal/kg) is required before the gas enters the throttle valve? (10 points)*

Cooling is $(H_5 – H_4)$

H_4 is at P = 100 bar and T = 240 °C; $H_4 = 250$ kcal/kg from diagram

H_5 is at P = 100 bar and T not known, but $H_5 = H_6$ as $\Delta H = 0$ for 5->6 (throttle valve)

H_6 is at P = 15 bar and x = 0.7; $H_6 = 105$ kcal/kg from diagram

Thus, cooling = $(H_5 – H_4)$ = 105 kcal/kg – 250 kcal/kg = -145 kcal/kg

d) What are the temperatures of the gas as it i) enters (with 5 °C) and ii) exits (within 2 °C) the throttle valve? (10 points)

At 5 (entering throttle), P = 100 bar and $H_5 = H_6 = 105$ kcal/kg. Thus, T at 5 is 35 °C

At 6 (exiting throttle), P = 15 bar and L+V, from table, T at P=15 bar = 235 K or –38 °C.

Problem 2 (25 points)

A steam power plant operates on the Rankine cycle. In this process, 145 lb_m/s of liquid water at 950 psia is heated by a combusted fuel to 1000 °F, requiring an input of 192,800 BTUs. This superheated gas enters a turbine ($\eta = 0.78$) at 950 psia and exits at 14.7 psia.

a) *Determine the work produced by the turbine in BTUs. (20 points)*

Entering turbine is 145 lb_m/s water at P = 950 psia and T = 1000 °C.

From steam table, $H_{in} = 1507.0$ BTU/lb_m and $S_{in} = 1.6595$ BTU/lb_m-°R

Exiting turbine is 145 lb_m/s water at P = 14.7 psia

Actual work by the turbine will be the efficiency times the isentropic work that could be produced for this expansion = 0.78 x $(\Delta H)_S$

Need to determine exiting condition if isentropic.

Output = P = 14.7 psia and $S_{out} = S_{in} = 1.6595$ BTU/lb_m-°R

For P = 14.7 psia, $S^{lig} = 0.3121$ BTU/lb_m-°R and $S^{vapor} = 1.7568$ BTU/lb_m-°R

Thus, output is a mixture of liquid + vapor

$S_{out} = (1 - x)\, S^{liq} + x\, S^{vapor}$

$1.6595 = (1 - x)(0.3121) + x(1.7568)$ leads to

$x = 0.9327$

For x = 0.933 & P = 14.7 psia, $H_{out} = (1 - x)\, H^{liq} + x\, H^{vapor}$

$H_{out} = (1 - 0.9327)(180.17) + (0.9327)(1150.5)$ BTU/lb_m

$= 12.1$ BTU lbm $+ 1073.1$ BTU/lbm $= 1085.2$ BTU/lb_m

Work produced by turbine = $(\eta_{turbine})(\Delta H)_S = (0.78)(1085.2 - 1507.0) = -329$ BTU/lb_m

Work produced by turbine in BTUs = (329 BTU/lb_m) (145 lb_m/s) = 47,700 BTU/s

b) *If the work required by the pump is 3.9 BTU/lb_m, determine the overall efficiency of the power plant. Note: if you could not solve part a), you may assume that the work generated by the turbine was 400 BTU/lb_m. (10 points)*

Overall efficiency = (desired output)/(required input) = (net output work)/(required heat input)

Net output work = work by turbine – work by pump

= 47,700 BTU/s - (145 lb_m/s)(3/9 BTU/lb_m)

= 47,700 BTU/s - 565.5 BTU/s = 47,135 BTU/s

Required heat input = 192,800 BTU/s from problem statement

Overall efficiency = (net output work)/(required heat input)

= (47,135 BTU/s)/(192,800 BTU/s) = 0.244

If a work term of 400 BTU/lb_m is used for the turbine, the overall efficiency = 0.298

Problem 3 (30 points)

Propane gas undergoes a change in state from an initial condition of 4.25 bar and 25 °C to 34 bar and 100 °C. Using generalized correlations, determine the value of DS for this change of state. The molar heat capacity of propane in the ideal state is given by

C_p^{ig} *(cal/mol-K) = 5.49 + 0.0424 T (°K).*

Three step process to get ΔS for real gas: transition gas first from real to ideal state, determine ΔS for an ideal gas between the two ideal state, and transition gas from ideal to real state

$\Delta S = -S^R_1 + \Delta S^{ig} + S^R_2$

S^R_1 and S^R_2 are determined using generalized correlations

For propane, P_c = 42.48 bar, T_c = 369.8 K, and ω = 0.152

For P_1 = 4.25 bar and T_1 = 25 °C = 298 K,

$P_{r,1}$ = (4.25 bar)/(42.48 bar) = 0.1 and $T_{r,1}$ = (298 K)/(369.8 K) = 0.8

From the generalized correlations charts for $P_{r,1}$ = 0.1 and $T_{r,1}$ = 0.8,

$S^R{}_1{}^0/R = -0.13$ and $S^R{}_1{}^1/R = -0.25$

Thus, $S^R{}_1/R = S^R{}_1{}^0/R + (\omega)S^R{}_1{}^1/R = (-0.13) + (0.152)(-0.25) = -0.17$

For P_2 = 34 bar and T_2 = 100 °C = 373 K,

$P_{r,2}$ = (34 bar)/(42.48 bar) = 0.8 and $T_{r,2}$ = (373 K)/(369.8 K) = 1.0

From the generalized correlations charts for $P_{r,2}$ = 0.8 and $T_{r,2}$ = 1.0,

$S^R{}_2{}^0/R = -0.85$ and $S^R{}_2{}^1/R = -0.95$

Thus, $S^R{}_2/R = S^R{}_2{}^0/R + (\omega)S^R{}_2{}^1/R = (-0.85) + (0.152)(-0.95) = -0.99$

$$\Delta S^{ig} = \int_{T_1}^{T_2} \frac{C_p^{ig}}{T} dT - R \ln \frac{P_2}{P_1} \quad \text{where } C_p^{ig} = 5.49 + 0.0424\,T$$

$$\Delta S^{ig} = \int_{298K}^{373K} \left[\frac{5.49}{T} + 0.0424\right] dT - R \ln \frac{34\text{ bar}}{4.25\text{ bar}} = 5.49 \ln \frac{373}{298} + 0.0424(373 - 298) - R \ln 8$$

$$\Delta S^{ig} = [(5.49)(0.2245) + 0.0424(75) - (2.079)(1.987)]\text{ cal/mol - K}$$

$$\Delta S^{ig} = [1.23 + 3.18 - 4.13]\text{ cal/mol - K} = 0.28\text{ cal/mol - K}$$

$\Delta S = -S^R_1 + \Delta S^{ig} + S^R_2$

= -(-0.17 R) + (0.28 cal/mol-K) + (-0.99 R)

= (0.17-0.99)(1.987 cal/mol-K) + 0.28 cal/mol-K = -1.35 cal/mol-K or –5.64 cal/mol-K

Note: for comparison, the NIST website give ΔS for this change as –5.38 cal/mol-K

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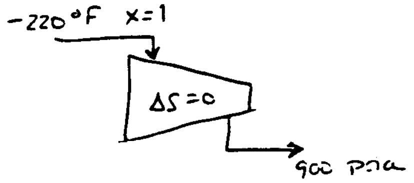
Exam II, Solutions

1) (a)



$$\dot{m}\,\Delta H = \dot{W} + \underset{0}{\dot{Q}}$$

$-220\ ^\circ F = T_1$
$5.0 = P_1$
$106 \simeq H_1$
$0.625 \simeq S_1$

$\longrightarrow$

$P_2 = 900$ psia

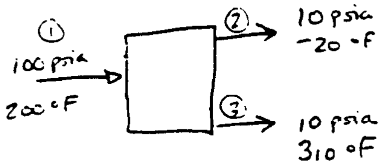
$S_2 = S_1 = 0.625$

$T_2 \simeq 150\ ^\circ F$

$\therefore\ H_2 = 150\ \frac{Btu}{lb}$

$$\frac{\dot{W}}{\dot{m}} = \Delta H, \qquad W = \Delta H = (150 - 106)\ \frac{Btu}{lb} = 44\ \frac{Btu}{lb}$$

(b)



$$\dot{m}\,\Delta H = \underset{0}{\dot{W}} + \underset{0}{\dot{Q}}$$

Just flow in & out
no work or heat in/out.

$H_1 \simeq 167 \qquad H_2 \simeq 130 \qquad H_3 \simeq 190$

$$-\dot{m}_1(167) + \dot{m}_2(130) + \dot{m}_3(190) = 0 \qquad [1^{st}\ Law]$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \qquad [\text{Flow Mass Balance}]$$

set basis $\dot{m}_1 = 1$ lb/min

$$\therefore\ \dot{m}_2 = 0.383\ \frac{lb}{min}, \qquad \dot{m}_3 = 0.617\ \frac{lb}{min}$$

$S_1 \simeq 0.70 \qquad S_2 \simeq 0.68 \qquad S_3 \simeq 0.78$

$$\dot{m}\,\Delta S = -1\ \tfrac{lb}{min}\,(0.70) + 0.383\ \tfrac{lb}{min}\,(0.68) + 0.617\ \tfrac{lb}{min}\,(0.78) = 0.0417\ \frac{Btu}{^\circ F\ min}$$

$$\Delta\dot{S} = 0.0417\ \frac{Btu}{^\circ F\cdot min} \quad \text{or} \quad \Delta S = 0.0417\ \frac{Btu}{^\circ F\cdot lb_m}$$

2a) $\eta_{turb} = \dfrac{\Delta H}{(\Delta H)_S}$ ← ΔH for process as is
← ΔH for $\Delta S = 0$

for numerator: Steam at $T_1 = 225°C$ → $P_2' = 50\,kPa$ sat'd

$P_1 = 300\,kPa$ → $T_2' = 81.35°C$

$H_1 = 2916.9\,kJ/kg$ $H_2' = 2646.0\,kJ/kg$

$S_1 = 7.4177\,kJ/kg \cdot K$ $S_2' = 7.5947\,kJ/kg\,K$

$$\Delta H = H_2 - H_1 = -270.9\ kJ/kg$$

if $\Delta S = 0$, final condition would be $P_2 = 50\,kPa$

$S_2 = 7.4177\,kJ/kg\text{-}K$

value of S_2 is below that for sat'd vapor

$$S_2 = x^V S^V + (1 - x^V) S^L$$

$$7.4177 = x^V (7.5947) + (1 - x^V)(1.0912)$$

$$x^V = 0.9728$$

For steam with $x^V = 0.9728$ need H_2

$$H_2 = x^V H^V + (1 - x^V) H^L$$
$$= (0.9728)(2646.0) + (0.0272)(340.564)$$
$$= 2583.3\ kJ/kg$$

Thus $(\Delta H)_S = 2583.3 - 2916.9 = -333.6\ kJ/kg$

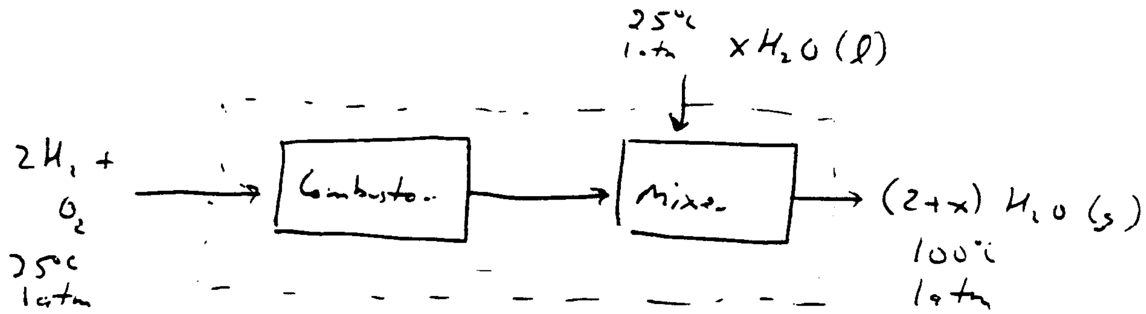
$$\eta = \frac{\Delta H}{(\Delta H)_S} = \frac{-270.9}{-333.6} = 0.81$$

For throttle, $\Delta H = 0$ so that $H_2 = 2916.9$ which is greater than H for sat'd steam at 50 kPa

→ superheated

As $\Delta H = 0$, $\eta = 0$ (no useful energy obtained)

2b



Consider 2 moles H_2 as basis;

For system; $\Delta H = 0$

heat produced by reaction must warm all water to final condition.

$$\Delta H_{rxn} = 2\Delta H_f^\circ(H_2O, l) - 2H_f^\circ(H_2) - H_f^\circ(O_2)$$
$$= 2(-285{,}830\ \text{J/mol}) - 2(0) - 0 = -571{,}660\ \text{J}$$

this calculation assumes an output of 25°C liquid H_2O (like our feed).

What is ΔH to take 25°C liq H_2O → 100° steam?

from tables: H 104.8 kJ/kg, H 2676.0 kJ/kg

$$\Delta H = 2676.0 - 104.8 = 2571.2\ \text{kJ/kg}$$

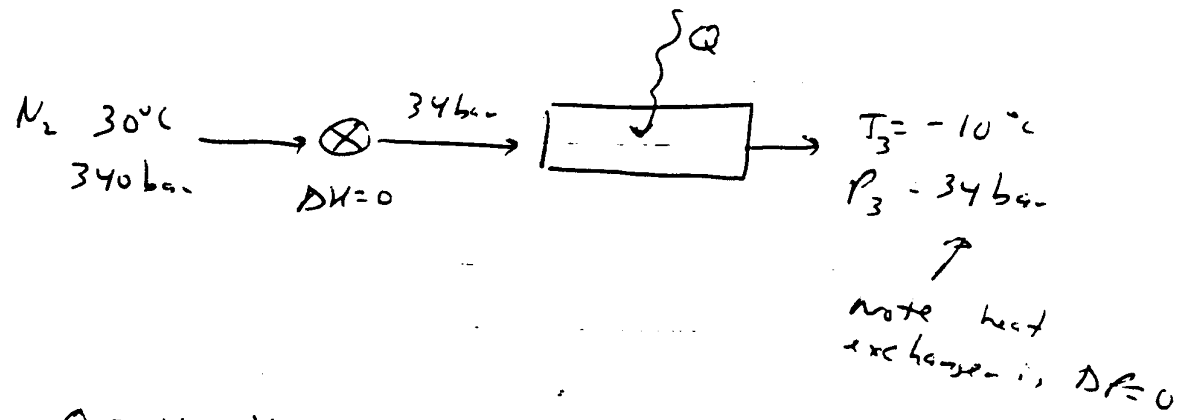
thus for 2 moles of water [illegible] produced by combustion, the −571.66 kJ will heat

$$(2+x) = \frac{+571.66\ \text{kJ}}{2571.2\ \text{kJ/kg}} = \frac{0.222\ \text{kg}}{18\ \text{g/mol}} = 12.35\ \text{moles}$$

~~be vaporized~~

x = 10.35 moles of H_2O per 2 moles of H_2

3a)



$Q = H_3 - H_2$

as ΔH for throttle = 0, $H_2 - H_1 = 0$

thus $Q = H_3 - H_2$

$= \Delta H^{is}_{1\to3} - H_1^R + H_3^R$

$C_p(T_3 - T_1)$

$3.5R(-10°C - 30°C)$

$-140R$

$T_c = 126.2\,K \Rightarrow T_{r_1} = 2.4,\ T_{r_3} = 2.1$

$P_c = 33.9$ bar $\Rightarrow P_{r_1} = 10,\ P_{r_3} = 1$

$\omega = 0.040$

~~For ①~~

	$\frac{H_R^0}{RT_c}$	$\frac{H_R^1}{RT_c}$	$\frac{S_R^0}{R}$	$\frac{S_R^1}{R}$
For ①	-1.06	0.65	-0.5	-0.25
For ③	-0.25	0.1	-0.1	-0.03

$\frac{H_R}{RT_c}_{①} = -1.034$ $\quad \frac{H_R}{RT_c}_{③} = -0.246$

$\Delta H^R = 826.8$ J/mol

$\Delta H = -140(8.314\ J/mol\ K) + 826.8\ J/mol = \overset{Q}{-337.16}$ kJ/kg

for $\Delta S = \Delta S^{is} - S_1^R + S_3^R$

$= C_p \ln\frac{263}{303} - R\ln\left(\frac{34}{340}\right) + \Delta S^R$

$= -20$ J/mol K