

16.61 - Aerospace Dynamics Spring 2006

Homework 1

Version: 1.0

Prof. Charles P. Coleman

Date Out: Friday 10 February 2006

Date Due: Friday 17 February 2006 2pm

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	

16.61 - Aerospace Dynamics Spring 2006

Homework 1

Version: 1.0

Prof. Charles P. Coleman

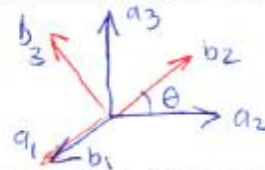
Date Out: Friday 10 February 2006

Date Due: Friday 17 February 2006 2pm

Problem 1

Two right-handed orthogonal frames $A : \{a_1, a_2, a_3\}$ and $B : \{b_1, b_2, b_3\}$ have collocated origins and are initially aligned with each other. The B frame is rotated through an angle θ about the a_1 axis of frame A . Show that the resulting transformation matrix R_A^B , which transforms vectors from the B frame to the A frame, is given by:

$$R_A^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

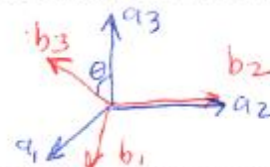


References: [1] Spong & Vidyasagar, Robot Dynamics and Control, Wiley, 1989. [2] Mortenson, Geometric Transformations, Industrial Press, 1995.

Problem 2

Two right-handed orthogonal frames $A : \{a_1, a_2, a_3\}$ and $B : \{b_1, b_2, b_3\}$ have collocated origins and are initially aligned with each other. The B frame is rotated through an angle θ about the a_2 axis of frame A . Show that the resulting transformation matrix R_A^B , which transforms vectors from the B frame to the A frame, is given by:

$$R_A^B = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$



References: [1] Spong & Vidyasagar, Robot Dynamics and Control, Wiley, 1989. [2] Mortenson, Geometric Transformations, Industrial Press, 1995.

Problem 3

Given the following proper rotation matrix

$$R = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Show by direct calculation that

$$\dot{R}(\theta) = S(\omega)R(\theta)$$

where $\omega = [0, 0, \dot{\theta}]^T$

Problem 4

Given a matrix A , and a scalar t , define the exponential of the matrix At as

$$e^{At} = I + At + \frac{t^2}{2!}A^2 + \frac{t^3}{3!}A^3 + \dots$$

Let $\theta(t) = \omega t$. Show by direct calculation that

$$R(\theta)(t) = e^{S(\omega)t}$$

where

$$R(\theta)(t) = \begin{pmatrix} \cos \omega t & \sin \omega t & 0 \\ -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$S(\omega)t = \begin{pmatrix} 0 & -\omega t & 0 \\ \omega t & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Problem 5

Consider the pendulum in the figure on the next page in which the hinged support is allowed to move with a vertical displacement $y(t)$ along a frictionless vertical line and is acted on by and external for u . The A frame is fixed in inertial space. The B frame is rotating with respect to (w.r.t.) the A frame, and its instantaneous orientation with respect to the A frame is given by θ . The location of the origin of the B frame expressed in the A frame is given by $r^{B/A} = -ya_1$. In the B frame, the location of the pendulum mass m is given by $r_B = lb_1$.

We want to derive the equations of motion of the pendulum. To do this we need to know the inertial accelerations and inertial forces that are acting on the pendulum mass m . We can algorithmically derive the equations of motion of this pendulum in several steps:

Part I: Kinematics of Rotating and Translating Frames

Since the B is rotating and translating w.r.t. the A frame we need to use the kinematic relationship we developed in class to calculate the inertial acceleration of the pendulum mass \ddot{r}_A

1. SETUP the pendulum problem for kinematic analysis using the quantities defined above r_B, θ and any other you need to define.
2. WRITE DOWN the kinematic equation needed to calculate the inertial acceleration \ddot{r}_A .
3. CALCULATE the quantities that you need to evaluate the kinematic equation.
4. PLUG 'N CHUG! to arrive at an expression for the inertial acceleration on mass m kinematically caused by rotation and translation of the moving frame B

Part II: Newton's (Inertial) Dynamics

We can now use Newton's dynamical equation to calculate the equations of motion. Newton's dynamical equation relates the inertial forces to the inertial accelerations that produced the observed dynamic (time) behavior.

$$F_A = \frac{d}{dt}(mv_A)$$

5. DRAW a free-body diagram and use Newton's dynamical equation to bookkeep the forces and accelerations in the pendulum problem.
6. VERIFY that the resulting equations of motion for the pendulum are:

$$a_1 : m\ddot{y} + ml\ddot{\theta} \sin \theta + ml\dot{\theta}^2 \cos \theta = u - mg$$

$$b_2 : ml\ddot{\theta} + m\ddot{y} \sin \theta + mg \sin \theta = 0$$

Reference: [1] Wie, Space Vehicle Dynamics and Control, AIAA, 1998.

