16.61 - Aerospace Dynamics Spring 2006
Homework 3

Version: 1.0
Prof. Charles P. Coleman

Date Out: Friday 7 April 2006
Date Due: Friday 14 April 2006 5pm

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	

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Problem 1

As shown in Figure 1, a mass m is attached to one end of a light rod of length l. The other end of the rod is pivoted so that the rod can swing in a plane. The pivot rotates in the same plane at angular velocity ω in a circle of radius R. Assume that gravity g acts in the negative y direction. Use Lagrangian analysis to find the equations of motion of the mass.
Hint: Take θ as the generalized coordinate. Find the position of the mass (x, y) as a function of θ. Then calculate the kinetic energy T and potential energy V of the mass.

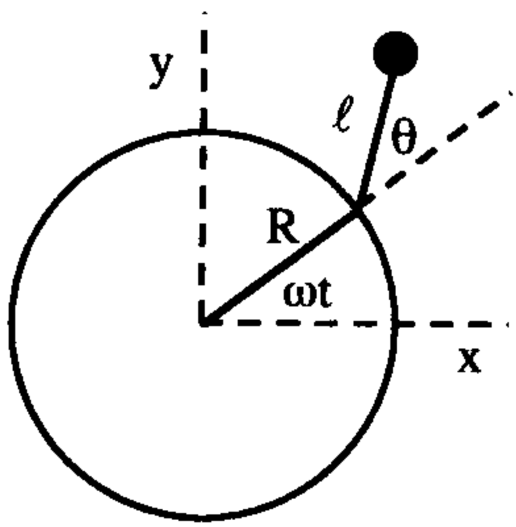

Figure 1: Pivoted Pendulum

References:
[1] Calkin, Lagrangian and Hamiltonian Mechanics, Chapter III, Problem 3, p 55, World Scientific, 1996.
[2] Calkin, Lagrangian and Hamiltonian Mechanics: Solutions to the Exercise, Chapter III, Exercise 3.03, pp 53-54, World Scientific, 1999.
[3] Greenwood, Principles of Dynamics, Chapter 6, Problem 6-6, p 276-77, Prentice-Hall, 1965.
[4] 16.61 OCW, Assignment #6, Problem 3, 2003.

Problem 2

As show in Figure!2, a spring pendulum is attached to a rotating shaft by an arm of length d. The spring stiffness is k, the pendulum mass is m, and gravity acts in the minus Z direction. The rest length of the pendulum is $r_0 = L$. The shaft is rotating with constant angular velocity Ω. Let the generalized coordinates q be $q = (\theta, r)$, where θ is the angle the pendulum makes with the vertical, and r is the distance from the pendulum pivot to the location of the mass. Use Lagrangian techniques to find the equations of motion.

References:

[1] 16.61 OCW, Assignment #3, Problem 1, 2003.

[2] 16.61 OCW, Assignment #4, Problem 1, 2003.

[3] 16.61 OCW, Assignment #8, Problem 1, 2003.

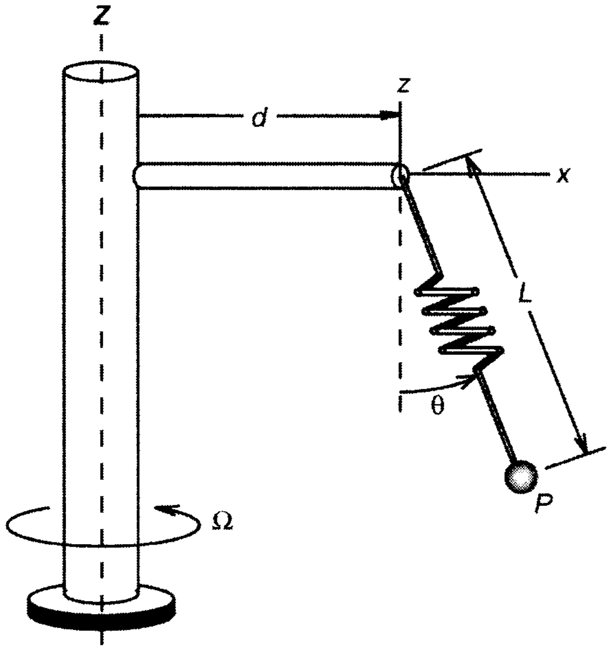

Figure 2: Swinging Pendulum

Problem 3

A sphere of mass M and radius R rolls without slipping down a triangular block of mass m that is free to move on a frictionless horizontal surface, as shown in Figure 3. The center of the sphere is initially located a distance ξ_0 from the top of the block, and the center of the sphere is initially at a height H above the horizontal surface. The sphere's moment of inertia is I. Find the Lagrangian and Lagrange's equations of motion for this system.

Hint: Take (x, θ) as the generalized coordinates. Find the position of the center of the sphere (x, y)as a function of x, ξ_0, R, θ and H. Then calculate the kinetic energy T and potential energy V of the sphere (translational AND rotational!) and the block.

References:

[1] Lim, Problems and Solutions on Mechanics, Part II, Section 1, Problem 2022 (UC Berkeley Physics PhD Exam Question), p 506-508, World Scientific, 1994.

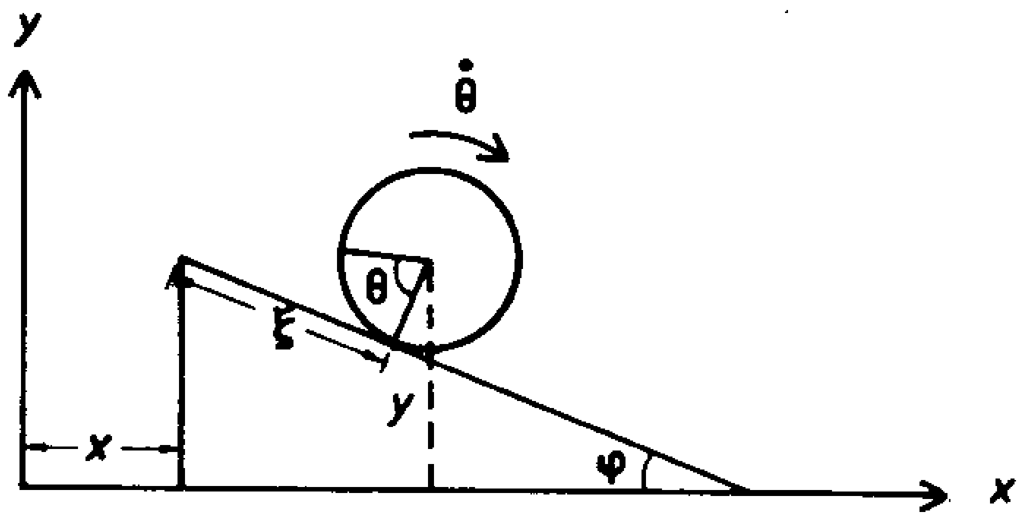

Figure 3: Sphere on Wedge